Q1 $\frac{2\sqrt{3}+2i}{1-\sqrt{3}i}=\frac{4cis\left(\frac{\pi}{6}\right)}{2cis\left(-\frac{\pi}{3}\right)}=2cis\left(\frac{\pi}{6}+\frac{\pi}{3}\right)=2cis\left(\frac{\pi}{2}\right)$

Q2a $f(z)=z^3-(\sqrt{5}-i)z^2+4z-4\sqrt{5}+4i$
$=z^2(z-(\sqrt{5}-i))+4(z-(\sqrt{5}-i))$
$=(z-(\sqrt{5}-i))(z^2+4)$
$\therefore f(\sqrt{5}-i)=0$. Hence $\sqrt{5}-i$ is a solution of
$z^3-(\sqrt{5}-i)z^2+4z-4\sqrt{5}+4i=0$

Q2b The other two solutions are:
$z^2+4=0$, $z=-2i$ or $2i$

Q3 By implicit differentiation:
$3x^2-4xy-2x^2\frac{dy}{dx}+4y\frac{dy}{dx}=0$,
$\frac{dy}{dx}=\frac{3x^2-4xy}{2x^2-4y}$.
At $P(2,3)$, $\frac{dy}{dx}=3$.
Equation of tangent: $y-3=3(x-2)$, $\therefore y=3x-3$

Q4 $V=\int_{-\frac{1}{2}}^{0}\pi y^2dx=\pi\int_{-\frac{1}{2}}^{0}\frac{1}{1-x^2}dx$
$=\frac{\pi}{2}\int_{-\frac{1}{2}}^{0}\left(\frac{1}{1-x}+\frac{1}{1+x}\right)dx$ (partial fractions)
$=\frac{\pi}{2}\left[-\log_e(1-x)+\log_e(1+x)\right]_{-\frac{1}{2}}^{0}$
$=\frac{\pi}{2}\left[\log_e\frac{1+x}{1-x}\right]_{-\frac{1}{2}}^{0}=\frac{\pi}{2}\left(-\log_e\frac{1}{3}\right)=\frac{\pi}{2}\log_e 3$.

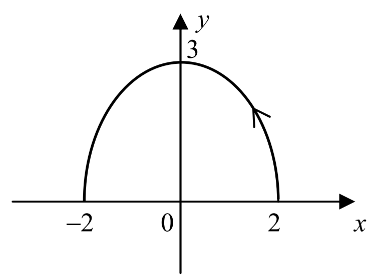
Q5a $u={}^{+}4$, $s={}^{+}3$, $v=0$, use $v^2=u^2+2as$ to find a.
$\therefore a=\frac{v^2-u^2}{2s}=\frac{0-16}{6}=-\frac{8}{3}$ ms^{-2}

Q5b Newton's second law: $R=ma$,
$-\mu N=6\left(-\frac{8}{3}\right)$, $-\mu(6g)=-6\left(\frac{8}{3}\right)$, $\mu=\frac{8}{3g}$.

Q6a $\boldsymbol{r}(t)=\int(-4\sin(2t)\boldsymbol{i}+6\cos(2t)\boldsymbol{j})dt$,
$\boldsymbol{r}(t)=2\cos(2t)\boldsymbol{i}+3\sin(2t)\boldsymbol{j}+\boldsymbol{c}$.
Given $\boldsymbol{r}(0)=2\boldsymbol{i}$, $\boldsymbol{c}=\boldsymbol{0}$. $\therefore \boldsymbol{r}(t)=2\cos(2t)\boldsymbol{i}+3\sin(2t)\boldsymbol{j}$

Q6b $x=2\cos(2t)$, $y=3\sin(2t)$.
$\therefore \frac{x^2}{4}=\cos^2(2t)$ and $\frac{y^2}{9}=\sin^2(2t)$
$\therefore \frac{x^2}{4}+\frac{y^2}{9}=1$.
Given $0\le t\le\frac{\pi}{2}$, when $t=0$, $x=2$, $y=0$;
when $t=\frac{\pi}{4}$, $x=0$, $y=3$;
when $t=\frac{\pi}{2}$, $x=-2$, $y=0$.
$\therefore -2\le x\le 2$ and $0\le y\le 3$.

Q6c



Q7a
$x_0=1$, $y_0=1$
$x_1=1.1$, $y_1\approx 1+0.1\times\frac{1}{1}=1+0.1=1.1$
$x_2=1.2$, $y_2\approx 1.1+0.1\times\frac{1}{1.1}=1.1+\frac{0.1}{1.1}=\frac{1.31}{1.1}=\frac{131}{110}$.

Q7b $\frac{dy}{dx}=\frac{1}{x}$, $y=\int\frac{1}{x}dx=\log_e x+c$, where $x>0$.
Since $y(1)=1$, $\therefore c=1$ and $y=\log_e x+1$.
When $x=1.2$, $y=\log_e(1.2)+1$.

Q8a

y	–2	–1	0	1	2
$\frac{dy}{dx}$	2.5	1	0.5	1	2.5

Note: $\frac{dy}{dx}=\frac{1+y^2}{2}$ is independent of x.
Graph in part c.

Q8b $\frac{dx}{dy}=\frac{2}{1+y^2}$, $x=2\int\frac{1}{1+y^2}dy$, $x=2\tan^{-1}y+c$.

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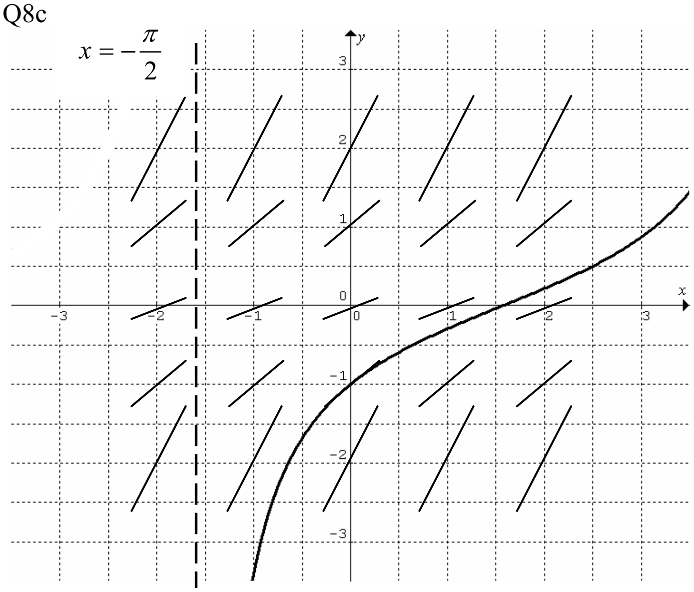
Given $y=-1$ when $x=0$,

$0=2\tan^{-1}(-1)+c$, $0=2\left(-\frac{\pi}{4}\right)+c$.

$\therefore c=\frac{\pi}{2}$ and $x=2\tan^{-1}y+\frac{\pi}{2}$.

Hence $y=\tan\frac{1}{2}\left(x-\frac{\pi}{2}\right)$.

Q8c



Q9 $\boldsymbol{r}=x\boldsymbol{i}+y\boldsymbol{j}$, $\boldsymbol{v}=\frac{d}{dt}\boldsymbol{r}=\frac{dx}{dt}\boldsymbol{i}+\frac{dy}{dt}\boldsymbol{j}$.

Given $\boldsymbol{v}=-y\boldsymbol{i}+x\boldsymbol{j}$, $\therefore\frac{dx}{dt}=-y$ and $\frac{dy}{dt}=x$.

$\boldsymbol{a}=\frac{d}{dt}\boldsymbol{v}=-\frac{dy}{dt}\boldsymbol{i}+\frac{dx}{dt}\boldsymbol{j}$.

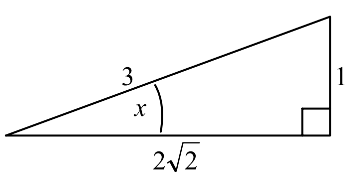
$\therefore\boldsymbol{a}=-x\boldsymbol{i}-y\boldsymbol{j}=-(x\boldsymbol{i}+y\boldsymbol{j})$.

Hence $\boldsymbol{a}=-\boldsymbol{r}$.

Q10 $\tan(2x)=\frac{4\sqrt{2}}{7}$, $\frac{2\tan x}{1-\tan^2 x}=\frac{4\sqrt{2}}{7}$. where $0\le x<\frac{\pi}{4}$.

$\therefore\tan^2 x+\frac{7}{2\sqrt{2}}\tan x-1=0$, where $0\le x<\frac{\pi}{4}$.

$$\tan x=\frac{-\frac{7}{2\sqrt{2}}+\sqrt{\frac{49}{8}+4}}{2}=\frac{1}{2\sqrt{2}}.$$ (Quadratic formula)



From diagram, $\sin x=\frac{1}{3}$.