Specialist Mathematics

Written examination 1

2005 Trial Examination

SOLUTIONS

PART I: Multiple-choice questions (1 mark each)

1.	A	11.	C	21.	C
2.	B	12.	E	22.	B
3.	A	13.	D	23.	D
4.	C	14.	D	24.	E
5.	D	15.	C	25.	A
6.	D	16.	D	26.	C
7.	D	17.	E	27.	E
8.	A	18.	D	28.	C
9.	B	19.	E	29.	D
10.	A	20.	A	30.	A

PART II: Short Answer Questions.

Question 1

$y = x\sin x \Rightarrow y' = \sin x + x\cos x$

$\Rightarrow y'' = \cos x + (\cos x - x\sin x) = 2\cos x - x\sin x$

Substituting into the differential equation, we obtain

$(2\cos x - x\sin x) + m(x\sin x) = n\cos x$

$\Rightarrow (m-1)x\sin x + 2\cos x = n\cos x$

$\Rightarrow m - 1 = 0 \;\&\; n = 2 \Rightarrow m = 1 \;\&\; n = 2$

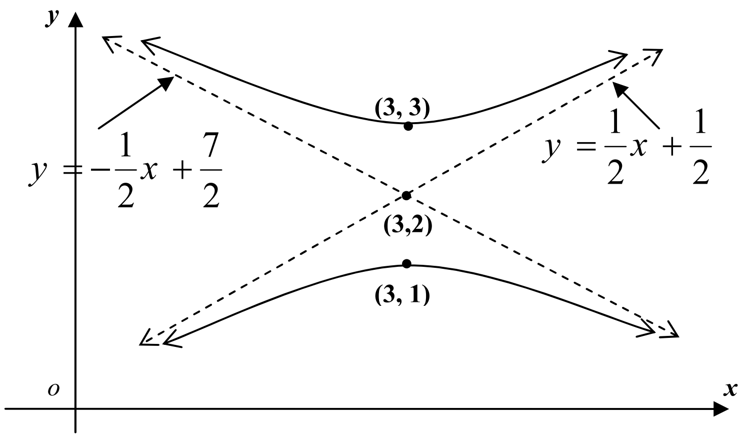
3 marks

Question 2

Centre: $(3,2)$ & $a = 2, b = 1 \Rightarrow$ Vertices: $(3, 2\pm 1) \Rightarrow (3,1)$ & $(3,3)$.

Asymptotes are $y - 2 = \pm\frac{1}{2}(x-3) = \pm\frac{1}{2}x + 2 \mp \frac{3}{2}$

$$\Rightarrow y = \frac{1}{2}x + \frac{1}{2} \;\&\; \Rightarrow y = -\frac{1}{2}x + \frac{7}{2}$$



4 marks

Question 3

$$\int_0^{\pi} 8\sin^4 x\,dx = 8\int_0^{\pi}(\sin^2 x)^2 dx = 8\int_0^{\pi}[\frac{1}{2}(1-\cos 2x)]^2 dx$$

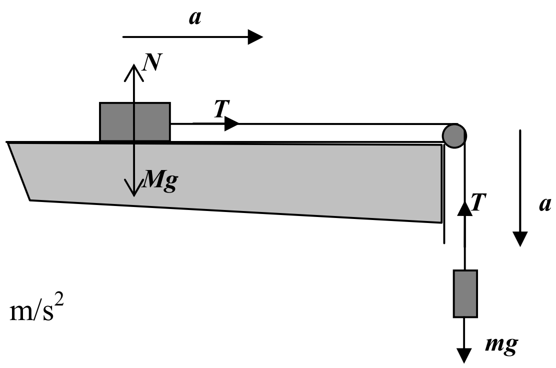
$$= 2\int_0^{\pi}(1-2\cos 2x + \cos^2 2x)dx = 2\int_0^{\pi}[1-2\cos 2x + \frac{1}{2}(1+\cos 4x)]dx$$

$$= \int_0^{\pi}(2-4\cos 2x + 1 + \cos 4x)dx = \int_0^{\pi}(3-4\cos 2x + \cos 4x)dx$$

$$= [3x - 2\sin 2x + \frac{1}{4}\sin 4x]_0^{\pi} = (3\pi) - (0) = 3\pi$$

3 marks

Question 4



a. The mass M:

$Ma = T \text{ --------- } (1)$

The mass m:

$ma = mg - T \text{ --- } (2)$

Adding (1) and (2):

$$(M+m)a = mg \Rightarrow a = \frac{mg}{M+m} \text{ m/s}^2$$

2 marks

b. Substitute into (1): $T = \dfrac{Mmg}{M+m}$ N

1 mark

Question 5

$$V = \pi\int_0^h x^2 dy = \pi\int_0^h (e^y)^2 dy = \pi\int_0^h e^{2y} dy = \frac{\pi}{2}[e^{2y}]_0^h = \frac{\pi}{2}(e^{2h}-1) \text{ cubic units}$$

3 marks

Question 6

$z^4 = -16 = 16cis(\pi) = 2^4 cis(\pi + 2k\pi),\ k = 0,1,2,3.$

$z = 2cis\left(\dfrac{\pi + 2k\pi}{4}\right),\ k = 0,1,2,3.$

$k = 0 \Rightarrow z_1 = 2cis\left(\dfrac{\pi}{4}\right) = 2[\cos(\dfrac{\pi}{4}) + i\ \sin(\dfrac{\pi}{4})] = 2[\dfrac{\sqrt{2}}{2} + i\ \dfrac{\sqrt{2}}{2}] = \sqrt{2} + \sqrt{2}i$

$k = 1 \Rightarrow z_2 = 2cis\left(\dfrac{3\pi}{4}\right) = 2[\cos(\dfrac{3\pi}{4}) + i\ \sin(\dfrac{3\pi}{4})] = 2[-\dfrac{\sqrt{2}}{2} + i\ \dfrac{\sqrt{2}}{2}] = -\sqrt{2} + \sqrt{2}i$

$k = 2 \Rightarrow z_3 = 2cis\left(\dfrac{5\pi}{4}\right) = 2[\cos(\dfrac{5\pi}{4}) + i\ \sin(\dfrac{5\pi}{4})] = 2[-\dfrac{\sqrt{2}}{2} - i\ \dfrac{\sqrt{2}}{2}] = -\sqrt{2} - \sqrt{2}i$

$k = 3 \Rightarrow z_1 = 2cis\left(\dfrac{7\pi}{4}\right) = 2[\cos(\dfrac{7\pi}{4}) + i\ \sin(\dfrac{7\pi}{4})] = 2[\dfrac{\sqrt{2}}{2} - i\ \dfrac{\sqrt{2}}{2}] = \sqrt{2} - \sqrt{2}i$

4 marks