THE SCHOOL FOR EXCELLENCE (TSFX)
UNIT 4 SPECIALIST MATHEMATICS 2006

WRITTEN EXAMINATION 1

Reading Time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
8	8	40

Students are permitted to bring into the examination rooms: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are **NOT** permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Students are **NOT** permitted to bring mobile phones and/or any electronic communication devices into the examination room.

All written responses must be in English.

COMPLIMENTS OF THE SCHOOL FOR EXCELLENCE

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Instructions

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- In questions where more than 1 mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the acceleration due to gravity to have magnitude $g\ m/s^2$ where $g = 9.8$

QUESTION 1

The points $A(-2, 4)$, $B(6, -2)$, $C(5, 5)$ are the vertices of triangle ΔABC and D is the mid-point of AB.

a. Find $\overrightarrow{CD}$.

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2 marks

b. Hence show that CD is perpendicular to AB.

2 marks

Total 4 marks

QUESTION 2

The region enclosed by the x-axis and the curve with equation $y = x - x^2$ is rotated about the x-axis to form a solid of revolution. Express the volume of this solid as a definite integral and **hence** find the exact volume of this solid.

2 marks

QUESTION 3

Consider the function f with rule $f(x) = \log_e(2x + 4)$.

a. Find a rule for the inverse function f^{-1} of f.

1 mark

b. Write, but do not evaluate, a definite integral that gives the area enclosed by $f(x)$ and the coordinate axes.

1 mark

c. Hence find the exact value of the area enclosed by $f(x)$ and the coordinate axes.

3 marks

Total 5 marks

QUESTION 4

A particle moves along a curve defined by the position vector $\underset{\sim}{r} = (4\sin^3 t)\underset{\sim}{i} + (\cos(2t))\underset{\sim}{j}$,

where $0 \leq t \leq \frac{\pi}{4}$.

a. Write parametric equations for the particle's motion.

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_____

1 mark

b. Find $\frac{dy}{dx}$ in terms of t.

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2 marks

c. Hence find the Cartesian equation of the normal to the curve at the point when $t = \frac{\pi}{6}$.

3 marks

Total 6 marks

QUESTION 5

Oil is leaking from the bottom of a cylindrical tank with horizontal circular base. At time t minutes the depth of oil in the tank is h metres. It is known that $h = 10$ when $t = 0$ and that $h = 5$ when $t = 40$.

a. Sara assumes that the rate of change of h with respect to t is constant.

(i) Find an expression for h in terms of t under Sara's assumption.

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1 mark

(ii) Hence find the exact value of h under Sara's assumption when $t = 60$.

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1 mark

b. James assumes that the rate of change of h with respect to t is proportional to h.

(i) Find an expression for h in terms of t under Jame's assumption.

3 marks

(ii) Hence find in the form $\frac{a}{\sqrt{b}}$, where a and b are positive integers, the exact value of h under Jame's assumption when $t = 60$.

1 mark

Total 6 marks

QUESTION 6

Let $u = -2 + 2i$.

a. If $uw = 4i$, find w in Cartesian form.

2 marks

b. Express u and w in polar form.

2 marks

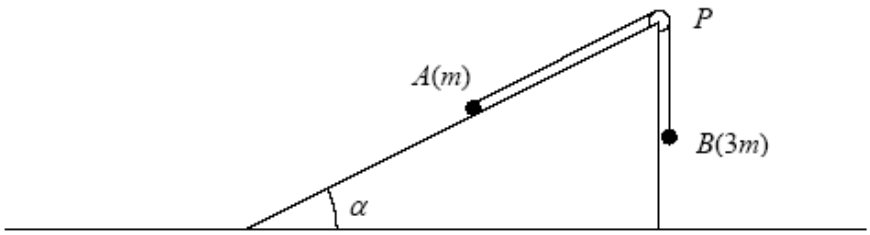
c. Find $a \in C$ so that $ua + 4i = \sqrt{2}cis\left(\frac{3\pi}{4}\right)$.

3 marks

Total 7 marks

QUESTION 7

Two particles *A* and *B* of masses m and $3m$ respectively are connected by a light inelastic string that passes over a smooth light pulley P as shown in the diagram. Particle *A* rests on a rough plane inclined at an angle α to the horizontal, where $\tan(\alpha) = \frac{3}{4}$. Particle B hangs freely below P. The particles are released from rest with the string taut and particle B moves downwards with an acceleration of magnitude $\frac{g}{2}$.



a. Find the tension in the string.

2 marks

b. Find the coefficient of friction between particle A and the plane.

3 marks

Total 5 marks

QUESTION 8

Use the substitution $x = \cos t$ to evaluate $\int_0^{\frac{1}{2}} \frac{x^2}{\left(1-x^2\right)^{\frac{1}{2}}}\, dx$.

5 marks

End of Paper