Year 2007

VCE

Specialist Mathematics

Trial Examination 1



KILBAHA MULTIMEDIA PUBLISHING PO BOX 2227 KEW VIC 3101 AUSTRALIA TEL: (03) 9817 5374 FAX: (03) 9817 4334 chemas@chemas.com www.chemas.com

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Victorian Certificate of Education 2007

STUDENT NUMBER

		_				Letter
Figures						
Words						

Latter

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question. In questions where more than one mark is available, appropriate working **must** be shown.

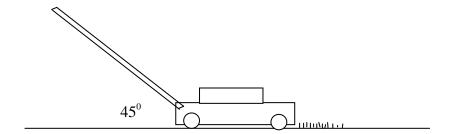
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude g m/s², where g = 9.8

A	4
Question	
Question	-

Question 1
Consider the relation $\frac{x}{e^{2y}} - 5y + 6x^2 + 3 = 0$
Find an expression for $\frac{dy}{dx}$ in terms of x and y.

A man pushes down along the handle of a motor-mower with a force of T newtons. The handle of the motor-mower is inclined at an angle of 45° to the horizontal lawn. The motor-mower has a mass of 10 kg and the coefficient of friction between the uncut grass and the motor-mower is $\frac{1}{7}$.



a. On the diagram above, mark in all the forces on the motor-mower.

1 mark

b. Show that if $T = 12\sqrt{2}$, the motor-mower will not move.

c.	Find the acceleration of the motor-mower if $T = \frac{49\sqrt{2}}{2}$.				

Question 3			
Solve the differential equation	$\frac{dy}{dx} = \frac{3x - 5}{\sqrt{9 - 4x^2}}$	given that	y = 0 when $x = 0$.

Consider the cubic equation $z^3 + pz^2 + qz + 15 = 0$, where p and q are real numbers. One root of this equation is 1-2i.

a.	State another root.	
		1 mark
b.	Find the values of p and q and all the roots.	

a.	Show that $\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$	
		3 marks

υ.	Find $\operatorname{Arg}\left(-1+\left(\sqrt{3}-2\right)t\right)$				

1 mark

a.	Show that	$\frac{d}{dx} \left(\tan^2 \frac{d}{x} \right)$	$-1\left(\sqrt{\frac{3}{x}}\right) = \frac{-1}{2\sqrt{x}}$	$\frac{-\sqrt{3}}{(x+3)}$	for $x > 0$.	
----	-----------	--	--	---------------------------	---------------	--

b.	Hence, find the exact value of	$\int_{1}^{9} \frac{1}{\sqrt{x^3 + 6x^2 + 9x}} dx$
b.	Hence, find the exact value of	$\int_{1}^{9} \frac{1}{\sqrt{x^3 + 6x^2 + 9x}}$

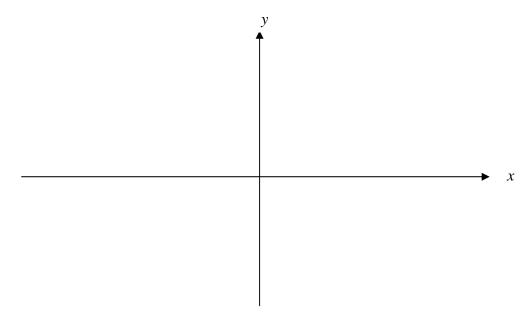
2 marks

The position vector of a moving particle is given by $\underline{r}(t) = \frac{3}{2} (e^{2t} + e^{-2t}) \underline{i} + \frac{5}{2} (e^{2t} - e^{-2t}) \underline{j}$ for $t \ge 0$.

a. If the Cartesian equation of the path can be expressed in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ find the values of a and b, where a and b are positive real constants.

3 marks

b. Sketch the path of the particle on the axes provided.



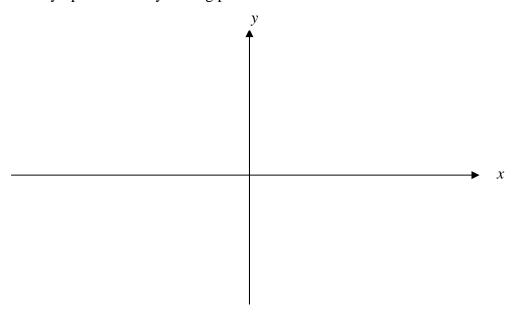
1 mark

The area enclosed by the curve with the equation	$y = 6\sin\left(\frac{\pi x}{2}\right)$	is rotated about the x-axis,
to form a solid of revolution between $0 \le x \le b$, v	where b is a real	constant and $0 < b < 4$.

a.	Find the volume of the solid of revolution, in terms of b .	
		3 mark
b.	Find the value of b if the volume is $9(7\pi + 2)$.	

1 mark

a. Sketch the graph with the equation $y = \frac{4}{x^2 - 4x}$, clearly indicating the location of all asymptotes and any turning points.



2 marks

b. Find the exact area bounded by $y = \frac{4}{x^2 - 4x}$, the *x*-axis and the lines x = 1 and x = 2.

EXTRA WORKING SPACE		

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

 $\pi r^2 h$ volume of a cylinder:

 $\frac{1}{2}\pi r^2 h$ volume of a cone:

volume of a pyramid: $\frac{1}{2}Ah$

 $\frac{4}{3}\pi r^3$ volume of a sphere:

 $\frac{1}{2}bc\sin(A)$ area of triangle:

 $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ sine rule:

 $c^2 = a^2 + b^2 - 2ab\cos(C)$ cosine rule:

Coordinate geometry

 $\frac{(x-h)^2}{x^2} + \frac{(y-k)^2}{h^2} = 1$ hyperbola: $\frac{(x-h)^2}{x^2} - \frac{(y-k)^2}{h^2} = 1$ ellipse:

Circular (trigonometric) functions

 $\cos^2(x) + \sin^2(x) = 1$ $1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \csc^2(x)$

 $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y) \qquad \sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$ $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \qquad \cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

 $\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

 $\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

 $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$ $\sin(2x) = 2\sin(x)\cos(x)$

function	sin ⁻¹	cos ⁻¹	tan ⁻¹
domain	[-1,1]	$\begin{bmatrix} -1,1 \end{bmatrix}$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$\left[0,\pi\right]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\cos(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \cos(\theta_1 + \theta_2)$$

$$z^n = r^n \cos(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg}(z) \le \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \cos(\theta_1 - \theta_2)$$

Vectors in two and three dimensions

$$\begin{aligned}
& \underset{\sim}{r} = x \underline{i} + y \underline{j} + z \underline{k} \\
& |\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r \\
& \underset{\sim}{\dot{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}
\end{aligned}$$

Mechanics

momentum: p = my

equation of motion: R = ma

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$$v = u + at$$
 $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2}(u + v)t$

acceleration:
$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dt} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Calculus

$$\frac{d}{dx}(x^{n}) = nx^{n-1} \qquad \int x^{n} dx = \frac{1}{n+1}x^{n+1} + c , n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_{e}(x)) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_{e}(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax) \qquad \int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax) \qquad \int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a\sec^{2}(ax) \qquad \int \sec^{2}(ax) dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^{2}}} \qquad \int \frac{dx}{\sqrt{a^{2}-x^{2}}} = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^{2}}} \qquad \int \frac{1}{\sqrt{a^{2}-x^{2}}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^{2}} \qquad \int \frac{a}{a^{2}+x^{2}} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

quotient rule:
$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

END OF FORMULA SHEET