Specialist Mathematics

2006

Trial Examination 1

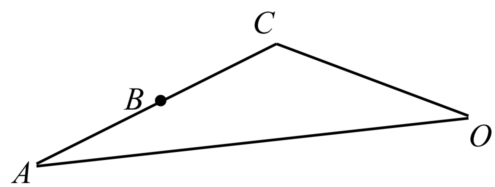
Instructions

Answer **all** questions. Do **not** use calculators.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

B is a point on the side $\overline{AC}$ of ΔOAC. Use vectors to prove that ΔOAC is isosceles if $\overline{OB}$ is a perpendicular bisector of $\overline{AC}$.

4 marks



Question 2

Solve $z^3 - 3iz^2 + 3z + 9i^3 = 0$ over C. 3 marks

Question 3

Consider the relation $xy - y^2 = 2$.

a. Find an expression for $\frac{dy}{dx}$ in terms of x and y. 3 marks

b. Hence find the two exact gradients of the relation at $x = 3$. 2 marks

Question 4

Evaluate the definite integral $\int_0^{\sqrt{e-1}} \left(\frac{2x \log_e\left(x^2+1\right)}{x^2+1} \right) dx$. 3 marks

Question 5

Consider $y = 1 + \left(x^2 - 2x + 2\right)\tan^{-1}(x-1)$ for $x \in R$.

a. Show that $\frac{dy}{dx} = 1 + 2(x-1)\tan^{-1}(x-1)$ for $x \in R$. 2 marks

b. Hence show that $(1,1)$ is a point of inflection of $y = 1 + \left(x^2 - 2x + 2\right)\tan^{-1}(x-1)$. 2 marks

Question 6

The position of a particle is given by $\boldsymbol{r} = \sin(2t)\boldsymbol{i} + \cos(2t)\boldsymbol{j} + (10 - t^2)\boldsymbol{k}$.

a. Find the initial velocity of the particle. 2 marks

b. Show that the acceleration of the particle is constant in magnitude. 2 marks

Question 7

The curve $y = \dfrac{2}{\sqrt{2 - x^2}}$, $0 \le x \le 1$ is rotated about the x-axis. Determine the exact volume of the solid of revolution. 4 marks

Question 8

Solve the differential equation $\frac{dy}{dx} = \sin(2x)\sqrt{1+\sin(x)}$ given that $y = \frac{7}{15}$ when $x = 0$. 4 marks

Question 9

Find p and q such that $\left\{z : \frac{(\operatorname{Re} z)^2}{p} + \frac{(\operatorname{Im} z)^2}{q} = 1\right\} = \{z : |z+i| + |z-i| = 4\}$. 4 marks

Question 10

A 10-kg block is placed on a rough surface inclined at 30° to the horizontal. The block is then pulled up the slope with a $10g$-newton force at an upward angle of 30° to the inclined surface. The coefficient of sliding friction between the block and the inclined surface is 0.20.

a. Show that the normal reaction of the inclined plane on the block is $5(\sqrt{3}-1)g$. 2 marks

b. Hence find the acceleration of the 10-kg block in terms of g. 3 marks

End of Exam 1