VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2006

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

SPECIALIST MATHEMATICS

Written examination 2

Monday 30 October 2006

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 23 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The hyperbola with equation $\frac{(x-2)^2}{9} - \frac{(y-1)^2}{4} = 1$ has asymptotes given by

A. $2x + 3y = 1$, and $2x - 3y = 7$
B. $3x + 2y = 1$, and $3x - 2y = -1$
C. $2x - 3y = 1$, and $2x + 3y = 7$
D. $3y - 2x = 7$, and $2y + 3x = 1$
E. $4x - 9y = 5$, and $4x + 9y = 13$

Question 2

The graph of the function with rule $f(x) = \frac{1}{(x-4)(x+2)}$ over its maximal domain has

A. asymptotes $x = -4$ and $x = 2$ and a turning point at $(1, -5)$

B. asymptotes $x = -4$ and $x = 2$ and a turning point at $\left(-1, -\frac{1}{5}\right)$

C. asymptotes $x = -4$ and $x = 2$ and a turning point at $\left(1, -\frac{1}{9}\right)$

D. asymptotes $x = 4$ and $x = -2$ and a turning point at $(1, -9)$

E. asymptotes $x = 4$ and $x = -2$ and a turning point at $\left(1, -\frac{1}{9}\right)$

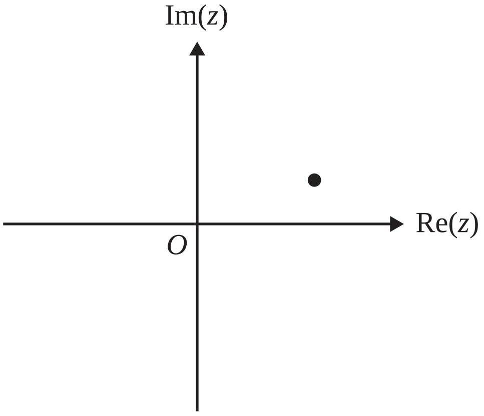
Question 3

The position vector of a particle at time $t \geq 0$ is given by $\underset{\sim}{r} = (1+t)\underset{\sim}{i} + (1-t)\underset{\sim}{j}$.
The path of the particle has equation

A. $y = x - 2$
B. $y = x + 2$
C. $y = -x - 2$
D. $y = -x + 2$
E. $y = x - 1$

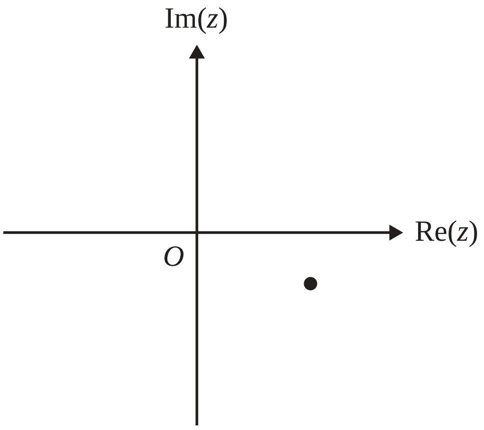
Question 4

The complex number $a + bi$, where a and b are real constants, is represented in the following diagram.

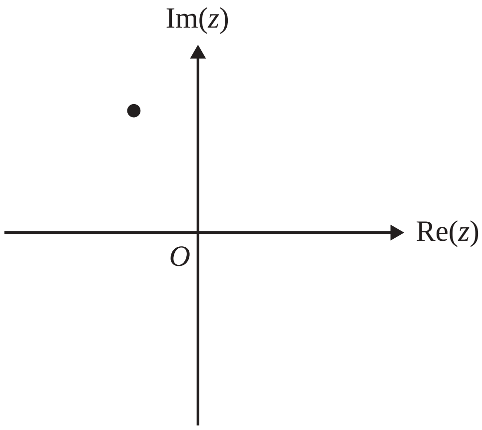


All axes below have the same scale as in the diagram above. The complex number $-i\,(a + bi)$ could be represented by

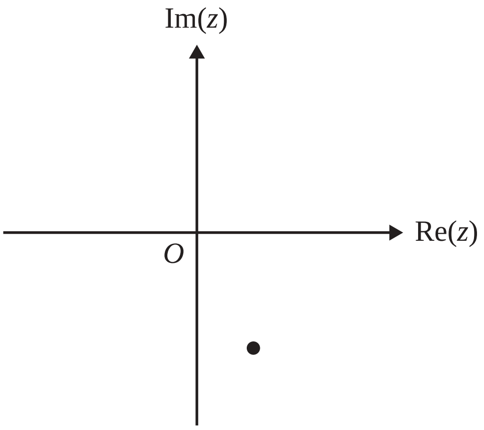
A.



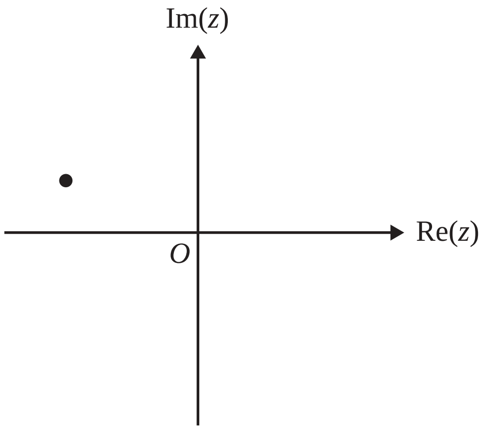
B.



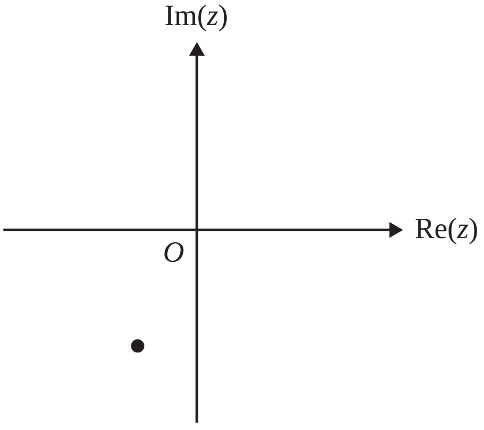
C.



D.



E.



Question 5

One of the complex solutions to $z^5 = -a$, where a is a positive real constant, is $a^{\frac{1}{5}}\text{cis}\left(\frac{\pi}{5}\right)$.
One of the other solutions is a real number and is equal to

A. $a^{\frac{1}{5}}\text{cis}\left(\frac{3\pi}{5}\right)$

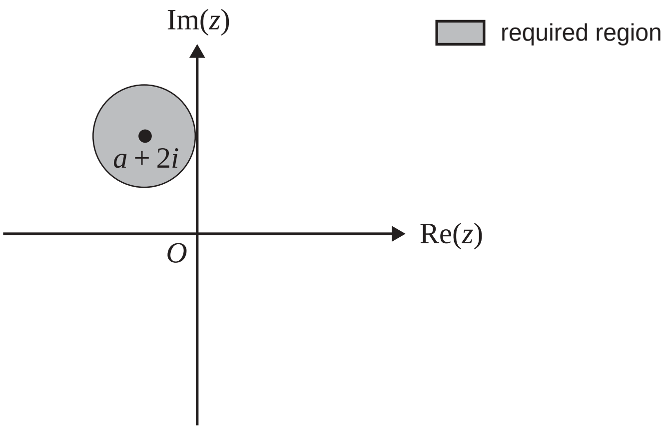
B. $-a^{\frac{1}{5}}$

C. $a^{\frac{1}{5}}$

D. $a^{\frac{1}{5}}\text{cis}\left(\frac{7\pi}{5}\right)$

E. $a^{\frac{1}{5}}\text{cis}\left(\frac{9\pi}{5}\right)$

Question 6



The region represented on the above Argand diagram, where a is a real constant, could be defined by

A. $|z-(a+2i)| \geq 1$

B. $|z-(a+2i)| \leq 1$

C. $|z-(-a+2i)| \leq 1$

D. $|z-(a+2i)| \leq 2$

E. $|z+a-2i| \geq 1$

Question 7

Which one of the following relations does **not** have a graph that is a straight line passing through the origin?

A. $z + \bar{z} = 0$

B. $3\ \text{Re}(z) = \text{Im}(z)$

C. $z = i\bar{z}$

D. $\text{Re}(z) - 2\ \text{Im}(z) = 0$

E. $\text{Re}(z) + \text{Im}(z) = 1$

Question 8

The slope of the curve $2x^3 - y^2 = 7$ at the point where $y = -3$ is

A. -4

B. -2

C. 2

D. 4

E. $\frac{27}{2}$

Question 9

Using a suitable substitution, $\int_a^b x(x^2+1)^5\, dx$ is equal to

A. $\frac{1}{2}\int_a^b u^5\, du$

B. $2\int_a^b u^5\, du$

C. $\frac{1}{2}\int_{a^2+1}^{b^2+1} u^5\, du$

D. $2\int_{a^2+1}^{b^2+1} u^5\, du$

E. $\frac{1}{12}b^6 - \frac{1}{12}a^6$

Question 10

A chemical dissolves in a pool at a rate equal to 5% of the amount of **undissolved** chemical. Initially the amount of undissolved chemical is 8 kg and after t hours x kilograms has dissolved.

The differential equation which models this process is

A. $\frac{dx}{dt} = \frac{x}{20}$

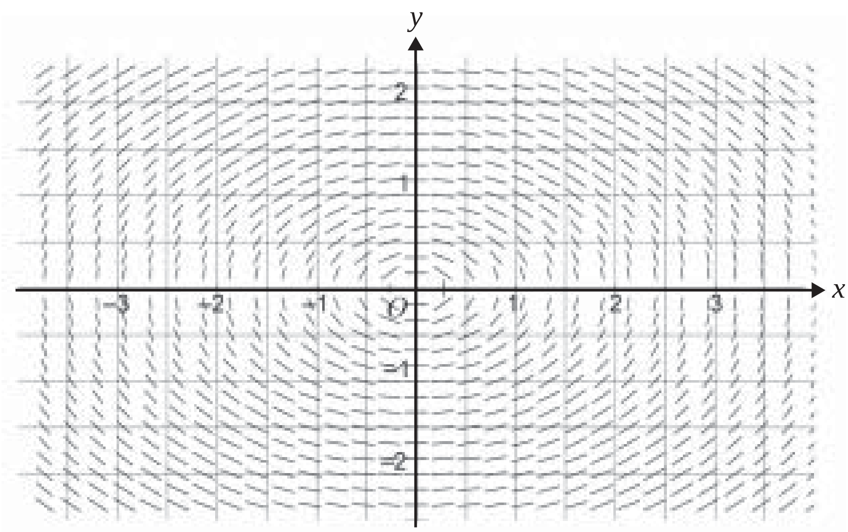
B. $\frac{dx}{dt} = \frac{8-x}{20}$

C. $\frac{dx}{dt} = \frac{x-8}{20}$

D. $\frac{dx}{dt} = -\frac{x}{20}$

E. $\frac{dx}{dt} = 8 - \frac{x}{20}$

Question 11



The direction (slope) field for a certain first order differential equation is shown above.

The differential equation could be

A. $\frac{dy}{dx} = \frac{x^2}{2} + y^2$

B. $\frac{dy}{dx} = x^2 + \frac{y^2}{2}$

C. $\frac{dy}{dx} = -\frac{x}{2y}$

D. $\frac{dy}{dx} = -\frac{y}{2x}$

E. $\frac{dy}{dx} = \frac{x}{2y}$

Question 12

A particle moves in a straight line such that its velocity v is given by $v = \sin(2x)$, when at a displacement x from the origin O.

The acceleration of the particle is given by

A. $2\cos(2x)$

B. $\sin(2x)\cos(2x)$

C. $-\frac{1}{2}\cos(2x)$

D. $\sin(4x)$

E. $2\cos(x)$

Question 13

Two particles, R and S, have position vectors $\underset{\sim}{r} = (2t-10)\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{s} = 2\underset{\sim}{i} + (t-1)\underset{\sim}{j}$ respectively at time t seconds, $t \geq 0$.

Then

A. R and S are in the same position when $t = 1$.

B. R and S are in the same position when $t = 4$.

C. R and S are in the same position when $t = 5$.

D. R and S are in the same position when $t = 6$.

E. R and S are never in the same position.

Question 14

The position vector of a particle at time t seconds, $t \geq 0$, is given by $\underset{\sim}{r}(t) = (3-t)\underset{\sim}{i} - 6\sqrt{t}\,\underset{\sim}{j} + 5\underset{\sim}{k}$.

The direction of motion of the particle when $t = 9$ is

A. $-6\underset{\sim}{i} - 18\underset{\sim}{j} + 5\underset{\sim}{k}$

B. $-\underset{\sim}{i} - \underset{\sim}{j}$

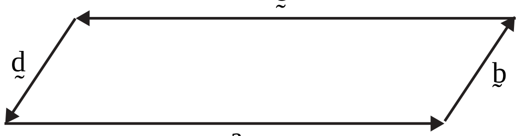
C. $-6\underset{\sim}{i} - \underset{\sim}{j}$

D. $-\underset{\sim}{i} - \underset{\sim}{j} + 5\underset{\sim}{k}$

E. $-13.5\underset{\sim}{i} - 108\underset{\sim}{j} + 45\underset{\sim}{k}$

Question 15

In the parallelogram shown, $|\underset{\sim}{a}| = 2|\underset{\sim}{b}|$.



Which one of the following statements is true?

A. $\underset{\sim}{a} = 2\underset{\sim}{b}$

B. $\underset{\sim}{a} + \underset{\sim}{b} = \underset{\sim}{c} + \underset{\sim}{d}$

C. $\underset{\sim}{b} - \underset{\sim}{d} = \underset{\sim}{0}$

D. $\underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{0}$

E. $\underset{\sim}{a} - \underset{\sim}{b} = \underset{\sim}{c} - \underset{\sim}{d}$

Question 16

A unit vector perpendicular to $5\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ is

A. $\frac{1}{4}\left(5\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}\right)$

B. $2\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}$

C. $\frac{1}{29}\left(2\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}\right)$

D. $\frac{1}{\sqrt{29}}\left(2\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}\right)$

E. $\frac{1}{\sqrt{30}}\left(5\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}\right)$

Question 17

Let $\underset{\sim}{u} = \underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{v} = \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$.

The angle between the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is

A. 0°

B. 45°

C. 30°

D. 22.5°

E. 90°

Question 18

A block of mass 20 kg lies on a plane inclined at an angle of 60° to the horizontal.

The normal reaction force of the plane on the block is R newtons. R is equal to

A. $10\,g$

B. $10\sqrt{3}\,g$

C. $\dfrac{g}{2}$

D. $\dfrac{\sqrt{3}\,g}{2}$

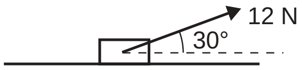
E. $20\,g$

Question 19

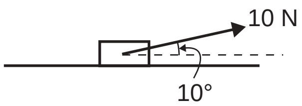
A block of mass 10 kg is pulled along a smooth horizontal plane by a force.

Under which one of the following sets of conditions will the mass have the largest acceleration?

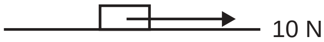
A.



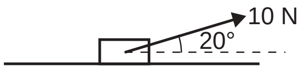
B.



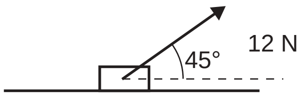
C.



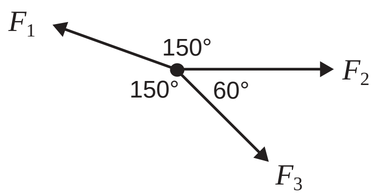
D.



E.



Question 20



If three co-planar forces, F_1, F_2 and F_3, act on a particle which is in equilibrium as shown in the above diagram, then

A. $F_1 = F_2 = \sqrt{3}\,F_3$

B. $F_2 = F_3 = \sqrt{3}\,F_1$

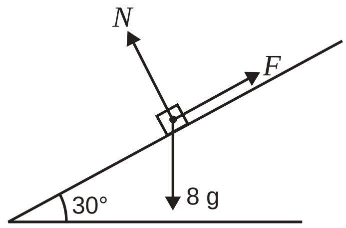
C. $F_1 = F_3 = \sqrt{3}\,F_2$

D. $F_2 = F_3 = \dfrac{\sqrt{3}}{3}\,F_1$

E. $F_1 = F_3 = \dfrac{\sqrt{3}}{3}\,F_2$

Question 21

A block of mass 8 kg is at rest on a plane inclined at an angle of 30° to the horizontal.



In the diagram, N newtons is the normal reaction of the plane on the block, and F newtons is the frictional force on the block up the plane.

For equilibrium to be maintained, the coefficient of friction between the plane and the block must be

A. at least $\dfrac{1}{\sqrt{3}}$

B. less than $\dfrac{1}{\sqrt{3}}$

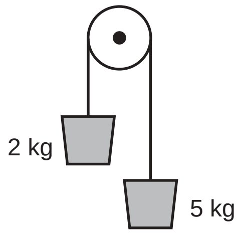
C. at least $\dfrac{1}{g\sqrt{3}}$

D. less than $\dfrac{1}{g\sqrt{3}}$

E. less than $\dfrac{g}{\sqrt{3}}$

Question 22

A light inextensible string passes over a smooth pulley. Particles of mass 5 kg and 2 kg are attached to each end of the string, as shown.



The acceleration of the 5 kg mass downwards is

A. $3g$

B. $\dfrac{5g}{7}$

C. $\dfrac{2g}{7}$

D. $\dfrac{5g}{3}$

E. $\dfrac{3g}{7}$

END OF SECTION 1
TURN OVER

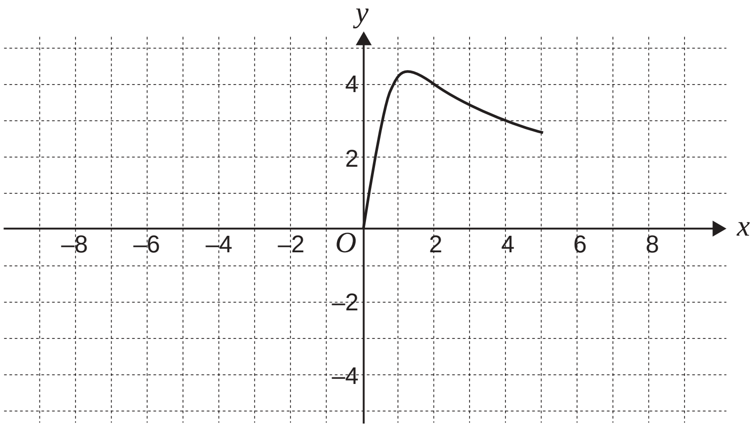
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The top part of a wine glass, while lying on its side, is constructed by rotating the graph of $y = \dfrac{6x}{\sqrt{1+x^3}}$ from $x = 0$ to $x = 5$ about the x-axis as shown below. All lengths are measured in centimetres.



a. Write down a definite integral which represents the volume, V cm^3, of the glass.

2 marks

b. Use the substitution $u = 1 + x^3$ to write down a definite integral which represents the volume of the glass in terms of u.

2 marks

c. Find the value of V correct to the nearest cm^3.

1 mark

At time $t = 0$ seconds wine begins to be poured into the upright glass so that its depth (x cm in the graph opposite) is increasing at a rate of 2 cm/sec.

d. Given that $\dfrac{dy}{dx} = \dfrac{6 - 3x^3}{(1 + x^3)^{\frac{3}{2}}}$, find an expression for $\dfrac{dy}{dt}$, the rate of change of the radius of the surface of the wine with respect to time, in terms of x.

1 mark

e. **Hence** find an expression for the rate of change of the area, A cm^2, of the surface of the wine in the upright glass with respect to t, in terms of x.

Give your answer in the form $\dfrac{dA}{dt} = \dfrac{ax^b(6 - 3x^3)}{(1 + x^3)^c}$ where a, b and c are constants.

3 marks

f. Find the exact value of the depth of the wine for which the area of its surface, A cm^2, is a maximum.

1 mark

Total 10 marks

Question 2

Point A has position vector $\underset{\sim}{a} = -\underset{\sim}{i} - 4\underset{\sim}{j}$, point B has position vector $\underset{\sim}{b} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$, point C has position vector $\underset{\sim}{c} = 5\underset{\sim}{i} - 4\underset{\sim}{j}$, and point D has position vector $\underset{\sim}{d} = 2\underset{\sim}{i} + 5\underset{\sim}{j}$ relative to the origin O.

a. Show that $\overrightarrow{AC}$ and $\overrightarrow{BD}$ are perpendicular.

2 marks

b. Use a vector method to find the cosine of $\angle ADC$, the angle between $\overrightarrow{DA}$ and $\overrightarrow{DC}$.

3 marks

c. Find the cosine of $\angle ABC$, and **hence** show that $\angle ADC$ and $\angle ABC$ are supplementary.

2 marks

Point P has position vector $\underset{\sim}{p} = 2\underset{\sim}{i}$.

d. Use the cosine of $\angle APC$ and an appropriate trigonometric formula to prove that $\angle APC = 2\angle ADC$.

3 marks

Total 10 marks

Question 3

a. A passenger jet of mass 48 000 kg moves from rest with constant acceleration along a runway due to a total thrust of 105 600 newtons supplied by its engines. Assume that air resistance and other frictional forces are negligible.

i. Show that the magnitude of the acceleration of the jet is 2.2 ms^{-2}.

ii. How many seconds, correct to one decimal place, does it take the jet to reach its lift-off speed of 70 ms^{-1}?

iii. What distance is needed, correct to the nearest metre, for the jet to take off?

1 + 2 + 2 = 5 marks

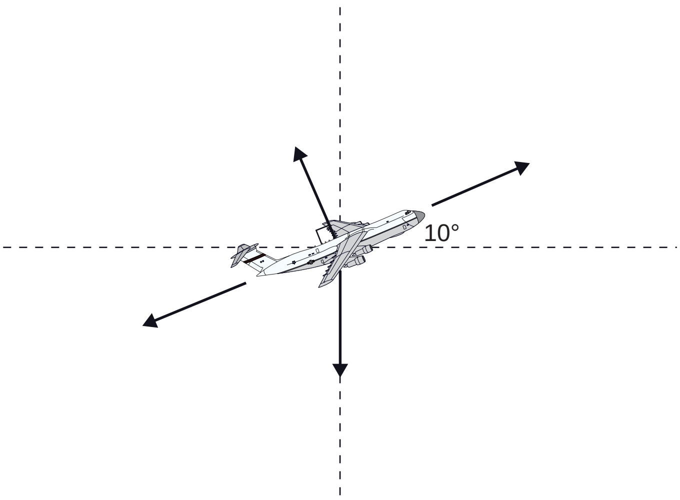
b. After lift-off the pilot eases back the thrust of the engines to 85 000 newtons and the plane climbs at an angle of 10° to the horizontal direction at a constant velocity for a short time. During this stage of the ascent the jet is subject to the following forces, all measured in newtons.

the thrust T of the engines

the lifting force L supplied by the wings

the weight force W

the drag on the plane R due to air resistance



i. On the diagram above, label clearly the forces acting on the jet.

ii. By resolving forces into perpendicular components, write down a pair of equations which would enable R and L to be found.

iii. Find L, the lift supplied by the wings, correct to the nearest newton.

1 + 2 + 1 = 4 marks

c. After a short time, the jet touches down on the runway with a horizontal speed of 80 ms^{-1}. The speed of the jet as it slows down is v ms^{-1} at time t seconds after touchdown. The jet is slowed by a reverse thrust of 80 000 newtons supplied by the engines, a force of $5v^2$ newtons, where v is the speed of the jet in ms^{-1}, supplied by the braking effect of the wing flaps and other frictional forces, and a force of $500(80–v)$ newtons supplied by the braking of the wheels.

i. Assuming that the mass of the jet is unchanged, write down the equation of motion of the jet while it is being slowed by the three forces listed above.

ii. **Hence** write down a definite integral which gives the distance the jet takes to slow down to a speed of 10 ms^{-1}.

iii. Find this distance, correct to the nearest metre.

1 + 3 + 1 = 5 marks

Total 14 marks

Question 4

A ball rolling along a horizontal plane has position vector $\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j}$, $t \geq 0$, $y > 0$ and velocity vector

$$\dot{\underset{\sim}{r}}(t) = \frac{1}{y(t)}\underset{\sim}{i} + \left(1 - y(t)\right)\underset{\sim}{j}.$$

a. The component of velocity in the $\underset{\sim}{j}$ direction gives the differential equation

$$\frac{dy}{dt} = 1 - y.$$

Use calculus to show that the solution to this differential equation is

$$\log_e |1 - y| = c - t,$$

where c is the constant of integration.

2 marks

b. Write down an equation for $\frac{dx}{dt}$ and **hence** use calculus to show that the gradient of the curve along which the ball rolls is given by $\frac{dy}{dx} = y(1 - y)$.

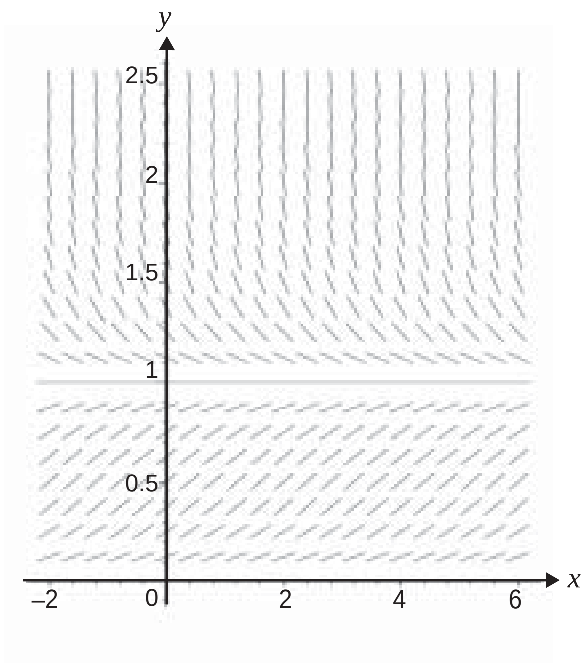
2 marks

c. **i.** Show that $\frac{d^2y}{dx^2} = (1 - 2y)y(1 - y)$.

ii. **Hence**, for $0 < y < 1$, find the y-coordinate of any points of inflection on the curve along which the ball rolls and verify that they are points of inflection.

1 + 2 = 3 marks

d. If the position of the ball at a particular time is given by $\underset{\sim}{r} = 0.5\underset{\sim}{j}$, sketch the path of the ball on the direction (slope) field below.



1 mark

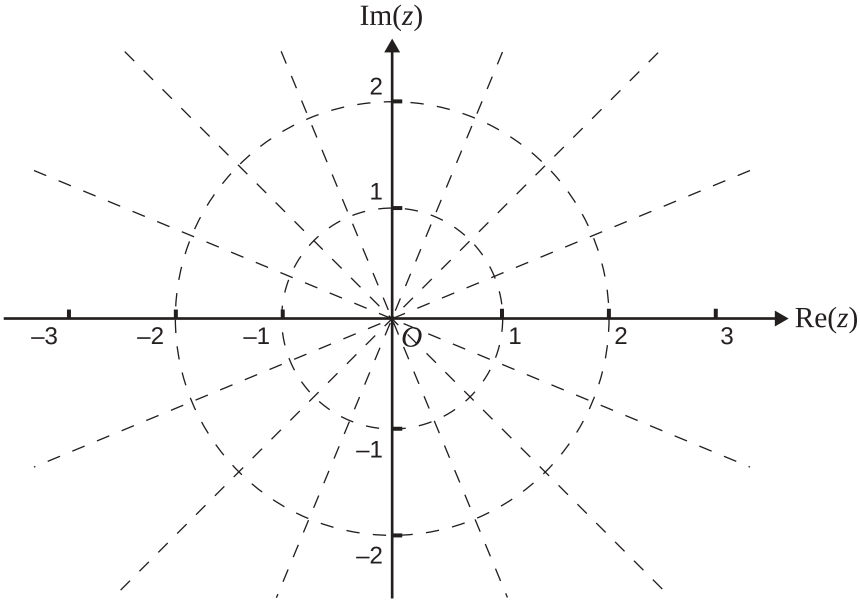
e. Given that $y = 2$ when $x = 0$, use Euler's method with a step-size of $\frac{1}{4}$ to estimate the value of y when $x = \frac{1}{2}$.

2 marks

Total 10 marks

Question 5

a. **i.** Let $z_1 = \text{cis}\left(\frac{\pi}{4}\right)$. Plot and label carefully the points $-z_1$, $\overline{z}_1$ and $-\overline{z}_1$ on the Argand diagram below.



ii. Write down the complex equation of the straight line which passes through the points z_1 and $-z_1$, in terms of $\overline{z}_1$.

2 + 1 = 3 marks

b. Use a double angle formula to show that the exact value of $\cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2+\sqrt{2}}}{2}$.
Explain why any values are rejected.

3 marks

c. **Hence** show that the exact value of $\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}$.

2 marks

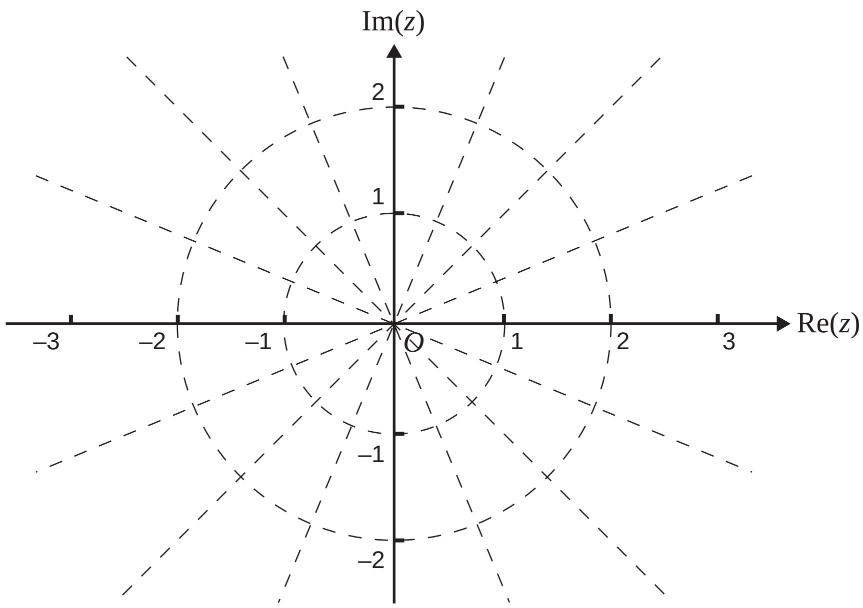
d. Evaluate $\left(\frac{\sqrt{2+\sqrt{2}}}{2} + \frac{\sqrt{2-\sqrt{2}}}{2}i\right)^7$, giving your answer in polar form.

2 marks

e. For what values of n is $\left(\frac{\sqrt{2+\sqrt{2}}}{2} + \frac{\sqrt{2-\sqrt{2}}}{2}i\right)^n$ a real number?

2 marks

f. Plot the roots of $z^8 = 1$ on the Argand diagram below.



2 marks

Total 14 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \text{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r \operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad \int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\qquad \int \dfrac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\qquad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\qquad \int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u+v)t$

TURN OVER

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

$$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET