Student Name: ______________________________

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2

2007 Trial Examination

Reading Time: 15 minutes
Writing Time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, an approved graphics calculator or CAS (memory does not have to be cleared) and, if desired, a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 25 pages.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks are **not** deducted for incorrect answers.
If more than 1 answer is completed for any question, no mark will be given.
Take the **acceleration due to gravity**, to have magnitude g m/s^2, where g = 9.8

Question 1

The equation $x^2 + y^2 - 6x - 7 = 0$ is

A. a circle with radius 7 and centre (-3, 0)
B. an ellipse with centre (0 , 0) and axes 6 and 7
C. a circle with radius 16 and centre (3, 0)
D. an ellipse with centre (3, 0) and axes $\sqrt{6}$ and $\sqrt{7}$
E. a circle with radius 4 and centre (3 , 0)

Question 2

The graph of the function with rule $f(x) = \dfrac{-1}{(x+a)^2 + b}$ has

A. a minimum at $(-a, -b)$
B. a maximum at $\left(-\frac{1}{a}, -\frac{1}{b}\right)$
C. a minimum at $\left(-a, -\frac{1}{b}\right)$
D. a maximum at $\left(-a, -\frac{1}{b}\right)$
E. a minimum at $\left(\frac{1}{a}, -\frac{1}{b}\right)$

SECTION 1- continued

Question 3
$P(z)$ is a polynomial in C of degree 5 with real coefficients.
Which one of the following statements **must** be **false?**

A. $P(z)=0$ has one real root and 4 non real roots
B. $P(z)=0$ has 2 real roots and 3 non real roots
C. $P(z)=0$ has 3 real roots and 2 non real roots
D. $P(z)=0$ has 5 real roots
E. $P(z)=0$ has one repeated real root and 3 other real roots

Question 4
The value of $\cos\left(2\cos^{-1}\left(\frac{1}{4}\right)\right)$ is

A. $-\frac{7}{8}$
B. $\frac{1}{2}$
C. $\frac{7}{8}$
D. $-\frac{1}{2}$
E. $\frac{1}{16}$

Question 5
If $z=-\sqrt{3}-i$ then $Arg\left(z^3\right)$ is

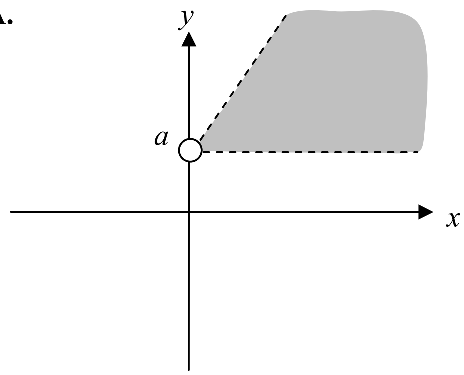
A. $-\frac{5\pi}{2}$
B. $-\frac{\pi}{2}$
C. $\frac{\pi}{2}$
D. $\frac{5\pi}{2}$
E. $-\frac{5\pi}{6}$

SECTION 1- continued
TURN OVER

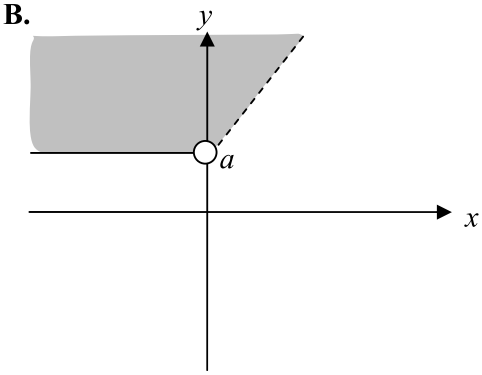
Question 6

The region defined by $Arg(z - ai) < \frac{\pi}{3}$, where a is a positive real constant can be represented by

A.



B.



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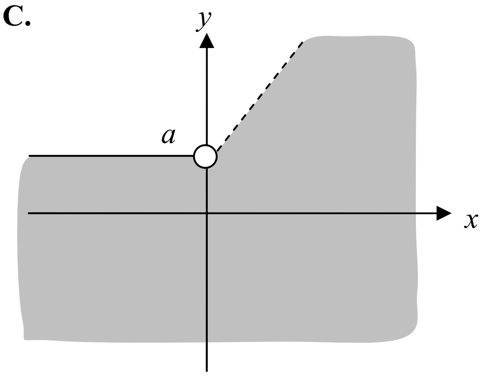
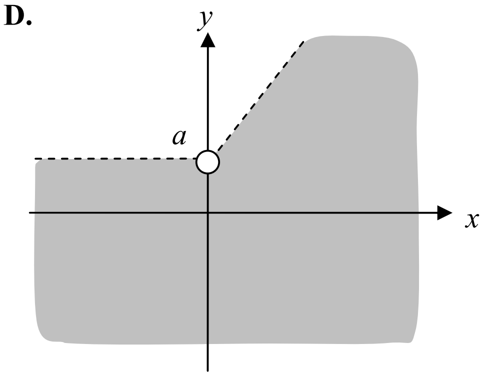


D.



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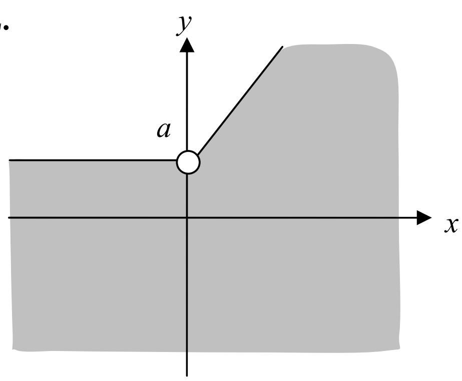


SECTION 1- continued

Question 7
The volume of a cube with side x is increasing at a rate of $6m^3/s$. The rate at which the side length is increasing when the side is 40cm is

A. $\frac{1}{8}m/s$

B. $\frac{72}{25}m/s$

C. $\frac{3}{8}m/s$

D. $\frac{25}{2}m/s$

E. $\frac{2}{25}m/s$

Question 8
The gradient of the tangent to the curve given by $\log_e y + xy = 2$ at (2,1) is

A. $-\frac{1}{2}$

B. $\frac{1}{3}$

C. 0

D. $\frac{2}{3}$

E. $-\frac{1}{3}$

SECTION 1- continued
TURN OVER

Question 9

Using a suitable substitution, $\int_0^{\frac{\pi}{6}} \sin^5(2x)\cos(2x)\,dx$ can be expressed as:

A. $\frac{1}{2}\int_0^{\frac{1}{2}} u^5\,du$

B. $-\frac{1}{2}\int_0^{\frac{1}{2}} u^5\,du$

C. $\frac{1}{2}\int_0^{\frac{\sqrt{3}}{2}} u^5\,du$

D. $2\int_0^{\frac{\sqrt{3}}{2}} u^5\,du$

E. $2\int_0^{\frac{1}{2}} u^5\,du$

Question 10

$\int \frac{4}{2+x^2}\,dx$ is equal to

A. $2\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}+c$

B. $2\sqrt{2}\tan^{-1}x+c$

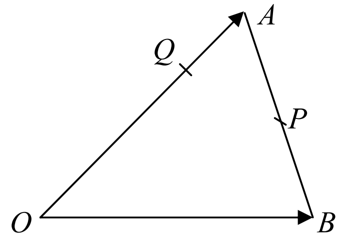
C. $4\log_e\left(2+x^2\right)+c$

D. $\frac{2}{x}\log_e\left(2+x^2\right)+c$

E. $8x\log_e\left(2+x^2\right)+c$

SECTION 1- continued

Question 11



In the triangle shown, P is the midpoint of AB and Q is a point on OA such that $OQ = \frac{3}{4}OA$.

If $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$, then $\overrightarrow{PQ} =$

A. $\frac{5}{4}\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{b}$

B. $\frac{1}{2}\underset{\sim}{b} - \frac{3}{4}\underset{\sim}{a}$

C. $\frac{1}{4}\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{b}$

D. $\frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

E. $\frac{1}{2}\underset{\sim}{b} - \frac{1}{4}\underset{\sim}{a}$

Question 12

A can of soft drink at 3°C is brought into a room at 28°C. After it has been in the room for t minutes, the temperature of the can of drink is T°C. The rate at which the can heats up is proportional to the excess of room temperature over the temperature of the can. If k is a positive constant, the differential equation involving T and t is

A. $\frac{dT}{dt} = kT - 28 \qquad T(0) = 3$

B. $\frac{dT}{dt} = -k(T - 3) \qquad T(0) = 28$

C. $\frac{dT}{dt} = -k(T + 28) \qquad T(0) = 3$

D. $\frac{dT}{dt} = -k(T - 28) \qquad T(0) = 3$

E. $\frac{dT}{dt} = k(T - 28) \qquad T(0) = 3$

SECTION 1- continued
TURN OVER

Question 13

A particle moves in a straight line with velocity v given by $v = e^{4x}$ when at a displacement x from the origin O. The acceleration of the particle is given by

A. $4e^{16x}$

B. $4e^{4x}$

C. $4e^{8x}$

D. $\frac{1}{4}e^{4x}$

E. $4e^{7x}$

Question 14

Particles at P and Q have position vectors $\underset{\sim}{r} = t\underset{\sim}{i} + (2t^2 - 8t + 9)\underset{\sim}{j}$ and $\underset{\sim}{s} = (8-t)\underset{\sim}{i} + (1+2t)\underset{\sim}{j}$ respectively at time t seconds, $t \geq 0$.

A. P and Q will be in the same position at the point (1, 3)

B. P and Q will be in the same position at the point (4, 9)

C. P and Q will be in the same position at the point (7, 3)

D. P and Q will be in the same position at the point (8, 1)

E. P and Q will never be in the same position

Question 15

The scalar resolute of the vector $-3\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ in the direction of $2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ is

A. $-\frac{1}{3}$

B. $\frac{1}{3}$

C. $-\frac{1}{9}$

D. $-\frac{1}{\sqrt{14}}$

E. $\frac{1}{\sqrt{14}}$

Question 16

The vectors $3\underset{\sim}{j} + \underset{\sim}{k}$ and $-\underset{\sim}{j} + m\underset{\sim}{k}$ are linearly dependent if

A. $m = 0$

B. $m = -3$

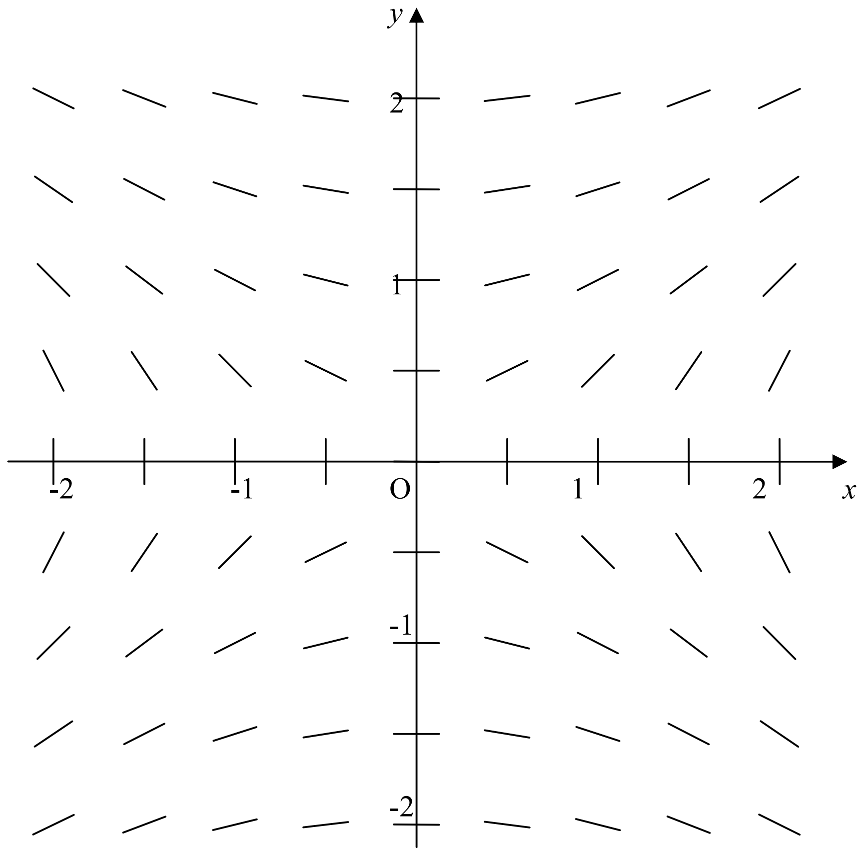
C. $m = 3$

D. $m = \frac{1}{3}$

E. $m = -\frac{1}{3}$

SECTION 1- continued

Question 17



The direction (slope) field for a certain first order differential equation is shown above.
The differential equation could be

A. $\frac{dy}{dx} = y^2 - \frac{x^2}{2}$

B. $\frac{dy}{dx} = \frac{y^2}{2} - x^2$

C. $\frac{dy}{dx} = -\frac{x}{2y}$

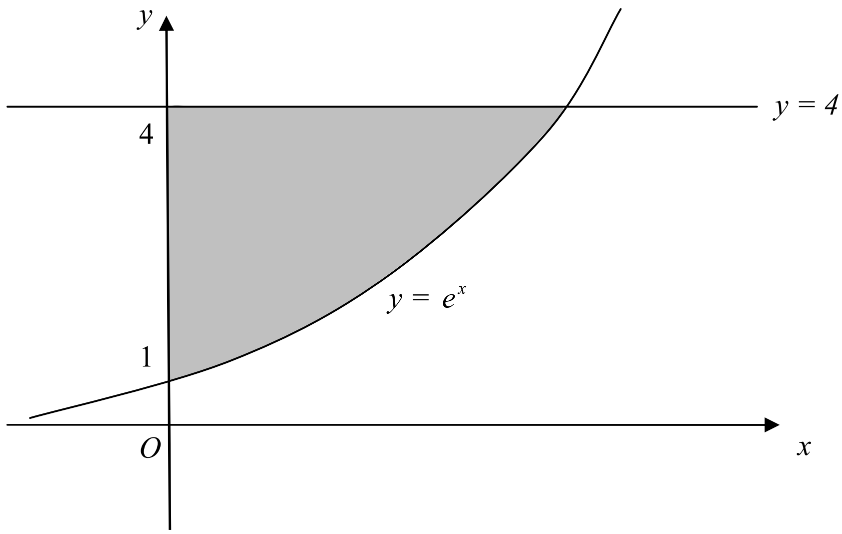
D. $\frac{dy}{dx} = \frac{x}{2y}$

E. $\frac{dy}{dx} = \frac{y}{2x}$

SECTION 1- continued
TURN OVER

Question 18

The shaded region of the diagram between the curve $y = e^x$ and $y = 4$ is rotated about the x axis.



The volume of the solid formed is given by

A. $\pi \int_0^4 (4 - e^x)dx$

B. $\pi \int_0^{\log_e 4} (4 - e^x)dx$

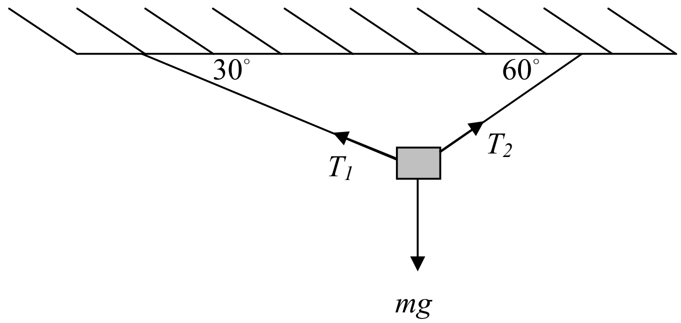
C. $\pi \int_0^4 (16 - e^{2x})dx$

D. $\pi \int_0^{\log_e 4} (16 - e^{2x})dx$

E. $\pi \int_0^{\log_e 4} (4 - e^x)^2 dx$

SECTION 1- continued

Question 19



A particle of mass m hangs from the ceiling on two strings. The forces acting on the particle are shown on the diagram. The tension force T_1 will have the value

A. $2mg$

B. $\frac{1}{2}mg$

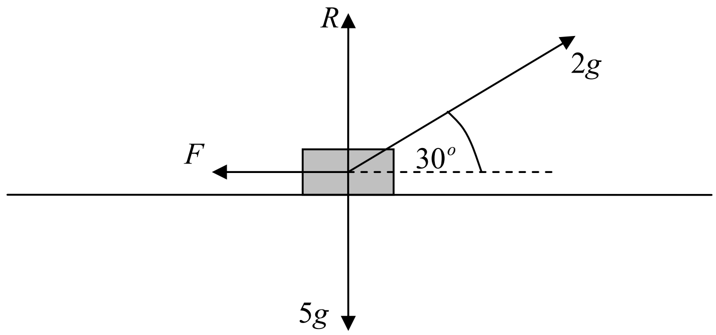
C. $\sqrt{3}\,mg$

D. $2\sqrt{3}\,mg$

E. $\frac{\sqrt{3}}{2}mg$

SECTION 1 –continued
TURN OVER

Question 20
A block of mass 5 kg lies on a rough horizontal surface. A force of $2g$ Newton is applied to it at an angle of 30° to the horizontal.



For equilibrium to be maintained, the coefficient of friction between the block and the surface must be

A. at least $\dfrac{\sqrt{3}}{4}$

B. less than $\dfrac{\sqrt{3}}{4}$

C. at least $\dfrac{\sqrt{3}}{5}$

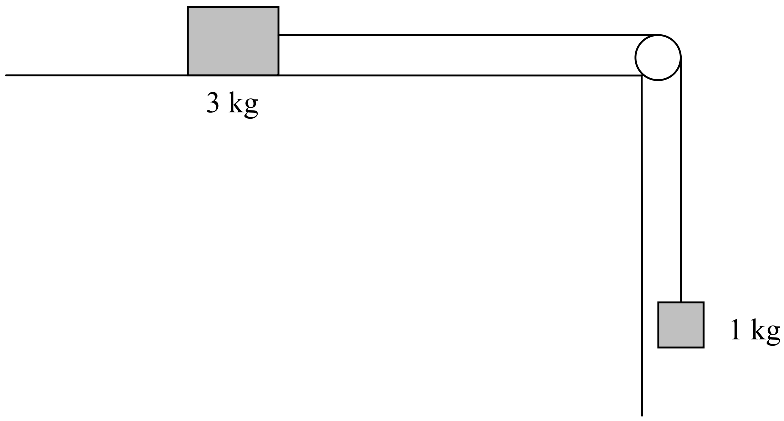
D. less than $\dfrac{\sqrt{3}}{5}$

E. at least $\dfrac{1}{5-\sqrt{3}}$

Question 21
A man of mass 70 kg is descending on a parachute which experiences air resistance of 546 Newton. The acceleration of the man downward is
A. -2 m/s^2
B. 2 m/s^2
C. 9.8 m/s^2
D. 7.8 m/s^2
E. 6.8 m/s^2

SECTION 1 –continued

Question 22
A 3 kg mass rests on a smooth horizontal table. It is connected by a light string passing over a smooth pulley to a 1 kg mass hanging freely. The system is released from rest. The acceleration of the system in m/s^2 is



A. $\frac{g}{3}$
B. $4g$
C. g
D. $\frac{3g}{4}$
E. $\frac{g}{4}$

END OF SECTION 1
TURN OVER

SECTION 2

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if the question specifically asks for an **exact** answer.
Questions worth more than one mark, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams are **not** drawn to scale.
Take the **acceleration due to gravity**, to have magnitude g m/s^2 , where $g = 9.8$

Question 1

a.

i. Express $u = 4cis\dfrac{5\pi}{6}$ in Cartesian form.

ii. Express $w = 1 - i$ in polar form.

iii. Find uw in both polar and Cartesian form.

SECTION 2- Question 1- continued

iv. *Hence*, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$ in the simplest form with a rational denominator.

1 + 1 + 2 + 2 = 6 marks

b. Solve the equation $z^4 = -1 - i$. Give the solutions in polar form.

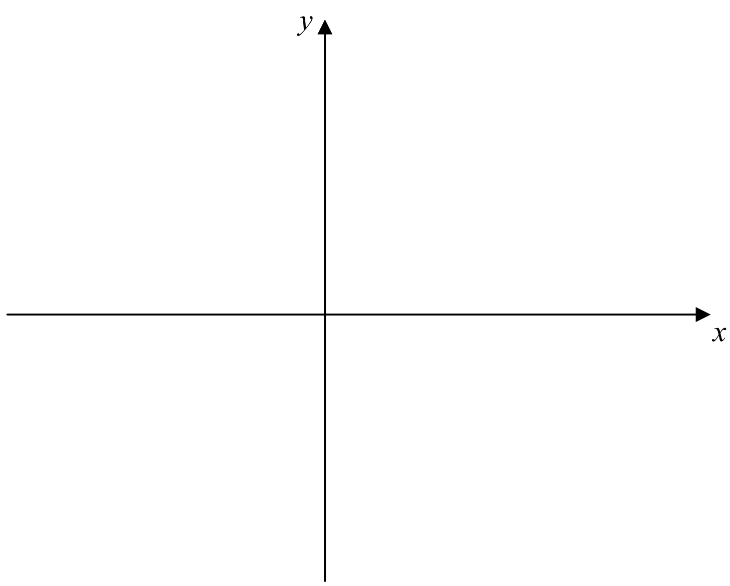
3 marks

SECTION 2- Question 1- continued
TURN OVER

c. One solution of the equation $z^3 + 4z^2 + z - 26 = 0$ is $-3 + 2i$. Find the other two solutions.

2 marks

d. Sketch in the complex plane $|z - 1 + i| = |z + 3 + i|$



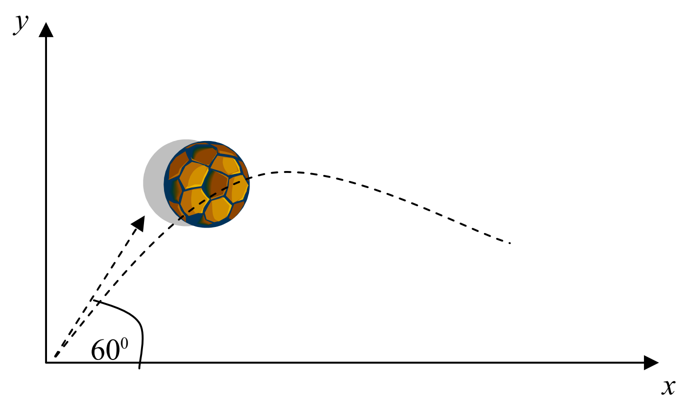
1 mark
Total 12 marks

SECTION 2- continued

Question 2

In this question give all answers accurate to ***two*** *decimal places.*

A football is placed on the ground. It is then kicked, so that it has an initial speed of 10 *m/s* at an angle of 60^0 to the ground. We assume that the mass of the ball and the air resistance are negligible. The acceleration vector of the ball is $\ddot{\underset{\sim}{r}} = -g\underset{\sim}{j}$.



a. Show that the velocity vector of the ball is $\dot{\underset{\sim}{r}} = 5\underset{\sim}{i} + (5\sqrt{3} - gt)\underset{\sim}{j}$.

2 marks

b. Find the position vector of the ball after *t* seconds.

1 mark

SECTION 2- Question 2- continued
TURN OVER

c. Show that the maximum height reached by the ball is 3.83 metres.

3 marks

d. After the ball has reached its maximum height, it hits the bar of a goal at a height of 2.44 metres.

i. How long does it take for the ball to hit the goal?

ii. Find the horizontal distance of the goal from the point where the ball was kicked.

3 + 1 = 4 marks

Total 10 marks

SECTION 2- continued

Question 3

A function $y = f(x)$ is defined by $f(x) = -3\cos^{-1}\left(\frac{x-1}{2}\right)$.

a. State the implied domain and the range of $f(x)$.

2 marks

b. Find the exact value of x when $f(x) = -\frac{\pi}{2}$.

2 marks

c. Show that $\frac{dy}{dx} = \frac{3}{\sqrt{3-x^2+2x}}$.

2 marks

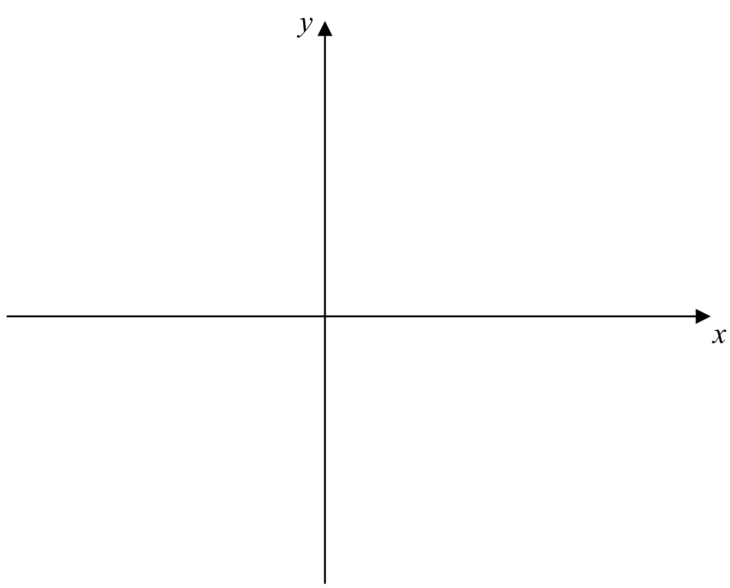
SECTION 2- Question 3-continued
TURN OVER

d. Find the point of inflexion of $f(x) = -3\cos^{-1}\left(\frac{x-1}{2}\right)$ by using second derivative.

2 marks

e. Sketch the graph of $y = -3\cos^{-1}\left(\frac{x-1}{2}\right)$.

Clearly label the axes intercepts and inflexion point.



1 mark

f. Find the area bounded by the x-axis and the curve over its implied domain.

1 mark

Total 10 marks

SECTION 2- continued

Question 4

A body of mass *m kg* moves in a straight line under the action of two forces. One of the forces accelerates the body and has a magnitude of $\frac{2m}{v}$ newtons whilst the second is a retarding force of magnitude $2mv$ newtons where v m/s is the velocity of the body at time t seconds. The body starts moving from the origin with a velocity of 0.5 *m/s.*

a. Show that $v = \frac{1}{2}\sqrt{4 - 3e^{-4t}}$.

5 marks

SECTION 2- Question 4-continued
TURN OVER

b.

i. How long will it take for the body to reach a velocity of $\frac{3}{4}$ *m/s*? Leave your answer in **exact** form.

ii. What is the body's terminal velocity?

2 + 1 = 3 marks

c. Find the displacement of the body as a function of its velocity, *v*.

5 marks

SECTION 2- Question 4 – continued

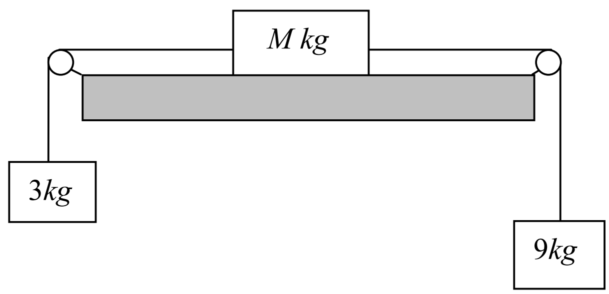
d. How far from the origin, to the nearest 0.01 metre, is the body when the velocity is $\frac{3}{4}$ *m/s*?

1 mark
Total 14 marks

SECTION 2- continued
TURN OVER

Question 5

a. A body of mass M *kg* rests on a smooth horizontal table. Two bodies of mass 3 *kg* and 9 *kg*, hanging freely, are attached to the first body by strings which pass over smooth pulleys at the edge of the table as shown in the diagram below. When the system is released from rest, it accelerates at 1.5 m/s^2.



i. On the diagram above, show all forces acting on these three bodies.

ii. Write the equations of motion and hence find the mass M and the tensions in the strings.

1 + 3 = 4 marks

SECTION 2- Question 5 – continued

b. A particle of mass m kg rests on a rough plane of inclination θ in limiting equilibrium.

i. Draw a diagram showing the forces acting on the particle.

ii. Show that the coefficient of friction is $\mu = \tan\theta$.

iii. If the inclination of the plane is increased to φ, show that the acceleration down the plane is $a = g\dfrac{\sin(\varphi - \theta)}{\cos\theta}$.

iv. Find the magnitude of angle φ in terms of θ so that $a = g$.

1 + 2 + 3 + 2 = 8 marks
Total 12 marks

END OF QUESTION AND ANSWER BOOK