Year 2007

VCE

Specialist Mathematics

Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING PO BOX 2227 KEW VIC 3101 AUSTRALIA	TEL: (03) 9817 5374 FAX: (03) 9817 4334 chemas@chemas.com www.chemas.com

Victorian Certificate of Education 2007

STUDENT NUMBER

Figures									Letter
Words									

SPECIALIST MATHEMATICS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 31 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the acceleration due to gravity to have magnitude g m/s^2, where $g = 9.8$

Question 1

Consider the graph of the function with the rule $f(x) = \dfrac{1}{x^2 + 2bx + 9}$ over its maximal domain, where b is a real number.
Which one of the following statements is **false?**

A. If $|b| < 3$ then the graph has no vertical asymptotes.

B. If $|b| > 3$ then the graph has two vertical asymptotes.

C. The x-axis is a horizontal asymptote.

D. The graph has a minimum turning point at $\left(-b, \dfrac{1}{9-b^2}\right)$

E. The graph crosses the y-axis at $y = \dfrac{1}{9}$

Question 2

If the hyperbola with the equation $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ has asymptotes given by $y = 3x + 11$ and $y = -3x - 7$ and has a vertex at the point $(-1, 2)$, then

A. $h = 3$, $k = -2$, $a = 2$, and $b = 6$

B. $h = 3$, $k = -2$, $a = 1$, and $b = 3$

C. $h = -3$, $k = 2$, $a = 1$, and $b = 3$

D. $h = -3$, $k = 2$, $a = 2$, and $b = 6$

E. $h = -3$, $k = 2$, $a = 1$, and $b = 9$

Question 3

If $\operatorname{cosec}(x) = \frac{\sqrt{5}}{2}$, $\frac{\pi}{2} < x < \pi$, then $\cot(2x)$ is equal to

A. $\frac{4}{3}$

B. $-\frac{4}{3}$

C. $\frac{3}{4}$

D. $-\frac{3}{4}$

E. $-\frac{\sqrt{5}}{2}$

Question 4

The position vector of a particle at a time $t \geq 0$ is given by $\underset{\sim}{r}(t) = \cos^2(2t)\underset{\sim}{i} + \cos(4t)\underset{\sim}{j}$.

The path of the particle is

A. a parabola.

B. an ellipse.

C. a hyperbola.

D. a circle.

E. a straight line.

Question 5

Given that $a, b \in R$ and that the graph of $y = \frac{ax^4 + b}{x^2}$ has two turning points, then

A. $a = -b$

B. $a > 0$ and $b < 0$

C. $a > 0$ and $b > 0$

D. $a < 0$ and $b > 0$

E. $a \neq 0$ and $b > a$

Question 6

$\sin(\theta)+i\cos(\theta)$ is equal to

A. $\text{sic}(\theta)$

B. $i\ \text{cis}(\theta)$

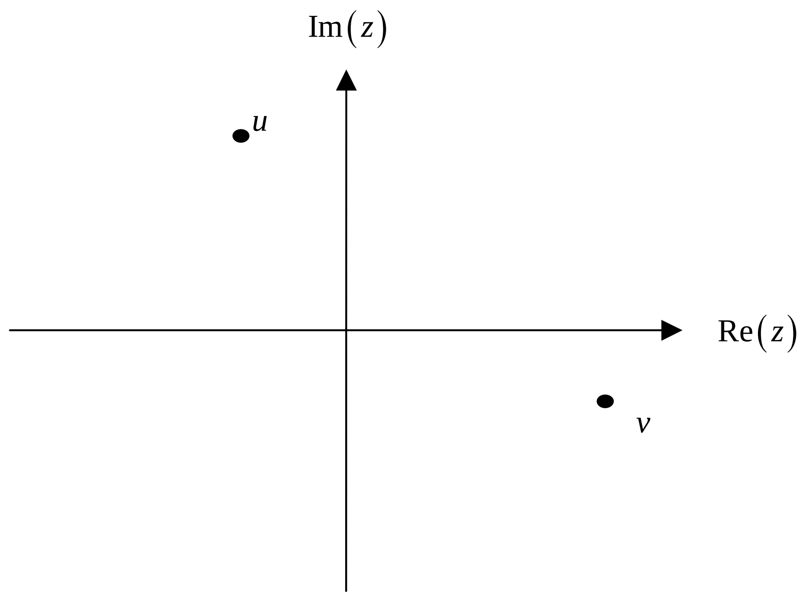
C. $i\ \text{cis}(-\theta)$

D. $-i\ \text{cis}(\theta)$

E. $-i\ \text{cis}(-\theta)$

Question 7

The diagram shows two complex numbers u, v.



The relationship between u and v is given by

A. $u=-i\overline{v}$

B. $v=-i\overline{u}$

C. $u=-iv$

D. $v=iu$

E. $u=i\overline{v}$

Question 8

If $\operatorname{Arg}(a+bi)=\tan^{-1}\left(\frac{b}{a}\right)$, then

A. $a \neq 0$ and $a, b \in R$

B. $a > 0$ and $b \in R$

C. $b > 0$ and $a \in R$

D. $a > 0$, $b > 0$ or $a < 0$, $b < 0$

E. $a > 0$, $b < 0$ or $a < 0$, $b > 0$

Question 9

Which one of the following relations is **not** the graph of a straight line passing through the origin with gradient -1?

A. $\{z : i(z+\overline{z})=\overline{z}-z\}$

B. $\{z : |z+1|=|z-i|\}$

C. $\{z : |z-1|=|z+i|\}$

D. $\{z : \operatorname{Re}(z)+\operatorname{Im}(z)=0\}$

E. $\{z : \operatorname{Arg}(z)=-\frac{\pi}{4}\} \cup \{z : \operatorname{Arg}(z)=\frac{3\pi}{4}\}$

Question 10

Two particles, P and Q have position vectors $\underset{\sim}{p}=(t^2-6t+8)\underset{\sim}{i}+(t^2-5t+6)\underset{\sim}{j}$ and $\underset{\sim}{q}=(t^2-7t+12)\underset{\sim}{i}+(t^2-4t+3)\underset{\sim}{j}$ respectively at a time t seconds, $t \geq 0$.
It follows that

A. both particles move on parabolic paths.

B. the particles paths intersect exactly once.

C. P and Q are in the same position when $t=2$.

D. P and Q are in the same position when $t=3$.

E. P and Q are never in the same position.

Question 11

The point *M* cuts the line segment *PQ* in the ratio of $3:4$ with *M* being closer to *P*. If the position vectors of *P* and *Q* are $\underset{\sim}{p}$ and $\underset{\sim}{q}$ respectively, then the position vector of *M* is

A. $\frac{1}{7}\left(3\underset{\sim}{p}+4\underset{\sim}{q}\right)$

B. $\frac{1}{7}\left(3\underset{\sim}{p}-4\underset{\sim}{q}\right)$

C. $\frac{1}{7}\left(4\underset{\sim}{q}-3\underset{\sim}{p}\right)$

D. $\frac{1}{7}\left(4\underset{\sim}{p}+3\underset{\sim}{q}\right)$

E. $\frac{1}{7}\left(4\underset{\sim}{p}-3\underset{\sim}{q}\right)$

Question 12

A vector of magnitude 10 units in the opposite direction to the vector $5\underset{\sim}{i}-4\underset{\sim}{j}+3\underset{\sim}{k}$ is given by

A. $\sqrt{2}\left(-5\underset{\sim}{i}+4\underset{\sim}{j}-3\underset{\sim}{k}\right)$

B. $\frac{5}{2}\left(5\underset{\sim}{i}-4\underset{\sim}{j}+3\underset{\sim}{k}\right)$

C. $-10\left(5\underset{\sim}{i}-4\underset{\sim}{j}+3\underset{\sim}{k}\right)$

D. $-2\left(5\underset{\sim}{i}-4\underset{\sim}{j}+3\underset{\sim}{k}\right)$

E. $-\frac{5}{2}\left(5\underset{\sim}{i}-4\underset{\sim}{j}+3\underset{\sim}{k}\right)$

Question 13

The scalar resolute of the vector $\underset{\sim}{r}$ in the direction of $-3\underset{\sim}{i}+12\underset{\sim}{j}-4\underset{\sim}{k}$ is -2.

The vector resolute of $\underset{\sim}{r}$ perpendicular to $-3\underset{\sim}{i}+12\underset{\sim}{j}-4\underset{\sim}{k}$ is $\frac{1}{13}\left(20\underset{\sim}{i}-2\underset{\sim}{j}-21\underset{\sim}{k}\right)$.

The vector $\underset{\sim}{r}$ is

A. $2\underset{\sim}{i}-2\underset{\sim}{j}-\underset{\sim}{k}$

B. $\frac{1}{13}\left(-3\underset{\sim}{i}+12\underset{\sim}{j}-4\underset{\sim}{k}\right)$

C. $-\frac{2}{13}\left(3\underset{\sim}{i}-12\underset{\sim}{j}+4\underset{\sim}{k}\right)$

D. $\frac{1}{13}\left(6\underset{\sim}{i}-24\underset{\sim}{j}+8\underset{\sim}{k}\right)$

E. $\frac{1}{13}\left(-14\underset{\sim}{i}-22\underset{\sim}{j}+29\underset{\sim}{k}\right)$

Question 14

Using a suitable substitution, $\int_0^2 \frac{x^3}{\sqrt{3x^2+4}}\,dx$ is equal to

A. $\frac{1}{18}\int_4^{16} \frac{u-4}{\sqrt{u}}\,du$

B. $2\int_4^{16} \frac{u-4}{\sqrt{u}}\,du$

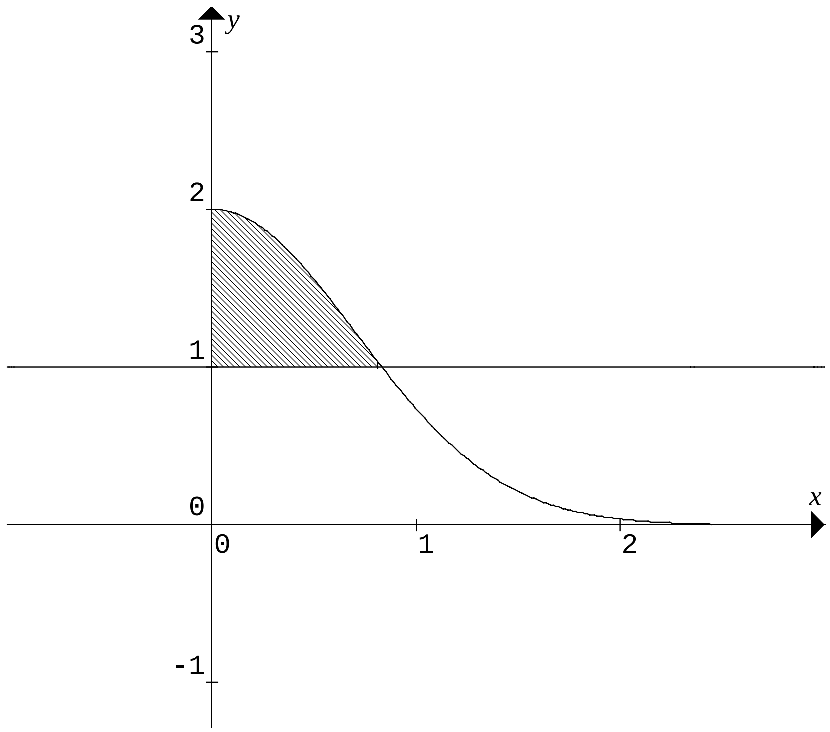
C. $\frac{1}{2}\int_0^{2} \frac{u}{\sqrt{3u+4}}\,du$

D. $2\int_0^{4} \frac{u}{\sqrt{3u+4}}\,du$

E. $\frac{1}{9}\int_2^{4} \frac{u^2-4}{u}\,du$

Question 15

The graph of the function $f:[0,\infty)\rightarrow R$ where $f(x)=2e^{-x^2}$ is shown below. The shaded region is the area bounded by the graph of f, the y-axis and the line with the equation $y=1$. The shaded region is rotated about the x-axis to form a solid of revolution.



The volume of the solid in cubic units is given by

A. $\pi\int_1^2\left(2e^{-x^2}-1\right)dx$

B. $\pi\int_1^2\log_e\left(\dfrac{2}{y}\right)dy$

C. $\pi\int_0^{\sqrt{\log_e(2)}}\left(4e^{-2x^2}-1\right)dx$

D. $\pi\int_0^{\sqrt{\log_e\left(\sqrt{2}\right)}}\left(4e^{-x^4}-1\right)dx$

E. $\pi\int_0^{\log_e\sqrt{2}}\left(2e^{-x^2}-1\right)^2dx$

Question 16

A particle travels in a straight line with velocity *v* at a time *t* and its displacement is *x*.
If $v^2 = 9x$ for $x > 0$, then the acceleration of the particle is given by

A. $\dfrac{2x}{3}$

B. 4.5

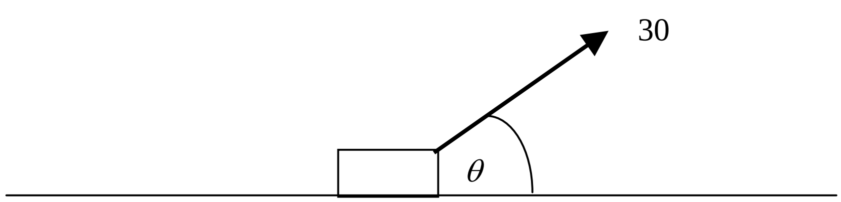
C. $2\sqrt{x^3}$

D. $6x^2$

E. $\dfrac{3}{2\sqrt{x}}$

Question 17

A box of mass 10 kg is on a horizontal plane. A rope makes an angle of θ^0 with the horizontal and exerts a tension of 30 newtons. If the coefficient of friction between the block and the surface is 0.2, which one of the following values of θ produces the largest acceleration of the block?



A. $\theta = 0$

B. $\theta = 5$

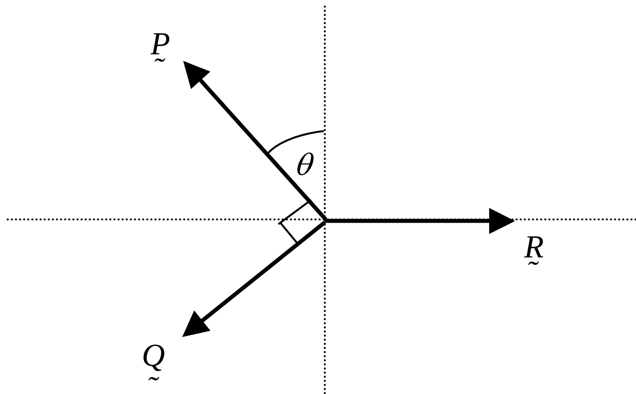
C. $\theta = 10$

D. $\theta = 15$

E. $\theta = 20$

Question 18

The following diagram shows a particle in equilibrium under the action of three concurrent coplanar forces $\underset{\sim}{P}$, $\underset{\sim}{Q}$ and $\underset{\sim}{R}$. The forces $\underset{\sim}{P}$, $\underset{\sim}{Q}$ and $\underset{\sim}{R}$ have magnitudes of *P*, *Q* and *R* respectively. Which one of the following statements is **not** correct?



A. $P\operatorname{cosec}(\theta) = Q\sec(\theta)$

B. $R^2 = P^2 + Q^2$

C. $R = P\sin(\theta) + Q\cos(\theta)$

D. $\cot(\theta) = \dfrac{P}{Q}$

E. $\underset{\sim}{P} + \underset{\sim}{Q} + \underset{\sim}{R} = \underset{\sim}{0}$

Question 19

Consider the curve $x^2 - 6xy - 16y^2 = 0$. Which one of the following is **false**?

A. The gradient of the tangent is given by $\dfrac{x}{3x+16y}$

B. At the point $(2,-1)$, the gradient of the normal is 2.

C. At the point $\left(2,\dfrac{1}{4}\right)$, the gradient of the tangent is $\dfrac{1}{8}$.

D. At the point $(-8,4)$, the gradient of the tangent is $-\dfrac{1}{2}$.

E. At the point $(-8,-1)$, the gradient of the normal is -8.

Question 20

In a chemical reaction, the velocity of the reaction is proportional to the products of the unused amounts of the substances A and B present. Initially, there is *a* grams of substance A and *b* grams of substance B. These combine in equal parts to form *x* grams of substance X after time *t* seconds. If k is a positive constant, then the differential equation which models the process is

A. $\frac{dx}{dt} = k(a-x)(b-x) \quad x(0)=0$

B. $\frac{dx}{dt} = -k(a-x)(b-x) \quad x(0)=0$

C. $\frac{dx}{dt} = k\left(a-\frac{x}{2}\right)\left(b-\frac{x}{2}\right) \quad x(0)=0$

D. $\frac{dx}{dt} = -k\left(a-\frac{x}{2}\right)\left(b-\frac{x}{2}\right) \quad x(0)=0$

E. $\frac{dx}{dt} = k(a+b-2x) \quad x(0)=0$

Question 21

A light inextensible string passes over a smooth pulley. Particles of masses m_1 and m_2 are attached to each end of the string as shown in the diagram. If the mass m_2 accelerates downwards at $\frac{g}{5}$ m/s^2, then the ratio $\frac{m_2}{m_1}$ is equal to

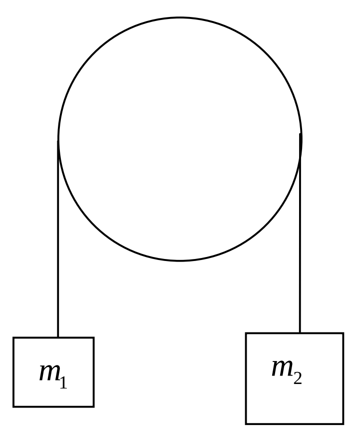
A. 1

B. $\frac{3}{2}$

C. $\frac{2}{3}$

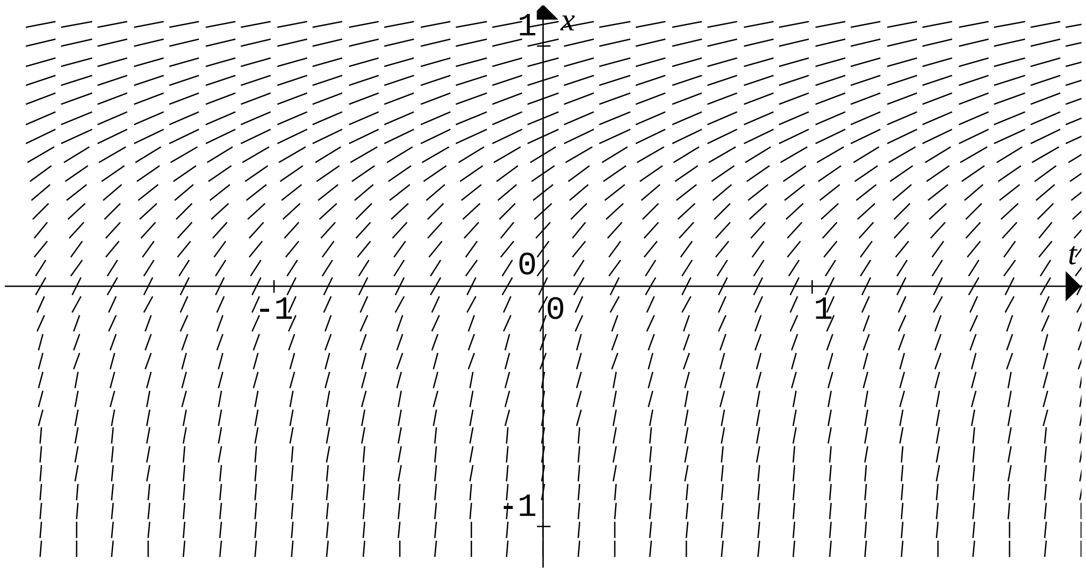
D. 5

E. $\frac{5}{4}$



Question 22

The direction (slope) field for a certain first order differential equation is shown below.



The differential equation could be

A. $\frac{dx}{dt} = -\frac{1}{2}e^{-2t}$

B. $\frac{dx}{dt} = -e^{-2t}$

C. $\frac{dx}{dt} = -2e^{-2t}$

D. $\frac{dx}{dt} = 2e^{-2t}$

E. $\frac{dx}{dt} = \frac{1}{2}e^{-2t}$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

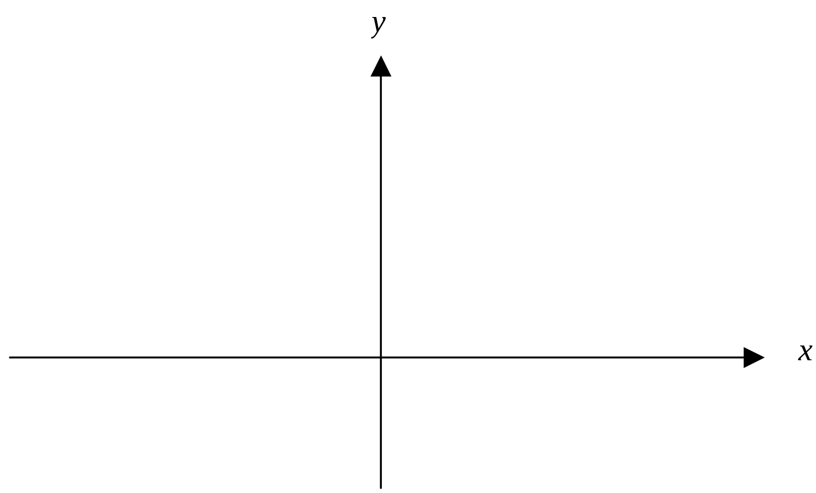
Consider the function f with the rule $f(x) = 50\sin^{-1}\left(\dfrac{x-10}{10}\right)$.

a. The gradient function can be expressed as $f'(x) = \dfrac{a}{\sqrt{x(b-x)}}$ for $0 < x < b$.

Find the values of a and b.

2 marks

b. A bowl is formed when the part of the curve $y = f(x)$ and the area bounded by the coordinate axes and the line $y = 30$ is rotated about the y-axis. Draw the shape of the bowl on the diagram below. The dimensions of the bowl are in centimetres.



1 mark

c. Find the diameter of the top of the bowl, giving your answer correct to four decimal places.

1 mark

d. **i.** Find a definite integral which gives the volume V of the bowl in cubic centimetres.

2 marks

ii. Find the volume V correct to three decimal places.

1 mark

e. The bowl is filled with water to a height of h centimetres, where $0 \le h \le 30$. However there is a small hole in the base and water leaks out at a rate of 10 cm^3/sec. Find the rate at which the water level is falling, when the height is 25 cm. Give your answer correct to three decimal places.

3 marks
Total 10 marks

Question 2

Point A has position vector $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$, point B has position vector $\underset{\sim}{b} = 5\underset{\sim}{i} + y\underset{\sim}{j} - 3\underset{\sim}{k}$, point C has position vector $\underset{\sim}{c} = 3\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$ and point D has position vector $\underset{\sim}{d} = -3\underset{\sim}{i} + 4\underset{\sim}{j} + 6\underset{\sim}{k}$ relative to the origin O.

a. Find the value of the scalar y if $\overrightarrow{AB}$ has a length of 13.

2 marks

b. Find the value of the scalar y if $\overrightarrow{AB}$ makes an angle of 135^0 with the z-axis.

2 marks

c. Find the value of the scalar y if $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CD}$.

2 marks

d. Find the value of the scalar y if $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$.

2 marks

e. Find the cosine of the angle between $\overrightarrow{CA}$ and $\overrightarrow{CD}$ and hence, (or otherwise) find the closest distance from the point A to the line CD.

3 marks
Total 11 marks

Question 3

Ashley is riding his motor-cycle along a roadway. The mass of Ashley and the motor-bike is 500 kg.

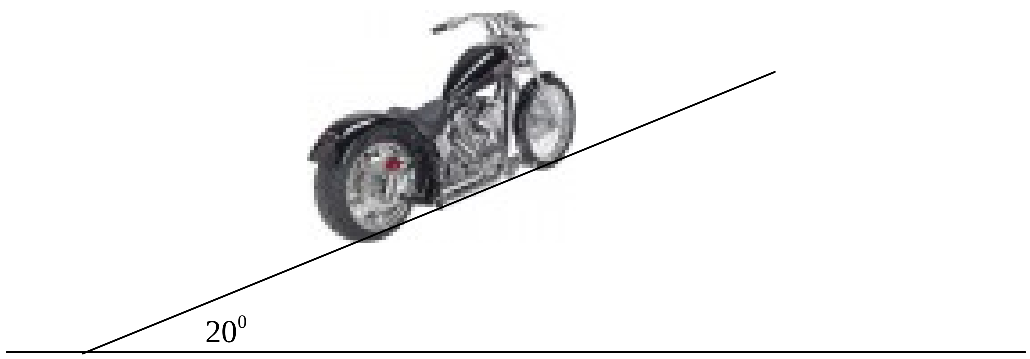
a. Ashley rides along a level road and brakes uniformly decreasing his speed from 22.5m/s to 17.5m/s, over a distance of 8 metres. Assuming constant acceleration and no resistive forces, find the acceleration and the time taken while breaking, giving all answers correct to one decimal place.

3 marks

b. Ashley now ascends a slope inclined at 20^0 and accelerates from rest.

The engine of the motor-bike produces a propulsive force of $\frac{P}{v}$ newtons, where P is the power output in watts of the engine and v is its speed in m/s. The wind and frictional resistance forces are $16v^2$ newtons.

i. On the diagram below, mark in all the forces acting on Ashley and the motor-bike as he rides his motor-bike up a slope inclined at 20^0 to the horizontal, when the power output of the motor-bike is 120,000 watts.



1 mark

ii. Write down the equation of motion of the motor-bike, while Ashley ascends the slope.

____

____

____

____

____

____

____

2 marks

iii. **Hence,** write down a definite integral, which gives the distance that Ashley covers in attaining a speed of 17.5 m/s from rest.

3 marks

iv. Find this distance in metres, giving your answer correct to two decimal places.

1 mark
Total 10 marks

Question 4

a. Given that $\cos\left(\frac{\pi}{5}\right)=\frac{\sqrt{5}+1}{4}$ and $\sin\left(\frac{\pi}{5}\right)=\frac{\sqrt{2\left(5-\sqrt{5}\right)}}{4}$,

use a double angle formula to show that $\cos\left(\frac{2\pi}{5}\right)=\frac{\sqrt{5}-1}{4}$.

2 marks

b. Hence, show that the exact value of $\sin\left(\frac{2\pi}{5}\right)=\frac{\sqrt{2\left(\sqrt{5}+5\right)}}{4}$.

2 marks

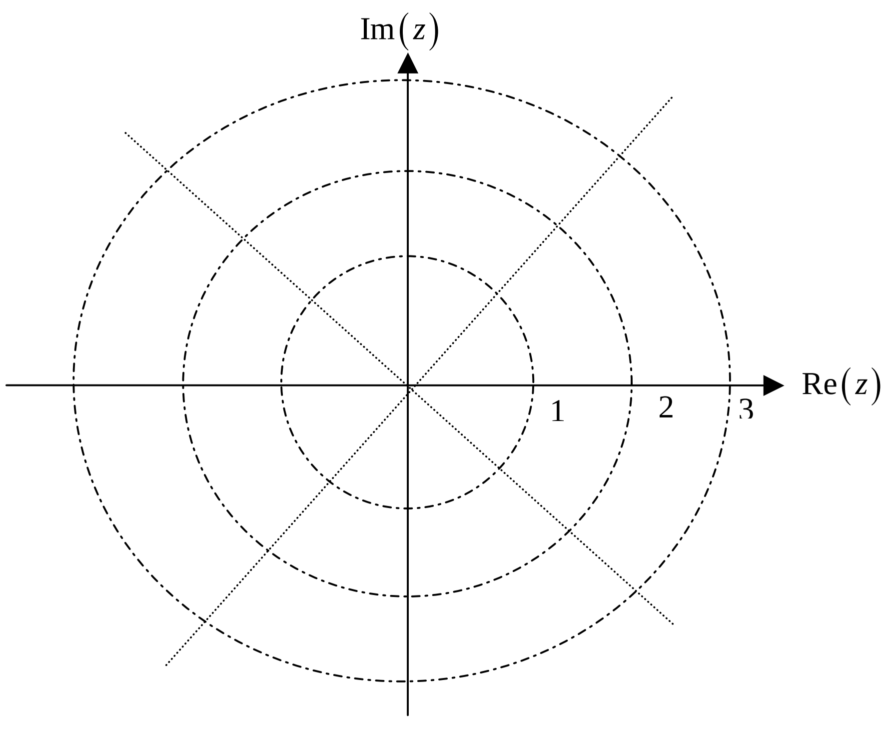
c. Evaluate $4\left(\sqrt{\frac{\sqrt{5}-1}{4}+\frac{\sqrt{2\left(5+\sqrt{5}\right)}}{4}\,i}\right)^{21}$ giving your answer in exact rectangular form.

3 marks

d. For what values of n is $\left(\sqrt{5}+1+\left(\sqrt{2\left(5-\sqrt{5}\right)}\right)i\right)^{n}$ a real number?

2 marks

e. Plot the roots of $z^5 + 32 = 0$ on the Argand diagram below, stating **three** of the roots in both exact rectangular and polar form. Comment on their relative positions.



4 marks
Total 13 marks

Question 5

A ball is thrown in a vertical plane and has a position vector given by $\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j}$ for $y \ge 0$ and $0 \le t \le T$, where t is the time in seconds, $\underset{\sim}{i}$ is a unit vector of one metre horizontally forward and $\underset{\sim}{j}$ is a unit vector of one metre vertically upwards, above ground level. Initially, the ball is thrown from a point 1.5 metres above the ground, and its velocity vector is given by $\dot{\underset{\sim}{r}}(t) = 10\underset{\sim}{i} + (10 - gt - 0.2y(t))\underset{\sim}{j}$.

a. The component of the velocity in the $\underset{\sim}{j}$ direction, gives the differential equation $\frac{dy}{dt} + ky = b - gt \quad y(0) = y_0$. Write down the values of k, b and y_0.

1 mark

b. Use Euler's method with a step size of $\frac{1}{5}$ to estimate the value of y when $t = 0.4$. Give your answer correct to four decimal places.

2 marks

c. Using differential calculus, verify that $y(t) = 295 - 293.5e^{-\frac{t}{5}} - 49t$ is a solution of the differential equation above in **a.**

2 marks

d. Show that the ball hits the ground when $T = 2.02676$ seconds.

1 mark

e. Find the range, that is, the horizontal distance that the ball travels before hitting the ground. Give your answer correct to three decimal places.

1 mark

f. Find the time when the ball reaches its maximum height. Find the maximum height reached and the horizontal distance travelled at this time. Give all answers correct to three decimal places.

4 marks

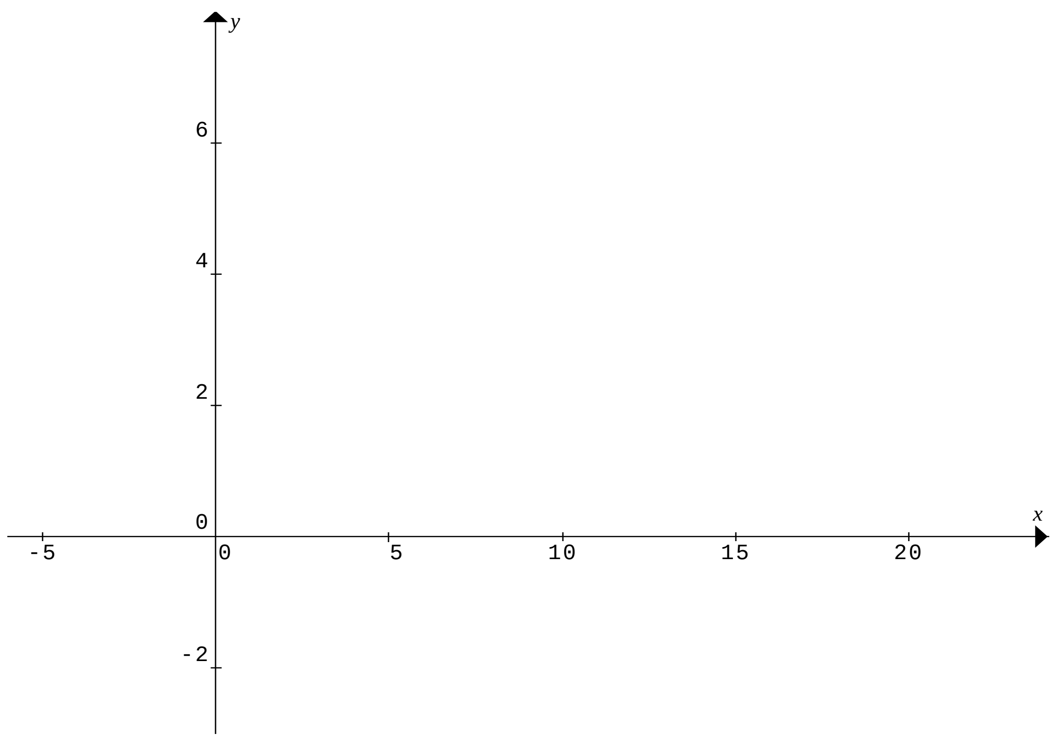
g. Find the speed at which the ball hits the ground. Give your answer correct to three decimal places.

1 mark

h. Find the angle in degrees and minutes at which the ball hits the ground.

1 mark

i. Draw the path of the ball y versus x on the following set of axes.



1 mark
Total 14 marks

END OF EXAMINATION

EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\mathrm{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r \qquad -\pi < \mathrm{Arg}(z) \le \pi$$

$$z_1 z_2 = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2) \qquad \frac{z_1}{z_2} = \frac{r_1}{r_2}\,\mathrm{cis}(\theta_1 - \theta_2)$$

$z^n = r^n \mathrm{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$$

$$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r \qquad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u+v)t$$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ | $\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ | $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$ | $\int \frac{1}{x} dx = \log_e(x) + c,\ \text{for}\ \ x > 0$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ | $\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ | $\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ | $\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$ | $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$ | $\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$ | $\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

Figures										Letter:
Words										

SIGNATURE ____

SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**