# Year 2007

# VCE

# Specialist Mathematics

# Trial Examination 1



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**Victorian Certificate of Education**
**2007**

**STUDENT NUMBER**

| | | | | | | | | | Letter |
|---|---|---|---|---|---|---|---|---|---|
| Figures | | | | | | | | | |
| Words | | | | | | | | | |

# SPECIALIST MATHEMATICS

# Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

## QUESTION AND ANSWER BOOK

**Structure of book**

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| 9 | 9 | 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

**Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**Instructions**

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude g m/s$^2$, where $g = 9.8$

**Question 1**

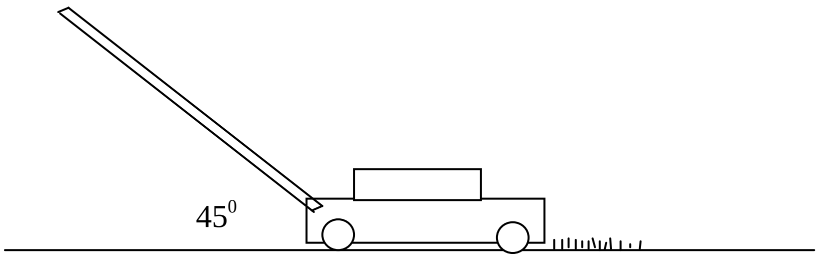
Consider the relation $\frac{x}{e^{2y}} - 5y + 6x^2 + 3 = 0$

Find an expression for $\frac{dy}{dx}$ in terms of $x$ and $y$.

3 marks

**Question 2**

A man pushes down along the handle of a motor-mower with a force of $T$ newtons.
The handle of the motor-mower is inclined at an angle of $45^0$ to the horizontal lawn.
The motor-mower has a mass of 10 kg and the coefficient of friction between the uncut grass and the motor-mower is $\frac{1}{7}$.




**a.** On the diagram above, mark in all the forces on the motor-mower.

1 mark

**b.** Show that if $T = 12\sqrt{2}$, the motor-mower will not move.

2 marks

**c.** Find the acceleration of the motor-mower if $T = \frac{49\sqrt{2}}{2}$.

3 marks

**Question 3**

Solve the differential equation $\frac{dy}{dx} = \frac{3x-5}{\sqrt{9-4x^2}}$ given that $y = 0$ when $x = 0$.

5 marks

**Question 4**

Consider the cubic equation $z^3 + pz^2 + qz + 15 = 0$, where $p$ and $q$ are real numbers.
One root of this equation is $1-2i$.

**a.** State another root.

1 mark

**b.** Find the values of $p$ and $q$ and all the roots.

3 marks

**Question 5**

**a.** Show that $\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$

3 marks

**b.** Find $\text{Arg}\left(-1 + \left(\sqrt{3} - 2\right)i\right)$

1 mark

**Question 6**

**a.** Show that $\dfrac{d}{dx}\left(\tan^{-1}\left(\sqrt{\dfrac{3}{x}}\right)\right)=\dfrac{-\sqrt{3}}{2\sqrt{x}\,(x+3)}$ for $x>0$.

2 marks

**b.** Hence, find the exact value of $\displaystyle\int_1^9 \frac{1}{\sqrt{x^3+6x^2+9x}}\,dx$.

2 marks

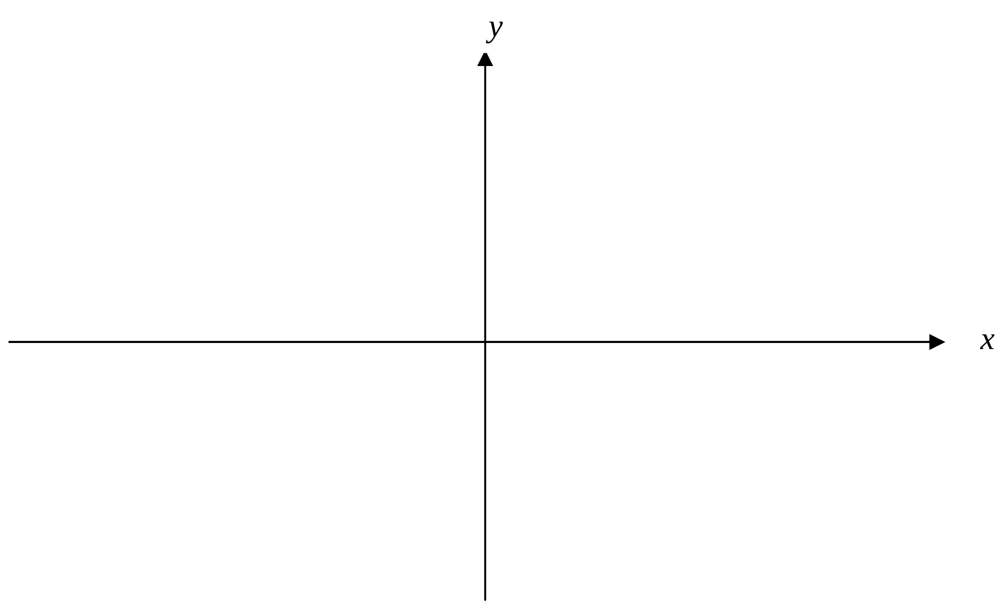
**Question 7**

The position vector of a moving particle is given by $\underset{\sim}{r}(t) = \frac{3}{2}\left(e^{2t} + e^{-2t}\right)\underset{\sim}{i} + \frac{5}{2}\left(e^{2t} - e^{-2t}\right)\underset{\sim}{j}$ for $t \geq 0$.

**a.** If the Cartesian equation of the path can be expressed in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ find the values of $a$ and $b$, where $a$ and $b$ are positive real constants.

3 marks

**b.** Sketch the path of the particle on the axes provided.




1 mark

**Question 8**

The area enclosed by the curve with the equation $y = 6\sin\left(\frac{\pi x}{2}\right)$ is rotated about the *x*-axis, to form a solid of revolution between $0 \le x \le b$, where *b* is a real constant and $0 < b < 4$.

**a.** Find the volume of the solid of revolution, in terms of *b*.

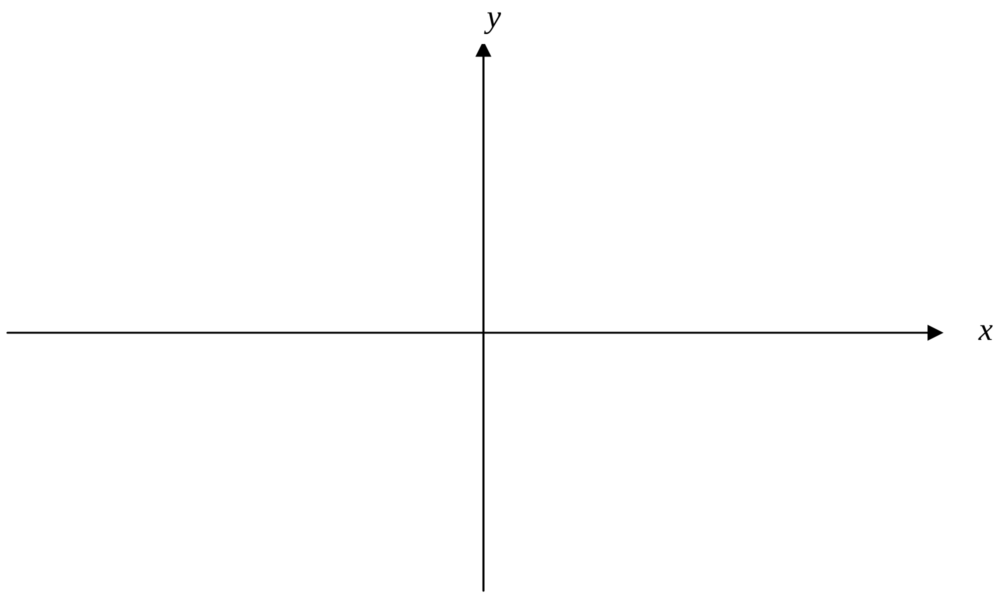
3 marks

**b.** Find the value of *b* if the volume is $9(7\pi + 2)$.

1 mark

**Question 9**

**a.** Sketch the graph with the equation $y = \dfrac{4}{x^2 - 4x}$, clearly indicating the location of all asymptotes and any turning points.




2 marks

**b.** Find the exact area bounded by $y = \dfrac{4}{x^2 - 4x}$, the $x$-axis and the lines $x = 1$ and $x = 2$.

4 marks

**EXTRA WORKING SPACE**

**END OF EXAMINATION**

# SPECIALIST MATHEMATICS

# Written examination 1

## FORMULA SHEET

### Directions to students

| Detach this formula sheet during reading time.<br><br>This formula sheet is provided for your reference. |
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# Specialist Mathematics Formulas

## Mensuration

| | |
|---|---|
| area of a trapezium: | $\frac{1}{2}(a+b)h$ |
| curved surface area of a cylinder: | $2\pi rh$ |
| volume of a cylinder: | $\pi r^2 h$ |
| volume of a cone: | $\frac{1}{3}\pi r^2 h$ |
| volume of a pyramid: | $\frac{1}{3}Ah$ |
| volume of a sphere: | $\frac{4}{3}\pi r^3$ |
| area of triangle: | $\frac{1}{2}bc\sin(A)$ |
| sine rule: | $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ |
| cosine rule: | $c^2 = a^2 + b^2 - 2ab\cos(C)$ |

## Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

## Circular ( trigonometric ) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \text{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

| function | $\sin^{-1}$ | $\cos^{-1}$ | $\tan^{-1}$ |
|---|---|---|---|
| domain | $[-1,1]$ | $[-1,1]$ | $R$ |
| range | $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ | $[0,\pi]$ | $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ |

## Algebra ( Complex Numbers )

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem )

## Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

## Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant ( uniform ) acceleration:

$v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

## Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ | $\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ | $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$ | $\int \frac{1}{x} dx = \log_e(x) + c,\ \text{for}\ \ x > 0$

$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ | $\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ | $\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ | $\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$ | $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$ | $\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$ | $\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x),\ x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

**END OF FORMULA SHEET**