

INSIGHT

Trial Exam Paper

2007

SPECIALIST MATHEMATICS

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 11 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

This trial examination produced by Insight Publications is NOT an official VCAA paper for the 2007 Specialist Mathematics written examination 1.

This examination paper is licensed to be printed, photocopied or placed on the school intranet and used only within the confines of the purchasing school for examining their students. No trial examination or part thereof may be issued or passed on to any other party including other schools, practising or non-practising teachers, tutors, parents, websites or publishing agencies without the written consent of Insight Publications.

Copyright © Insight Publications 2007

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation **will not** be accepted if an exact answer is required.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, diagrams in this book **are not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

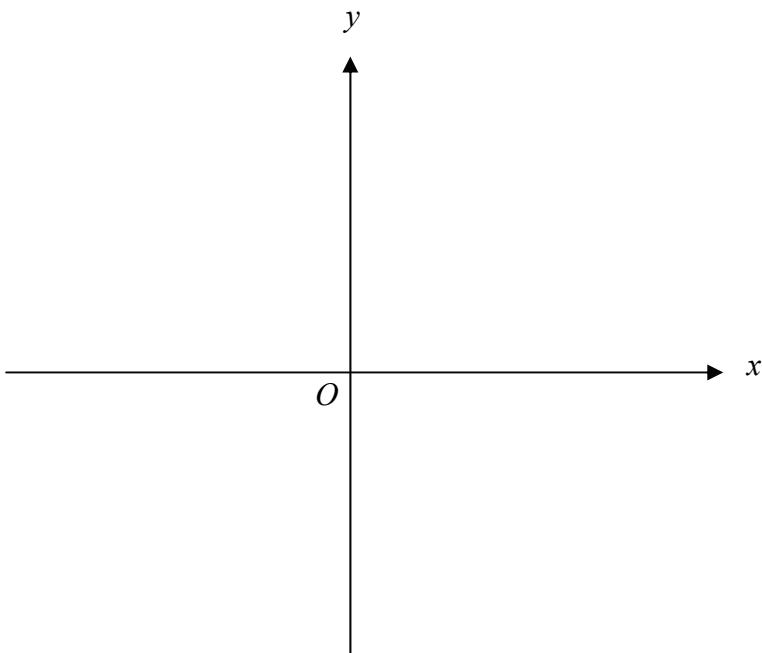
Question 1

Let $x = \sqrt{t+4}$ and $y = 1 - t$ for $-4 \leq t \leq 4$.

- a. Find the Cartesian equation of the curve.

2 marks

- b. Sketch a graph of the curve, showing all features clearly.



2 marks

Question 2

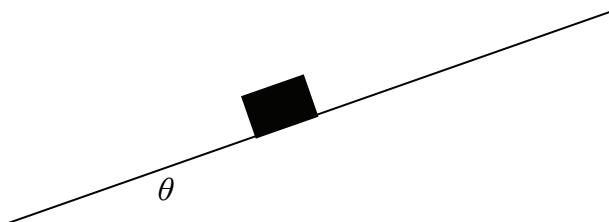
Express $(\sqrt{3} - i)^7$ in the form $x + iy$ where $x, y \in R$.

4 marks

Question 3

A 10 kg mass is pulled up a rough plane inclined at an angle of θ to the horizontal by a force of 120 newtons acting parallel to the plane.

The coefficient of friction between the mass and the plane is $\frac{1}{3}$, $\cos(\theta) = \frac{3}{5}$ and the acceleration due to gravity is $g \text{ m/s}^2$.



- a. Show all forces acting on the mass on the diagram above.

1 mark

- b. Find the acceleration of the mass up the plane in terms of g .

4 marks

Question 4

- a. Show that $\frac{\sin(x)}{1-\cos(x)} = \cot\left(\frac{x}{2}\right)$.

2 marks

- b. Hence or otherwise, solve the equation $\sin(x) = \cos(x) - 1$ over $0 \leq x \leq 2\pi$.

2 marks

Question 5

The position of a particle at time t seconds, $t \geq 0$, is given by the vector $\mathbf{r} = t\mathbf{i} + (1 - 2t)\mathbf{j} + (t - 6)\mathbf{k}$. Find the time when the particle's velocity vector is perpendicular to its position vector.

3 marks

Question 6

Consider the relation $xy + \frac{y^2}{x} = 2$.

- a. Find an expression for $\frac{dy}{dx}$ in terms of x and y .

3 marks

- b. Hence find the equations of the tangents to the curve when $x = 1$.

3 marks

TURN OVER

Question 7

$$f : D \rightarrow R, \quad f(x) = \arccos\left(\frac{1}{\sqrt{x}}\right)$$

- a. Determine the domain D of function f .

1 mark

- b. Find $f'(x)$.

3 marks

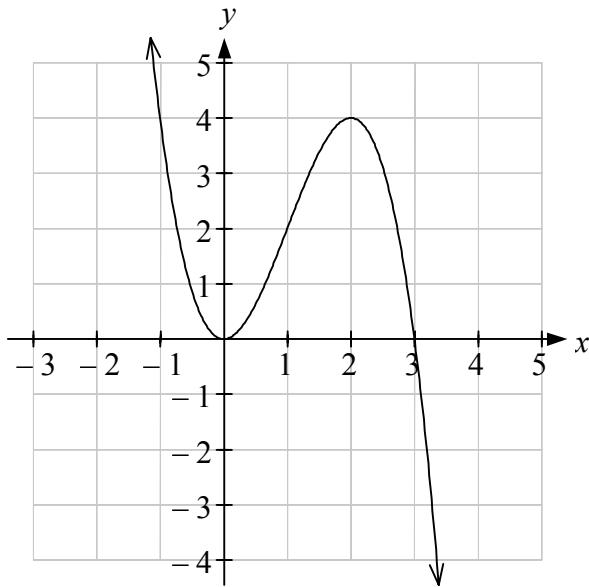
Question 8

Solve the differential equation $\frac{dt}{dx} = \frac{t^2 + 3}{t^2}$ given $x = 1$ when $t = 1$.

3 marks

Question 9

The graph of $f(x) = 3x^2 - x^3$ is shown on the axes below.



- a. Draw the graph of $g(x) = \frac{1}{3x^2 - x^3}$ on the axes above, showing all features clearly.

2 marks

- b. Given $\frac{1}{3x^2 - x^3} = \frac{Ax + B}{x^2} + \frac{C}{3 - x}$, find the exact values of A , B , and C .

2 marks

- c. Find the exact area between the graph of $g(x) = \frac{1}{3x^2 - x^3}$, the x -axis and the lines $x = 1$ and $x = 2$.

3 marks

END OF QUESTION AND ANSWER BOOK