Year 2006

VCE

Specialist Mathematics Solutions Trial Examination 2

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SECTION 1

ANSWERS

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
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7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
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10	**A**	**B**	**C**	**D**	**E**
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14	**A**	**B**	**C**	**D**	**E**
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16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**

SECTION 1

Question 1 **Answer D**

The graphs of all alternatives give vertical asymptotes at $x = 0$, $\frac{\pi}{2}$ and π however option D is wrong, it is the reflection in the *x*-axis

Question 2 **Answer D**

$$f\left(g\left(x\right)\right) = \frac{1}{-x^2 - \left(a-b\right)x + ab} = \frac{1}{-\left(x^2 + \left(a-b\right) - ab\right)} = \frac{1}{-\left(x-b\right)\left(x+a\right)}$$

So there are vertical asymptotes at $x = -a$ and $x = b$

Question 3 **Answer A**

$$y = \frac{ax^2 + b}{x} = ax + \frac{b}{x} = ax + bx^{-1}$$

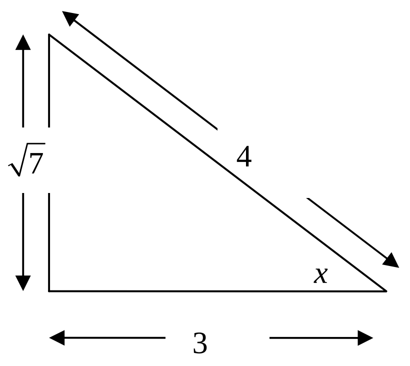
$\frac{dy}{dx} = a - bx^{-2} = a - \frac{b}{x^2}$ for turning points $\frac{dy}{dx} = 0$

$a = \frac{b}{x^2}$ $\quad x^2 = \frac{b}{a}$ $\quad x = \pm\sqrt{\frac{b}{a}}$ however there are no turning points, so there is no solutions for $x = \pm\sqrt{\frac{b}{a}}$ so a and b must have opposite signs the product $ab < 0$ so $a > 0$ and $b < 0$

Question 4 **Answer B**

$$\operatorname{cosec}\left(x\right) = \frac{1}{\sin\left(x\right)} = \frac{4\sqrt{7}}{7}$$

$$\sin\left(x\right) = \frac{7}{4\sqrt{7}} = \frac{\sqrt{7}}{4}$$



so from the triangles $\cos\left(x\right) = -\frac{3}{4}$ since $\frac{\pi}{2} < x < \pi$ x is obtuse and in the second quadrant $\sec\left(x\right) = \frac{1}{\cos\left(x\right)} = -\frac{4}{3}$

Question 5   **Answer E**

The ellipse $\frac{(y+1)^2}{18}+\frac{(x-4)^2}{8}=2$ in standard form is $\frac{(x-4)^2}{16}+\frac{(y+1)^2}{36}=1$

it has centre at $(4,-1)$ $a=4$ $b=6$ the domain is $4\pm 4=[0,8]$

and the range $-1\pm 6=[-7,5]$

Question 6   **Answer C**

Given $u=3\operatorname{cis}\left(\frac{\pi}{4}\right)$, $v=a\operatorname{cis}(b)$ and $\frac{u}{v}=\frac{3\operatorname{cis}\left(\frac{\pi}{4}\right)}{a\operatorname{cis}(b)}=\frac{3}{a}\operatorname{cis}\left(\frac{\pi}{4}-b\right)=-6=6\operatorname{cis}(\pi)$

so $\frac{3}{a}=6$ $a=\frac{1}{2}$ and $\frac{\pi}{4}-b=\pi$ $b=-\frac{3\pi}{4}$

Question 7   **Answer E**

$z=(\cos\theta+i\sin\theta)^3=\operatorname{cis}(3\theta)$ by DeMoivre's Theorem

$z^2=(\cos\theta+i\sin\theta)^6=\operatorname{cis}(6\theta)=\cos(6\theta)+i\sin(6\theta)$ and $z^2=a+bi$

Equating real and imaginary parts respectively gives $\cos(6\theta)=a$ and $\sin(6\theta)=b$

Question 8   **Answer C**

The circle has radius 2, one of the solutions is $u=2i$ and $u^3=8i^3=-8i$

but $P(u)=0$ so $P(z)=z^3+8i=0$

Question 9   **Answer A**

$P(z)=z^3+bz^2+cz+d$ and $P(-\alpha i)=0$ and $P(\beta)=0$ by the conjugate root theorem $P(\alpha i)=0$ so $z^2+\alpha^2$ is one factor, the other is $(z-\beta)$ expanding

$P(z)=(z^2+\alpha^2)(z-\beta)=z^3-\beta z^2+\alpha^2 z-\alpha^2\beta=z^3+bz^2+cz+d=0$ equating

$b=-\beta$ $c=\alpha^2$ $d=-\alpha^2\beta$

Question 10 **Answer C**

The inside of the circle of radius 4, can be represented by $\{z: |z| \le 4\}$ or $\{z: z\bar{z} \le 16\}$ or $\{z: \operatorname{Re}^2(z) + \operatorname{Im}^2(z) \le 16\}$ the upper half plane of the Argand diagram can be represented by $\{z: \operatorname{Im}(z) \ge 0\}$ or $\{z: 0 \le \operatorname{Arg}(z) \le \pi\}$ option C is the only correct intersection of all the possible correct representations.

Question 11 **Answer E**

When we have the repeated factor $(x+3)^2$ and the non-linear factor (x^2+3) we represent the expression $\dfrac{3x+1}{(x+3)^2(x^2+3)}$ in partial fractions as

$$\frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{Cx+D}{x^2+3}$$

Question 12 **Answer E**

$$\int_1^4 \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx = \int_1^4 \cos(\sqrt{x})\,\frac{1}{\sqrt{x}}\,dx$$

let $u = \sqrt{x} = x^{\frac{1}{2}}$ $\quad \dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$ $\qquad \dfrac{1}{\sqrt{x}}\,dx = 2\,du$

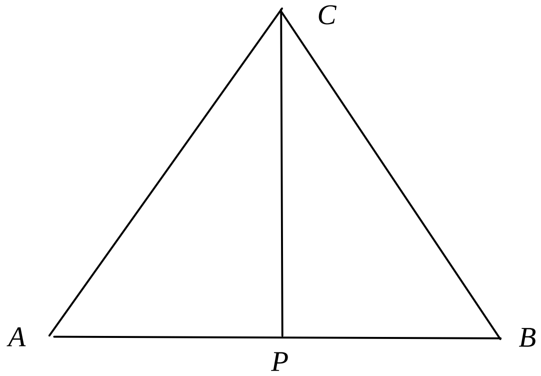
change the terminals when $x = 4$ $u = 2$ and when $x = 1$ $u = 1$

the definite integral becomes $2\int_1^2 \cos(u)\,du$

Question 13 **Answer D**

At $x = a$ and $x = e$ the gradient changes from negative to positive, this gives a minimum at the two points $x = a$ and $x = e$. At $x = c$ the gradient changes from positive to negative this gives a maximum at the point $x = c$. The graph of $F(x)$ is a quartic and will have two points of inflexions, one somewhere between $x = a$ and $x = c$ and another somewhere between $x = c$ and $x = e$.

Question 14   **Answer C**



Option A is true $\overrightarrow{AP}=\overrightarrow{PB}=\frac{1}{2}\overrightarrow{AB}$ since P is the mid-point of $\overrightarrow{AB}$

Option B is true since ABC is an equilateral triangle all side lengths are equal

$\left|\overrightarrow{AB}\right|=\left|\overrightarrow{BC}\right|=\left|\overrightarrow{AC}\right|$

Option C is false $\overrightarrow{AP}$ is perpendicular to $\overrightarrow{PC}$ so $\overrightarrow{AP}.\overrightarrow{PC}=0$

Option D is true the angle CAB is 60^0 $\cos\left(60^0\right)=\frac{1}{2}=\frac{\overrightarrow{AC}.\overrightarrow{AB}}{\left|\overrightarrow{AC}\right|\left|\overrightarrow{AB}\right|}$

Option E is true the angle PCB is 30^0 $\cos\left(30^0\right)=\frac{\sqrt{3}}{2}=\frac{\overrightarrow{CP}.\overrightarrow{CB}}{\left|\overrightarrow{CP}\right|\left|\overrightarrow{CB}\right|}$

Question 15   **Answer A**

$\underset{\sim}{a}=-2\underset{\sim}{i}+2\underset{\sim}{j}-\underset{\sim}{k}$ and $\underset{\sim}{b}=4\underset{\sim}{i}+y\underset{\sim}{j}+2\underset{\sim}{k}$ $\underset{\sim}{a}.\underset{\sim}{b}=-8+2y-2=2y-10$

$\left|\underset{\sim}{a}\right|=\sqrt{4+4+1}=3$ $\left|\underset{\sim}{b}\right|=\sqrt{16+y^2+4}=\sqrt{20+y^2}$

So the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is $\cos^{-1}\left(\frac{2y-10}{3\sqrt{\left(20+y^2\right)}}\right)$

If $y=-4$ then $-2\underset{\sim}{a}=\underset{\sim}{b}$ so the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are linearly **dependent**, option A is false

All the other alternatives are true.

Option C if $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$ then $2y-10=0$ $y=5$

Option D if $y>5$ then the angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is acute.

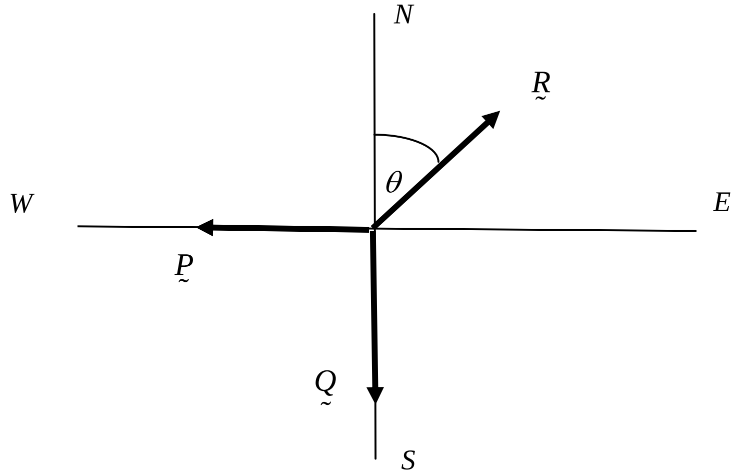
Option E if $y<5$ then the angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is obtuse.

Question 16 **Answer B**

The velocity vector is $\dot{r}(t) = 4e^{\frac{t}{2}}\underset{\sim}{i} - 2\sin\left(\frac{t}{2}\right)\underset{\sim}{j}$ integrating with respect to t to get the position vector $r(t) = \int 4e^{\frac{t}{2}}\,dt\,\underset{\sim}{i} - \int 2\sin\left(\frac{t}{2}\right)dt\,\underset{\sim}{j} = 8e^{\frac{t}{2}}\underset{\sim}{i} + 4\cos\left(\frac{t}{2}\right)\underset{\sim}{j} + \underset{\sim}{C}$ now using the initial conditions $r(0) = 3\underset{\sim}{i} - 3\underset{\sim}{j} = 8\underset{\sim}{i} + 4\underset{\sim}{j} + \underset{\sim}{C}$ so $\underset{\sim}{C} = -5\underset{\sim}{i} - 7\underset{\sim}{j}$ and

$$r(t) = \left(8e^{\frac{t}{2}} - 5\right)\underset{\sim}{i} + \left(4\cos\left(\frac{t}{2}\right) - 7\right)\underset{\sim}{j}$$

Question 17 **Answer A**



Option A is false if $\theta = 45$ then $P = Q = \frac{\sqrt{2}\,R}{2}$

All the other alternatives are true

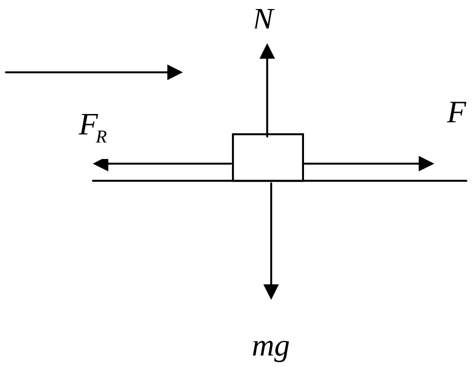
Option B $R^2 = P^2 + Q^2$

Option C $P = R\sin(\theta)$ and $Q = R\cos(\theta)$

Option D $\tan(\theta) = \frac{P}{Q}$

Option E as vectors $\underset{\sim}{P} + \underset{\sim}{Q} + \underset{\sim}{R} = \underset{\sim}{0}$

Question 18 **Answer E**



$m = 60\,\text{kg} \quad g = 9.8\,\text{m/s}^2 \quad \mu = 0.75$

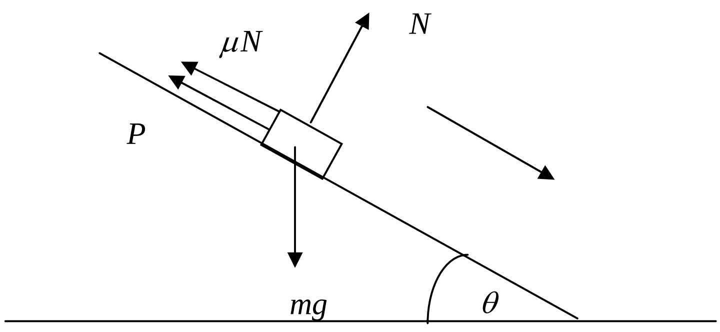
$N = mg = 588$ newtons

$\mu N = 441$ newtons

movement with constant acceleration

is only possible if $F > \mu N$

Question 19 **Answer B**



resolving the forces, perpendicular to the plane

$N - mg\cos(\theta) = 0 \;\Rightarrow\; N = mg\cos(\theta)$

resolving the forces up and parallel to the plane.

$P + \mu N - mg\sin(\theta) = 0$

$P + \mu mg\cos(\theta) - mg\sin(\theta) = 0$

$P = mg\sin(\theta) - \mu mg\cos(\theta)$

$P = mg\left(\sin\theta - \mu\cos\theta\right)$

Question 20 **Answer A**

$\tan^{-1}\left(\frac{1}{2t}\right)\frac{dt}{dy}=1$ $\frac{dy}{dt}=\tan^{-1}\left(\frac{1}{2t}\right)$ using a calculator program gives

t	y
1.00	2.0000
1.25	2.1159
1.50	2.2110

Question 21 **Answer B**

The solution curves are of the form $x=\sin(2t)+c$ so differentiating,

the differential equation is $\frac{dx}{dt}=2\cos(2t)$

Question 22 **Answer B**

The volume in the tank is $V = A\,h$ $\frac{dV}{dh}=A$

and $\frac{dV}{dt}=\text{inflow}-\text{outflow}=Q-c\sqrt{h}$

$\frac{dh}{dt}=\frac{dh}{dV}\frac{dV}{dt}$ so that $\frac{dh}{dt}=\frac{Q-c\sqrt{h}}{A}$ $h(0)=h_0$

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

a.i. $3x^2 + 18x - y^2 + 4y + 11 = 0$

$3\left(x^2 + 6x \quad\right) - \left(y^2 - 4y \quad\right) = -11$

$3\left(x^2 + 6x + 9\right) - \left(y^2 - 4y + 4\right) = -11 + 27 - 4$ M1

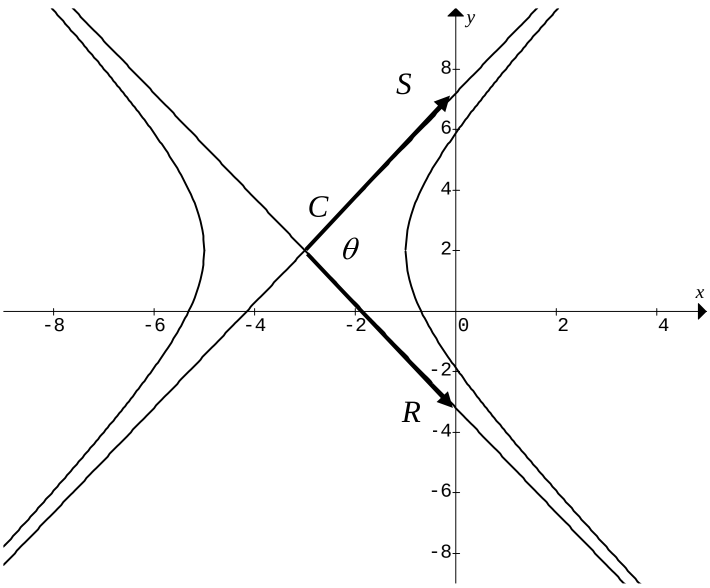
$3\left(x+3\right)^2 - \left(y-2\right)^2 = 12 \qquad \dfrac{\left(x+3\right)^2}{4} - \dfrac{\left(y-2\right)^2}{12} = 1$ A1

ii. centre $\left(h,k\right)$ where $h = -3 \quad k = 2 \quad a = 2 \quad b = 2\sqrt{3}$

hyperbola, centre $\left(-3,2\right)$ vertices $\left(-5,2\right)$ $\left(-1,2\right)$ A1

asymptotes $\dfrac{y-2}{2\sqrt{3}} = \pm\dfrac{x+3}{2} \qquad y = \pm\sqrt{3}\left(x+3\right) + 2$

$y = \sqrt{3}\,x + 2 + 3\sqrt{3}$ and $y = -\sqrt{3}\,x + 2 - 3\sqrt{3}$ A1



b.i $3x^2 + 18x - y^2 + 4y + 11 = 0$

differentiating implicitly

$\dfrac{d}{dx}\left(3x^2\right) + \dfrac{d}{dx}\left(18x\right) - \dfrac{d}{dx}\left(y^2\right) + \dfrac{d}{dx}\left(4y\right) + \dfrac{d}{dx}\left(11\right) = 0$

$6x + 18 - 2y\dfrac{dy}{dx} + 4\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}\left(2y - 4\right) = 6x + 18$

$\dfrac{dy}{dx} = \dfrac{3x+9}{y-2} = \dfrac{3\left(x+3\right)}{y-2}$ A1

ii. When the tangent is parallel to the y-axis. $\frac{dy}{dx} \to \infty \Rightarrow y-2=0 \quad y=2$

These are obviously the vertices $(-5,2)$ $(-1,2)$ A1

c. $R\left(0, 2-3\sqrt{3}\right)$, $S\left(0, 3\sqrt{3}+2\right)$ and $C(-3,2)$

$\overrightarrow{CS} = \overrightarrow{OS} - \overrightarrow{OC} = 3\underset{\sim}{i} + 3\sqrt{3}\,\underset{\sim}{j}$ $\quad$ $\overrightarrow{CR} = \overrightarrow{OR} - \overrightarrow{OC} = 3\underset{\sim}{i} - 3\sqrt{3}\,\underset{\sim}{j}$ M1

$\left|\overrightarrow{CS}\right| = \sqrt{9+27} = 6$ $\quad$ $\left|\overrightarrow{CR}\right| = \sqrt{9+27} = 6$

$\overrightarrow{CS} \cdot \overrightarrow{CR} = 9 - 9 \times 3 = -18$

The angle between $\overrightarrow{CS}$ and $\overrightarrow{CR}$ is θ where $\cos(\theta) = \frac{\overrightarrow{CS}.\overrightarrow{CR}}{\left|\overrightarrow{CS}\right|\left|\overrightarrow{CR}\right|} = \frac{-18}{36} = -\frac{1}{2}$

$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = 120^0$ A1

this angle represents the angle between the asymptotes at the centre of the hyperbola. A1

d. $m = -4 + yi$, $f = -7 + 2i$ and $z = x + yi$

$|z - f| = 2\,|z - m|$

$|(x+yi) - (-7+2i)| = 2\,|(x+yi) - (-4+yi)|$

$|(x+7) + (y-2)i| = 2\,|(x+4) + (y-y)i|$ M1

$\sqrt{(x+7)^2 + (y-2)^2} = 2\sqrt{(x+4)^2}$ squaring both sides and expanding M1

$x^2 + 14x + 49 + y^2 - 4y + 4 = 4\left(x^2 + 8x + 16\right) = 4x^2 + 32x + 64$

$3x^2 + 18x - y^2 + 4y + 11 = 0$

$\frac{(x+3)^2}{4} - \frac{(y-2)^2}{12} = 1$ the hyperbola again A1

e. $\underset{\sim}{r}(t) = \left(-3 + 2\sec(2t)\right)\underset{\sim}{i} + \left(2 + 2\sqrt{3}\tan(2t)\right)\underset{\sim}{j}$

$x = -3 + 2\sec(2t)$ $\quad$ $y = 2 + 2\sqrt{3}\tan(2t)$

$\sec(2t) = \frac{x+3}{2}$ $\quad$ $\tan(2t) = \frac{y-2}{2\sqrt{3}}$ M1

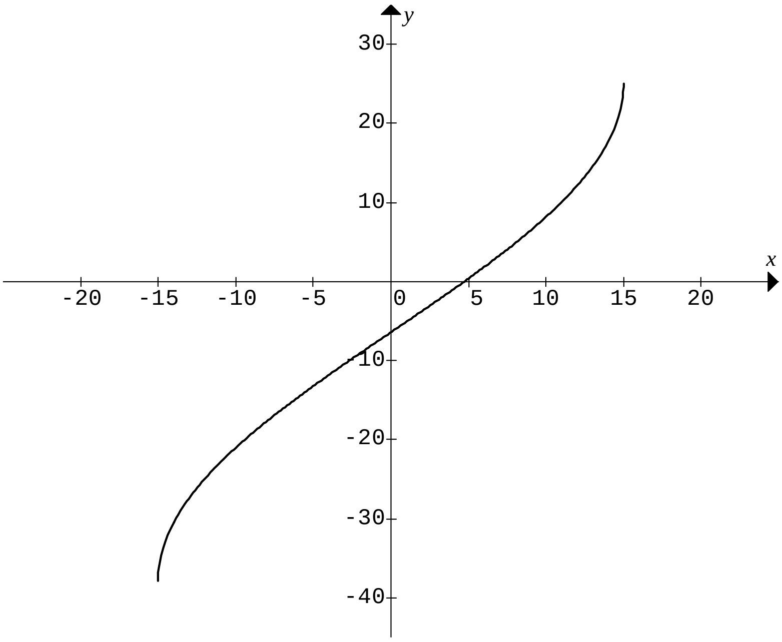
$\sec^2(2t) - \tan^2(2t) = 1$

$\frac{(x+3)^2}{4} - \frac{(y-2)^2}{12} = 1$ the hyperbola again A1

Question 2

a. domain $|x| \le 15 \quad = \quad [-15, 15]$ A1

b.



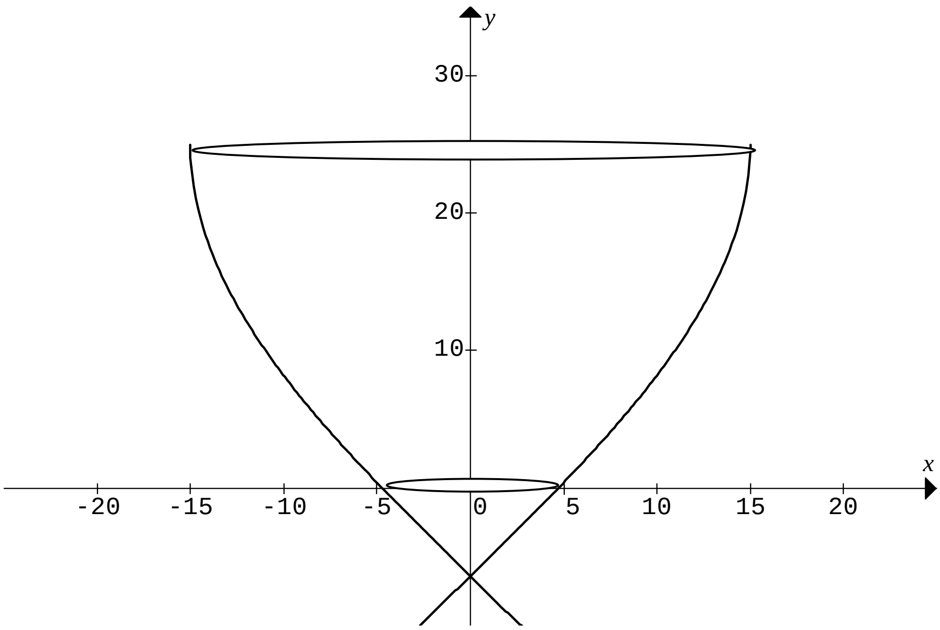
A1

c. $f(x) = 25 - 20\cos^{-1}\left(\frac{x}{15}\right) \qquad x \in [-15, 15]$

end points $(-15\,,\, 25-20\pi)\ (15\,,\, 25)$

$f'(x) = \frac{20}{\sqrt{225 - x^2}} \qquad x \in (-15, 15)$

the function is defined at the end-points
but the gradient is not defined at the end-points. A1



d. when $y = 0$ $\quad 25 - 20\cos^{-1}\left(\frac{x}{15}\right) = 0 \quad \Rightarrow x = 4.72984$

the diameter of the base of the vase is $2 \times 4.72984 = 9.460$ cm A1

e. $f'(10) = \dfrac{20}{\sqrt{225-100}} = \dfrac{4\sqrt{5}}{5} \approx 1.78885 = \tan(\theta)$ M1

$\theta = \tan^{-1}(1.789) = 60.794^0 = 60^0 48^{'}$ A1

f. i. $y = 25 - 20\cos^{-1}\left(\frac{x}{15}\right)$

$20\cos^{-1}\left(\frac{x}{15}\right) = 25 - y$

$\cos^{-1}\left(\frac{x}{15}\right) = \frac{25-y}{20}$ M1

$\frac{x}{15} = \cos\left(\frac{25-y}{20}\right)$

$x = 15\cos\left(\frac{25-y}{20}\right)$ M1

$V_y = \pi\int_a^b x^2\, dy$

$V = 225\pi\int_0^{25} \cos^2\left(\frac{25-y}{20}\right) dy$ A1

ii. $V = 225\pi\int_0^{25} \cos^2\left(\frac{25-y}{20}\right) dy = 10950.90 \text{ cm}^3$ A1

Question 3

a. $45a = -260\sqrt{v}$

$9a = -52\sqrt{v}$ A1

b. $a = \dfrac{dv}{dt} = -\dfrac{52\sqrt{v}}{9}$ inverting both sides

$\dfrac{dt}{dv} = -\dfrac{9}{52\sqrt{v}}$

$\displaystyle\int \frac{dv}{\sqrt{v}} = -\frac{52}{9}\int 1\,dt$ M1

$\displaystyle\int v^{-\frac{1}{2}}\,dv = -\frac{52t}{9} + C_1$

$2\sqrt{v} = -\dfrac{52t}{9} + C_1$ now when $t = 0$ $v = 9$

$2\sqrt{9} = C_1 = 6$

$2\sqrt{v} = 6 - \dfrac{52t}{9}$ now when $v = 1$

$2 = 6 - \dfrac{52t}{9}$

$\dfrac{52t}{9} = 4$

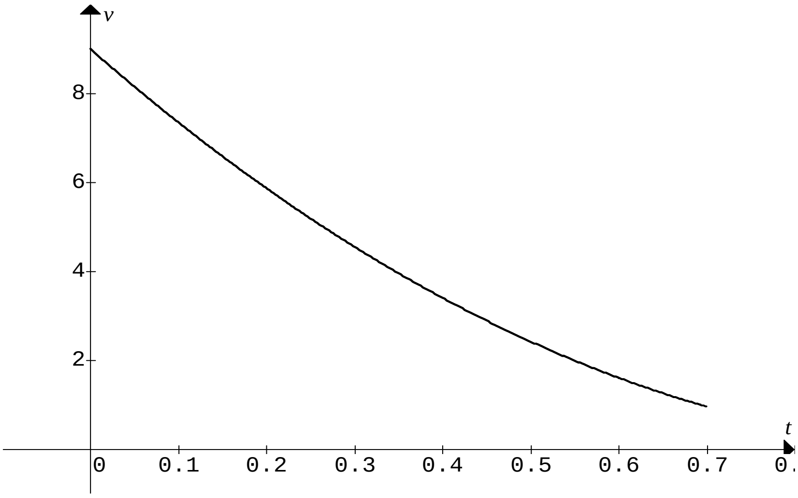
$t = \dfrac{36}{52} = \dfrac{9}{13}$

The head-wind blows for $\dfrac{9}{13}$ secs A1

c.i. $\sqrt{v} = 3 - \dfrac{26t}{9}$

$v = v(t) = \left(3 - \dfrac{26t}{9}\right)^2 = \dfrac{(27-26t)^2}{81}$ A1

ii.



Note the restricted domain for t as $\left[0, \frac{9}{13}\right]$, graph passes through the points

$(0, 9)$ and $\left(\frac{9}{13}, 1\right)$ A1

d. Since the area under the velocity time graph represents the displacement

$$D = \int_0^{\frac{9}{13}} \left(3 - \frac{26t}{9}\right)^2 dt = 3$$ A1

Pete rode his bike exactly 3 metres during the head-wind. A1

e. $45a = -260\sqrt{v}$ use $a = v\frac{dv}{dx}$

$$45v\frac{dv}{dx} = -260\sqrt{v}$$

$$\frac{dv}{dx} = -\frac{52}{9\sqrt{v}}$$ A1

f. $\frac{dv}{dx} = -\frac{52}{9\sqrt{v}}$ inverting both sides

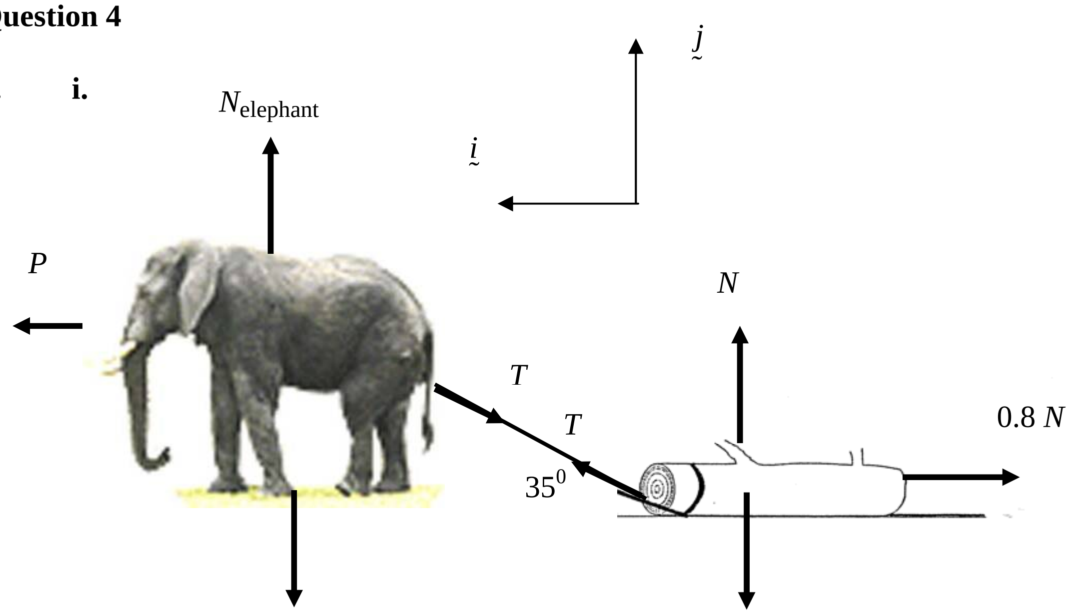
$\frac{dx}{dv} = -\frac{9\sqrt{v}}{52}$ integrating with respect to v M1

$$x = -\frac{9}{52}\int_9^1 \sqrt{v}\, dv = \frac{9}{52}\int_1^9 \sqrt{v}\, dv$$ A1

$$D = \frac{9}{52}\left[\frac{2}{3}v^{\frac{3}{2}}\right]_1^9 = \frac{3}{26}(27-1) = 3$$ A1

Question 4

a. **i.**



A1

ii Using Newtons 2nd Law of Motion

resolving horizontally around the log in the $\underset{\sim}{i}$ direction.

(1) $T\cos(35°) - 0.8\,N = 400 \times 0.1$

resolving vertically around the log in the $\underset{\sim}{j}$ direction. M1

(2) $T\sin(35°) + N - 400\,\mathrm{g} = 0$

(2) $\Rightarrow N = 400\,\mathrm{g} - T\sin(35°)$ into (1) M1

$T\cos(35°) - 0.8(400\,\mathrm{g} - T\sin(35°)) = 40$

$T(\cos(35°) + 0.8\sin(35°)) - 0.8 \times 400\,\mathrm{g} = 40$

$$T = \frac{40(1+10\times 0.8\times 9.8)}{\cos(35°)+0.8\sin(35°)} = 2485.11 = 2485 \text{ newtons}$$ A1

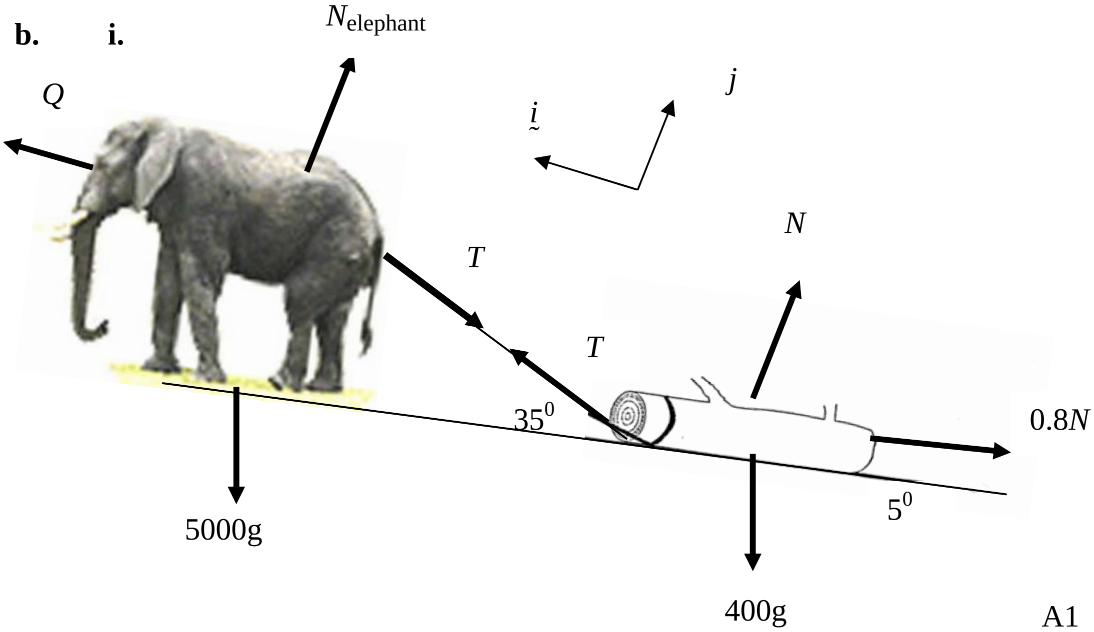
iii. resolving horizontally around the elephant in the $\underset{\sim}{i}$ direction.

(3) $P - T\cos(35°) = 5000 \times 0.1$

$P = T\cos(35°) + 500 = 2535.68$

$P = 2536$ newtons A1

b. **i.**



A1

ii. constant speed is zero acceleration. Note the 0.1 m/s is not used.

resolving up and parallel to the hill around the log in the $\underset{\sim}{i}$ direction.

(1) $T\cos(35°)-0.8N-400g\sin(5°)=0$

resolving perpendicular to the hill around the log in the $\underset{\sim}{j}$ direction

(2) $N+T\sin(35°)-400g\cos(5°)=0$ M1

(2) $\Rightarrow N=400g\cos(5°)-T\sin(35°)$ into (1)

$T\cos(35°)-0.8(400g\cos(5°)-T\sin(35°))-400g\sin(5°)=0$

$T(\cos(35°)+0.8\sin(35°))=400g(\sin(5°)+0.8\cos(5°))$ M1

$$T=\frac{400g(\sin(5°)+0.8\cos(5°))}{\cos(35°)+0.8\sin(35°)}=2711.801$$

$T=2712$ newtons A1

iii. resolving horizontally around the elephant in the $\underset{\sim}{i}$ direction.

(3) $Q-T\cos(35°)-5000g\sin(5°)=0$

$Q=T\cos(35°)+5000g\sin(5°)=6492.01$

$Q=6492$ newtons A1

Question 5 $\underset{\sim}{r}(t) = \left(-25t^2 + 52.5t\right)\underset{\sim}{i} + \left(2e^{-\frac{7t}{2}}\left|\cos\left(\frac{17\pi t}{10}\right)\right|\right)\underset{\sim}{j} \qquad t \geq 0$

a. $\underset{\sim}{r}(0) = 0\underset{\sim}{i} + 2\underset{\sim}{j} = 2\underset{\sim}{j}$

The ball was 2 metres above the ground when it left the bowlers hand. A1

b. differentiating using the product in the $\underset{\sim}{j}$ component M1

$$\dot{\underset{\sim}{r}}(t) = \left(-50t + 52.5\right)\underset{\sim}{i} + \left(-\frac{7}{2} \text{ x } 2e^{-\frac{7t}{2}}\left|\cos\left(\frac{17\pi t}{10}\right)\right| + 2e^{-\frac{7t}{2}} \text{ x } -\frac{17\pi}{10}\left|\sin\left(\frac{17\pi t}{10}\right)\right|\right)\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}}(t) = \left(-50t + 52.5\right)\underset{\sim}{i} - \left(e^{-\frac{7t}{2}}\left(7\left|\cos\left(\frac{17\pi t}{10}\right)\right| + \frac{17\pi}{5}\left|\sin\left(\frac{17\pi t}{10}\right)\right|\right)\right)\underset{\sim}{j}$$ A1

c. $\dot{\underset{\sim}{r}}(0) = 52.5\underset{\sim}{i} - 7\underset{\sim}{j} \qquad \left|\dot{\underset{\sim}{r}}(0)\right| = \sqrt{52.5^2 + (-7)^2} = 53.0\,\text{m/s}$ M1

$m = 155\,\text{gm} = 0.155\,\text{kg}$

magnitude of the momentum of the ball when it left the bowlers hand

$\left|\underset{\sim}{p}\right| = m\left|\dot{\underset{\sim}{r}}(0)\right| = 0.155 \text{ x } 53.0 = 8.21\ \text{kg m/s}$ A1

d. The ball hits the ground when $2e^{-\frac{7t}{2}}\left|\cos\left(\frac{17\pi t}{10}\right)\right| = 0$

$\cos\left(\frac{17\pi t}{10}\right) = 0$ M1

$\frac{17\pi t}{10} = \frac{\pi}{2}$

$t = \frac{5}{17}$ sec hits the ground after 0.294 seconds A1

$$\underset{\sim}{r}\left(\frac{5}{17}\right) = \left(-25\left(\frac{5}{17}\right)^2 + 52.5 \text{ x } \frac{5}{17}\right)\underset{\sim}{i} + \left(2e^{-\frac{7 \text{ x } \frac{5}{17}}{2}}\left|\cos\left(\frac{17\pi}{10} \text{ x } \frac{5}{17}\right)\right|\right)\underset{\sim}{j}$$

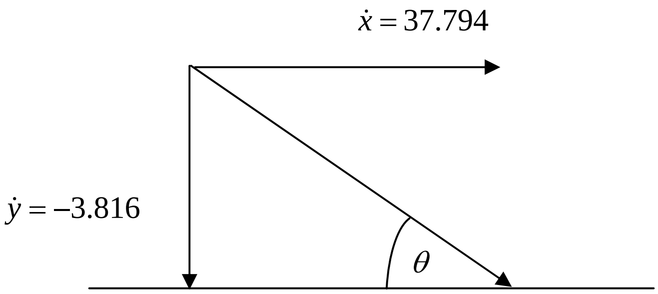
$\underset{\sim}{r}\left(\frac{5}{17}\right) = 13.278\underset{\sim}{i} + 0\underset{\sim}{j}$

$\underset{\sim}{r}\left(\frac{5}{17}\right) = 13.278\underset{\sim}{i} \qquad (20 - 13.278 = 6.72)$

The ball strikes the ground 6.72 metres from the batting stumps A1

e. $\dot{\underset{\sim}{r}}\left(\frac{5}{17}\right)=\left(-50 \times \frac{5}{17}+52.5\right)\underset{\sim}{i}+\left(e^{-\frac{7\frac{5}{17}}{2}}\left(-7\left|\cos\left(\frac{\pi}{2}\right)\right|-\frac{17\pi}{5}\left|\sin\left(\frac{\pi}{2}\right)\right|\right)\right)\underset{\sim}{j}$

$\dot{\underset{\sim}{r}}\left(\frac{5}{17}\right)=37.794\,\underset{\sim}{i}-3.8156\,\underset{\sim}{j}$



M1

$\tan(\theta)=\frac{\dot{y}}{\dot{x}}=\frac{3.8156}{37.794}$

$\theta=\tan^{-1}(0.101)=5.765^{0}=5^{0}\,46^{'}$

The ball strikes the ground at an angle of $5^{0}\,46^{'}$ A1

f. when $x=20$ $\quad 20=-25t^{2}+52.t$

$25t^{2}-52.5t+20=0$

$t=\frac{1}{2}=0.5$ seconds

Hits the batting stumps after 0.5 seconds A1

$\underset{\sim}{r}\left(\frac{1}{2}\right)=20\underset{\sim}{i}+\left(2e^{-\frac{7\times0.5}{2}}\left|\cos\left(\frac{17\pi}{20}\right)\right|\right)\underset{\sim}{j}$

$\underset{\sim}{r}\left(\frac{1}{2}\right)=20\underset{\sim}{i}+0.3097\,\underset{\sim}{j}$

hits the stumps 31 cm above the ground. A1

g. Total distance of 20 metres in 0.5 seconds, gives the average velocity of the ball as 40 m/s or $40 \times \frac{60 \times 60}{1000}=144\,\text{km/hr}$ A1

END OF SECTION 2 SUGGESTED ANSWERS