Insight Publications

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Trial Exam Paper

2007

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 2 hours

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, once bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring sheets of paper or white out liquid/tape into the examination.

Materials provided
- The question and answer book of 25 pages with a separate sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions

Instructions
- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam
- Place the multiple-choice answer sheet inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the multiple-choice answer sheet provided.
Choose the response that is **correct** for the question.
One mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.
Marks **are not** deducted for incorrect answers.
No marks will be awarded if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

If $z = i\left(2i + i^3 - 3\right)$, then $\text{Re}(z)$ is equal to

A. -3
B. -2
C. -1
D. 1
E. 3

Question 2

In polar form $-\frac{1}{\sqrt{2}}(1+i)$ is equivalent to

A. $\text{cis}\left(\frac{\pi}{4}\right)$

B. $\text{cis}\left(-\frac{3\pi}{4}\right)$

C. $\frac{1}{\sqrt{2}}\text{cis}\left(\frac{\pi}{4}\right)$

D. $\frac{1}{\sqrt{2}}\text{cis}\left(\frac{5\pi}{4}\right)$

E. $\frac{1}{\sqrt{2}}\text{cis}\left(-\frac{3\pi}{4}\right)$

Question 3

If $u = 3 - 4i$ and $v = 1 + 2i$ then $\left|\dfrac{u^2}{v^2}\right|$ is equal to

A. $\overline{u}$

B. $-u$

C. $|v|$

D. $|v|^2$

E. $\left(\dfrac{u}{v}\right)^2$

Question 4

The asymptotes of the curve $x^2 - 4y^2 + 6x + 16y = 11$ intersect at the point

A. $(-3, 2)$

B. $(-3, -8)$

C. $(-6, -4)$

D. $(3, 2)$

E. $(3, 8)$

Question 5

The range of the function $f : [0, \frac{7\pi}{12}] \to R$, $f(x) = 1 - 3\text{cosec}\left(x + \dfrac{\pi}{4}\right)$ is

A. R

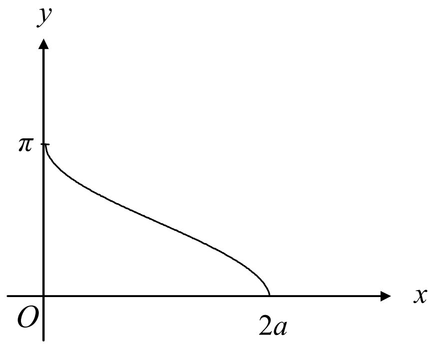
B. $(-\infty, -2] \cup [2, \infty)$

C. $(-\infty, -2]$

D. $[-5, -2]$

E. $\left[-5, 1 - 3\sqrt{2}\right]$

Question 6



The equation of the graph shown above could be

A. $y = \cos^{-1}(ax - 1)$

B. $y = \cos^{-1}(x - 2a)$

C. $y = \cos^{-1}\left(\frac{x}{2} - a\right)$

D. $y = \cos^{-1}\left(\frac{x-1}{a}\right)$

E. $y = \cos^{-1}\left(\frac{x}{a} - 1\right)$

Question 7

Given the function $f(x) = \frac{|\log_e(x)|}{\log_e|x| - 1}$, which one of the following statements is false?

A. f has an asymptote at $x = e$.

B. The point $(1, 0) \in f(x)$.

C. The inverse, f^{-1}, exists for $x \in (0, 1]$.

D. The range of f is $R \setminus [0, 1)$.

E. f has more than two asymptotes.

Question 8

At the point where $y = 2$, the gradient of the curve $y^2 + x^3 = 5$ is

A. -3

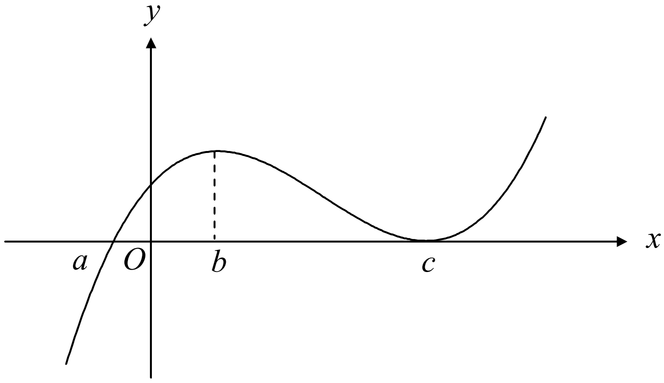
B. $-\frac{3}{4}$

C. $-\frac{3}{2}$

D. $\frac{1}{2}$

E. 2

Question 9



The graph of $y = f(x)$ is shown above.
Let $F(x)$ be an antiderivative of $f(x)$.

The graph of $y = F(x)$ has a

A. local maximum at $x = a$
B. stationary point at $x = b$
C. point of inflexion at $x = c$
D negative gradient for $b < x < c$
E. zero gradient at $x = 0$

Question 10

Using the substitution $u = 2 - x$, the integral $\int_2^4 x(2-x)^6\,dx$ is equal to

A. $\int_2^4 \left(2u^6 - u^7\right)du$

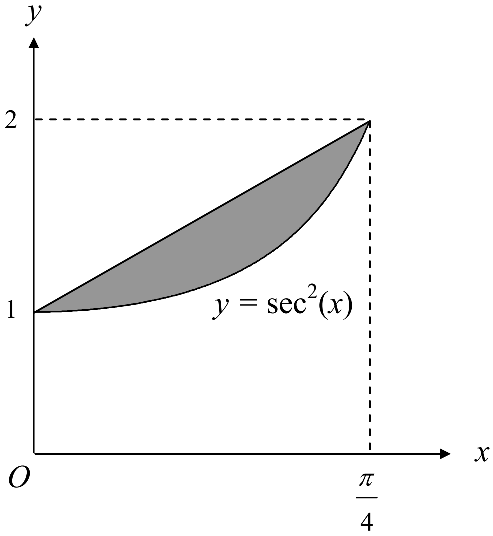
B. $\int_2^4 \left(u^7 - 2u^6\right)du$

C. $\int_{-2}^0 \left(u^7 - 2u^6\right)du$

D. $\int_0^{-2} \left(u^7 - 2u^6\right)du$

E. $\int_0^{-2} \left(2u^6 - u^7\right)du$

Question 11



The shaded region shown above is formed between the graph of $y = \sec^2(x)$ and the line joining the points $(0, 1)$ and $\left(\frac{\pi}{4}, 2\right)$.

The area of the shaded region would be

A. $\frac{3\pi - 8}{8}$

B. $\frac{3\pi - 4}{8}$

C. $\frac{3\pi - 1}{8}$

D. $\frac{3\pi}{8}$

E. $\frac{\pi}{4}$

Question 12

Given $f'(x) = \frac{e^x}{e^{-x} + e^x}$ and $f(0) = 2$.

Using Euler's method with increments of 0.1, an approximate value of $f(0.2)$ is

A. 2.0500
B. 2.0550
C. 2.1050
D. 2.1099
E. 2.1484

Question 13

A bowl of soup is heated to a temperature of 80°C. It is then left to cool in a room in which the air temperature is 20°C. The rate at which the temperature of the soup decreases is proportional to the difference between its temperature and the temperature of the room.

Let *S* °C be the temperature of the soup at any time *t* minutes after it is removed from the heat. Given *k* is a positive constant, the relationship between *S* and *t* may be modelled by the differential equation

A. $\frac{dS}{dt} = -k(S-20); \qquad t=0,\ S=60$

B. $\frac{dS}{dt} = -k(S-20); \qquad t=0,\ S=80$

C. $\frac{dS}{dt} = -k(S-60); \qquad t=0,\ S=80$

D. $\frac{dS}{dt} = -k(S-60); \qquad t=0,\ S=20$

E. $\frac{dS}{dt} = -k(S-80); \qquad t=0,\ S=20$

Question 14

The solution of the differential equation $\frac{dy}{dx} = e^{\sin(x)}$, given $y=3$ when $x=2$, is

A. $y = \int_2^x e^{\sin(u)}du + 3$

B. $y = \int_3^x e^{\sin(u)}du + 2$

C. $y = \int_2^3 e^{\sin(x)}dx$

D. $y = \int_2^3 e^{\sin(x)}dx + 3$

E. $y = \int_2^3 e^{\sin(x)}dx + 2$

Question 15

A particle moves in a straight line such that its velocity is given by

$v = \log_e\left|\cos\left(\frac{x}{4}\right)\right|,\ x \in [0, 2\pi)$, where x is its displacement from the origin O.

The particle's acceleration $\frac{4\pi}{3}$ units from O is

A. $-\frac{\sqrt{3}}{4}\log_e(2)$

B. $-\log_e(2)$

C. $-\frac{\sqrt{3}}{4}$

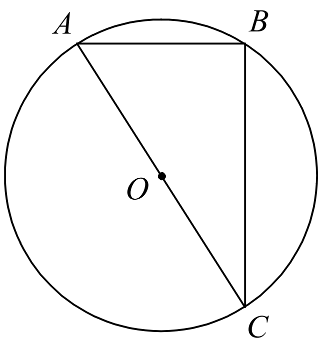
D. $\log_e(2)$

E. $\frac{\sqrt{3}}{4}\log_e(2)$

Question 16

A, B and C are three points on the circumference of a circle with centre O.

AC passes through O.



Which one of the following statements is **not** true?

A. $\overrightarrow{AB}.\overrightarrow{BC} = 0$

B. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \underset{\sim}{0}$

C. $\left(\overrightarrow{AB} + \overrightarrow{BC}\right).\overrightarrow{AC} = \left|\overrightarrow{AC}\right|^2$

D. $\overrightarrow{AB}.\overrightarrow{AC} = \left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\cos(A)$

E. $\left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$

Question 17

Let $\underset{\sim}{m} = 4\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{n} = \underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$.

A unit vector in the direction of $\underset{\sim}{m} - 2\underset{\sim}{n}$ would be

A. $\frac{1}{7}\left(2\underset{\sim}{i} - 3\underset{\sim}{j} + 6\underset{\sim}{k}\right)$

B. $\frac{1}{3}\left(2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right)$

C. $\frac{1}{\sqrt{17}}\left(2\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k}\right)$

D. $\frac{1}{\sqrt{41}}\left(6\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}\right)$

E. $\frac{1}{\sqrt{29}}\left(3\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k}\right)$

Question 18

The velocity of a particle at time *t*, $t \geq 0$ is given by $\underset{\sim}{v} = 4\sin(2t)\underset{\sim}{i} + 6\cos(3t)\underset{\sim}{j}$.

If the particle was initially at $\underset{\sim}{i} + 2\underset{\sim}{j}$, its position after $\frac{\pi}{2}$ seconds will be

A. $-3\underset{\sim}{i} + 4\underset{\sim}{j}$

B. $3\underset{\sim}{i} - 2\underset{\sim}{j}$

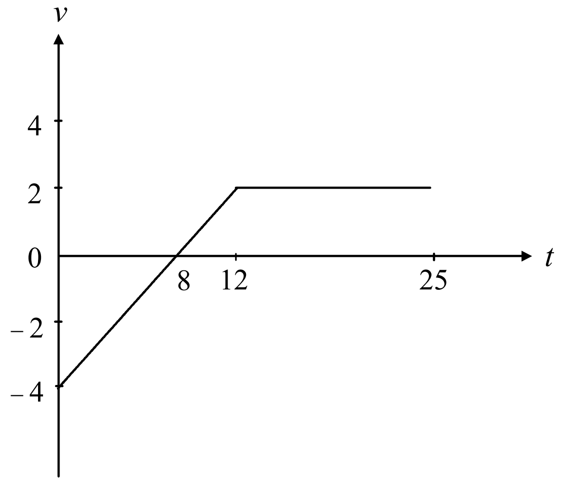
C. $3\underset{\sim}{i} + 3\underset{\sim}{j}$

D. $5\underset{\sim}{i}$

E. $5\underset{\sim}{i} - \underset{\sim}{j}$

Question 19

The graph below shows the velocity, v m/s, of a particle moving in a straight line for 25 s.

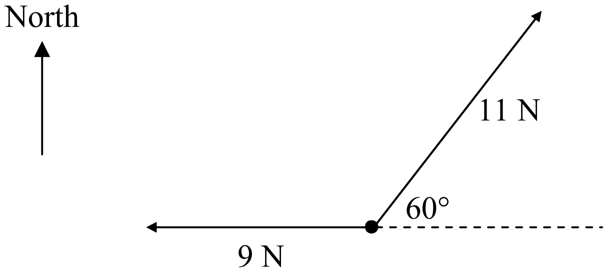


How many metres is the particle from its starting point after 25 s?

A. 6
B. 14
C. 30
D. 46
E. 50

Question 20

Two forces act simultaneously on a particle, as shown in the diagram below. One force of 9 N acts due west and another force of 11 N acts at an angle of 60° in an anticlockwise direction from due east.

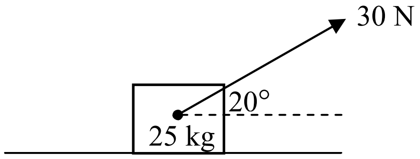


Correct to the nearest degree, the resultant force acting in an anticlockwise direction from due east will be

A. 50°
B. 70°
C. 110°
D. 120°
E. 130°

Question 21

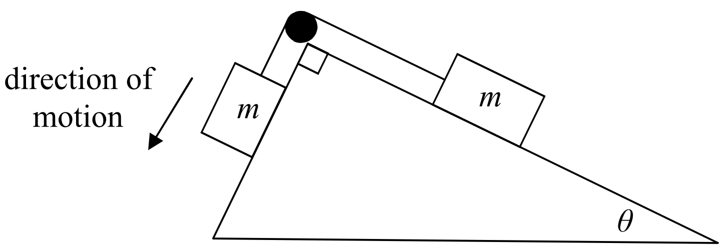
A mass of 25 kg is pulled across a smooth horizontal surface by a force of 30 newtons acting at and angle of 20° to the horizontal level.



The magnitude of the normal reaction of the surface on the mass, in newtons, is closest to

A. 215
B. 217
C. 235
D. 245
E. 255

Question 22



Two bodies each of mass, m kg, are connected on a back-to-back plane by a light string passing over a smooth pulley, as shown in the diagram. The coefficient of friction of the surface of each plane is μ.

If the system is on the point of moving in the direction shown, the value of μ will be

A. $\tan(\theta)$

B. $\cot(\theta)$

C. $\sin(\theta)-\cos(\theta)$

D. $\dfrac{\sin(\theta)-\cos(\theta)}{\sin(\theta)+\cos(\theta)}$

E. $\dfrac{\cos(\theta)-\sin(\theta)}{\cos(\theta)+\sin(\theta)}$

END OF SECTION A
TURN OVER

SECTION 2

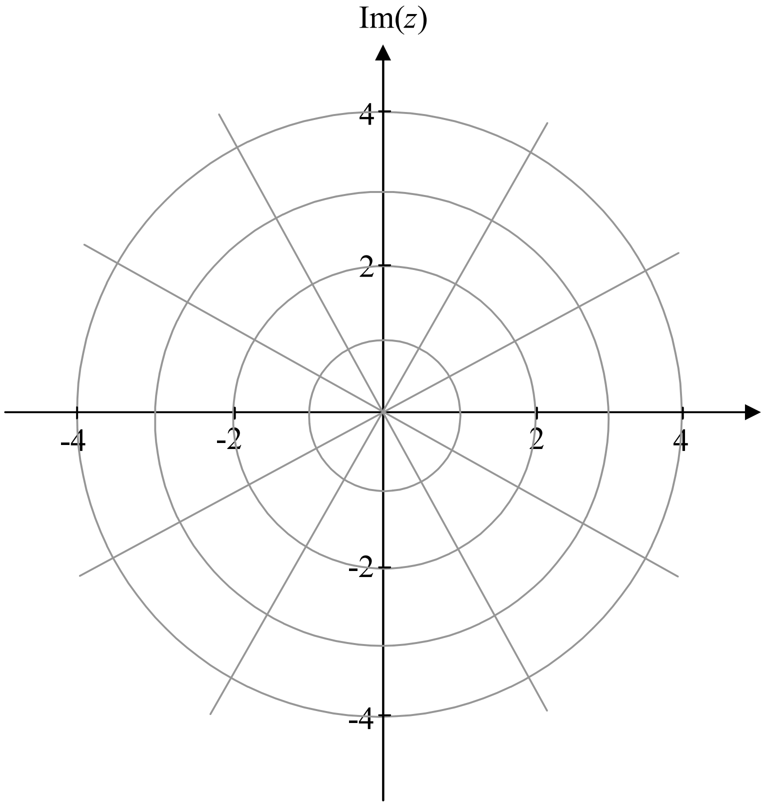
Instructions for Section 2

Answer all the questions in the spaces provided.
A decimal approximation will not be accepted if an exact answer is required to a question.
In questions where more than one mark is available, approximate working must be shown.
Unless otherwise indicated, the diagrams in this book have not been drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$

Question 1

Let $u = 2\text{cis}\left(\frac{\pi}{3}\right)$.

a. **i.** u, $\bar{u}$ and v are solutions of the equation $\{z : z^3 = k,\ z \in C\}$.
Plot u, $\bar{u}$ and v on the Argand plane below.



3 marks

ii. Determine the value of k.

___

1 mark

b. Show that $u \in \left\{z : \text{Re}(z) + \sqrt{3}\,\text{Im}(z) = 4\right\}$.

2 marks

c. **i.** Sketch $\left\{z : \text{Re}(z) + \sqrt{3}\,\text{Im}(z) = 4\right\}$ on the Argand plane given in part **a**, showing the exact intercepts with the axes.

1 mark

ii. **Hence,** shade the region represented by

$$\left\{z : \frac{\pi}{6} \le \text{Arg}(z) \le \frac{\pi}{3}\right\} \cap \left\{z : \text{Re}(z) + \sqrt{3}\,\text{Im}(z) \le 4\right\}.$$

1 mark

iii. Calculate the exact area of this region.

3 marks

Total 3 + 1 + 2 + 1 + 1 + 3 = 11 marks

Question 2

Given $f: R \to R,\ f(x) = \frac{x}{x^2+2} + 1$

a. **i.** Show that $f'(x) = \frac{2-x^2}{(x^2+2)^2}$.

1 mark

ii. **Hence,** determine the exact coordinates of any stationary points of f.

2 marks

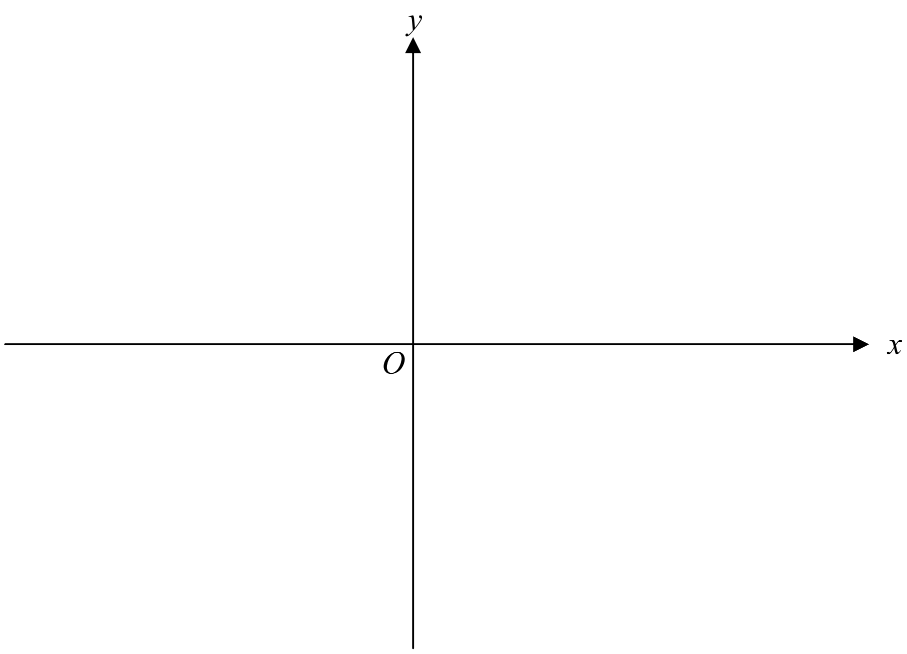
b. **i.** Use calculus to show that f has points of inflexion.

2 marks

ii. Find the exact coordinates of all points of inflexion and explain what each point represents.

2 marks

c. Sketch a graph of f on the axes below, labelling its features clearly.

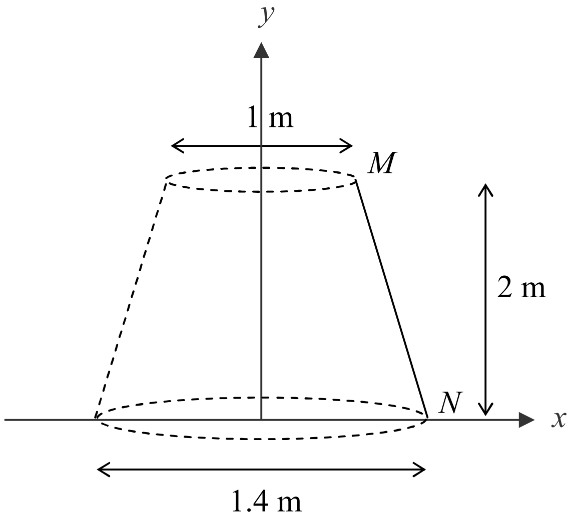


2 marks
Total 1 + 2 + 2 + 2 + 2 = 9 marks

SECTION B – continued
TURN OVER

Question 3

The tank shown below has the shape of a truncated cone with base diameter 1.4 m, top diameter 1 m and height 2 m. It may be modelled by rotating the line segment *MN* around the *y*-axis.



a. The line segment *MN* has equation $ax + by + c = 0$. Show that $a = 10$, $b = 1$ and $c = -7$.

2 marks

b. **i.** Show that the volume of the tank is $\frac{10\pi}{3}\left((h-7)^3+343\right)$ L, where h is the vertical height to which the tank is filled.

3 marks

ii. **Hence,** find the capacity of the tank when full. Write your answer correct to the nearest litre.

1 mark

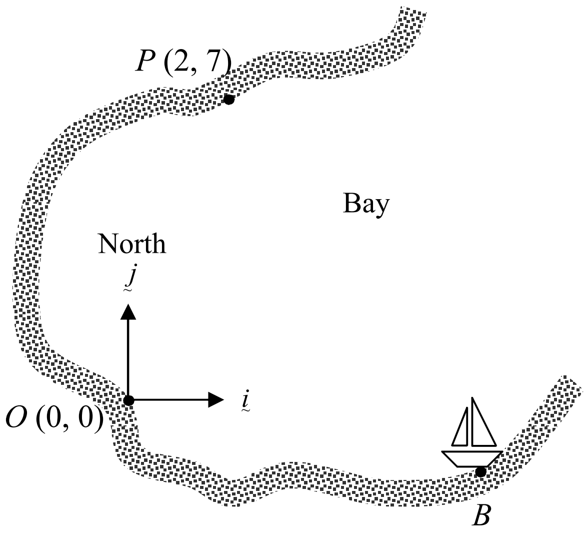
c. Suppose the tank is filled initially to a height of 1 m. Liquid then drains from a tap at the base of the tank at the rate of $2\sqrt{h}$ m^3/h. Use calculus to determine how long it takes before the tank is empty. Write your answer correct to the nearest minute.

5 marks

Total 2 + 3 + 1 + 5 = 11 marks

Question 4

A yacht, initially at point *B*, will sail to point *P* (2, 7) on the other side of the bay. Distances are measured in kilometres in relation to the origin, *O*.



a. Write a vector $\overrightarrow{OP}$ that gives the position of point *P*.

1 mark

The yacht leaves *B*, sailing with a velocity of $\underset{\sim}{v} = -2\underset{\sim}{i} + 2t\,\underset{\sim}{j}$ km/h. It reaches *P* after 3 h.

b. **i.** Determine the speed of the yacht when it reaches *P*.
Write your answer correct to 1 decimal place.

1 mark

ii. Show that the yacht's position at any time t hours is given by the vector $\underset{\sim}{r} = (8 - 2t)\underset{\sim}{i} + (t^2 - 2)\underset{\sim}{j}$.

2 marks

c. Write a vector $\overrightarrow{OB}$ that gives the position of point B.

1 mark

d. Determine $\angle BOP$ in degrees, correct to 1 decimal place.

2 marks

e. How long after the yacht commences sailing will it be on a bearing north-east of *O*? Write your answer in hours, correct to 2 decimal places.

2 marks

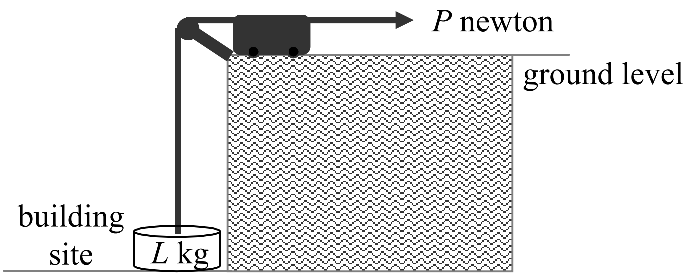
f. What is the closest distance the yacht comes to *O* when sailing from *B* to *P*? Give an exact answer.

3 marks

Total 1 + 1 + 2 + 1 + 2 + 2 + 3 = 12 marks

Question 5

A wire rope passing over a smooth pulley is used to transport materials to and from a building site situated below ground level. A 1 tonne engine at ground level applies a horizontal force of P newton along a track to pull a load of L kg upwards. The coefficient friction between the engine and the track is 0.25.



a. The load, L kg, is on the point of moving.

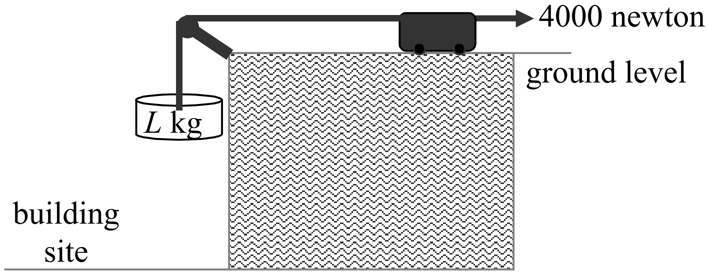
i. On the diagram above, show all forces acting.

1 mark

ii. Find P in terms of L.

4 marks

b. The engine exerts a horizontal force of 4000 newton. It pulls the load vertically upwards with an acceleration of 0.3 m/s^2.



i. Find the mass of load, L, in kg, correct to 1 decimal place.

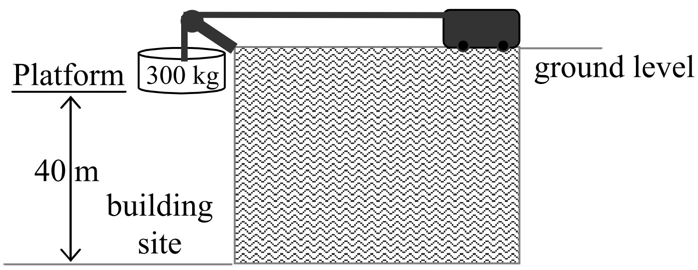
3 marks

ii. Determine the time taken, in seconds, to raise the load from rest to a point 15 m above the level of the building site.

1 mark

When the engine reaches the end of the track it applies its breaks so that the load will remain stationary at the loading platform.

The diagram below shows a 300 kg load suspended at the loading platform 40 m above the building site. It is stationary.



c. Find the minimum force that the engine's breaks need to apply in order to keep the 300 kg load stationary in this position.

3 marks

SECTION B – continued

d. The brakes are released when a load is to be lowered to the building site and the engine moves backwards from rest.

Unfortunately, a problem occurs when the 300 kg load is being lowered. At a point 30 m above the building site, the wire rope breaks and the load falls downwards. Find the speed of the load when it hits the building site. Write your answer in m/s, correct to 1 decimal place.

3 marks

Total 1 + 4 + 3 + 1 + 3 + 3 = 15 marks

END OF QUESTION AND ANSWER BOOK