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SPECIALIST MATHS TRIAL EXAMINATION 1 SOLUTIONS 2004

Part I – Multiple-choice answers

1.	**D**	**7.**	**C**	**13.**	**C**	**19.**	**D**	**25.**	**D**
2.	**D**	**8.**	**A**	**14.**	**B**	**20.**	**B**	**26.**	**C**
3.	**B**	**9.**	**E**	**15.**	**B**	**21.**	**D**	**27.**	**D**
4.	**E**	**10.**	**E**	**16.**	**C**	**22.**	**C**	**28.**	**D**
5.	**A**	**11.**	**D**	**17.**	**B**	**23.**	**D**	**29.**	**B**
6.	**A**	**12.**	**D**	**18.**	**C**	**24.**	**D**	**30.**	**A**

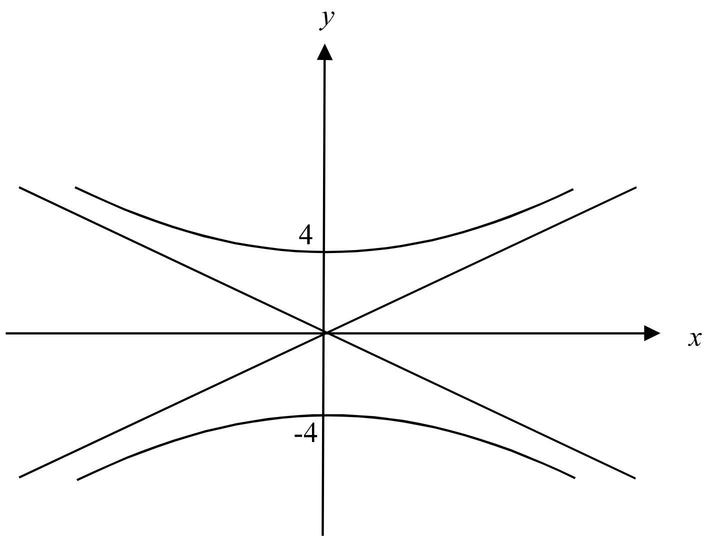
Part I- Multiple-choice solutions

Question 1

The graph of $y = 3x^2 + \frac{5}{x}$ has a vertical asymptote given by $x = 0$. Its other asymptote is given by $y = 3x^2$ so reject option E. The graph has a local minimum where $x = \sqrt[3]{\frac{5}{6}}$ and not where $x = 1$ so reject option B. The graph is defined for $y < 0$ so reject option C.
Also, the graph has an x-intercept but not at $x = -1$ so reject option A.
Only option D is correct since $x = 0$ is an asymptote and hence there are no y-intercepts.
The answer is D.

Question 2

The graph of the relation $\frac{y^2}{16} - \frac{x^2}{25} = 1$ is the hyperbola shown below.



It intersects with the y-axis twice.
The answer is D.

Question 3

$$y = \text{Cos}^{-1}\left(\frac{x}{2}\right)$$

$$= \text{Cos}^{-1}(u) \text{ where } u = \frac{x}{2} \text{ and so } \frac{du}{dx} = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} \text{ (Chain rule)}$$

$$= \frac{-1}{\sqrt{1-u^2}}.\frac{1}{2}$$

$$= \frac{-1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}$$

$$= \frac{-1}{2\sqrt{\frac{4-x^2}{4}}}$$

$$= \frac{-1}{\sqrt{4-x^2}}$$

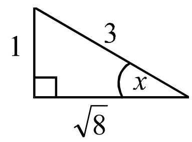
At $x = 0$

$$\frac{dy}{dx} = \frac{-1}{2}$$

The answer is B.

Question 4

$\sin(x) = -\frac{1}{3}, \quad \frac{3\pi}{2} \le x \le 2\pi$



$\cos(x) = \frac{\sqrt{8}}{3}$ since in the fourth quadrant cos is positive

So $\sec(x) = \frac{3}{2\sqrt{2}}$

$$= \frac{3\sqrt{2}}{4}$$

The answer is E.

Question 5

$u = 2 - i, \quad v = \overline{u} + 1$

$\quad\quad\quad = 2 + i + 1$

$\quad\quad\quad = 3 + i$

So, $\dfrac{u}{v} = \dfrac{2-i}{3+i}$

$= \dfrac{2-i}{3+i} \times \dfrac{3-i}{3-i}$

$= \dfrac{6-5i-1}{10}$

$= \dfrac{5-5i}{10}$

$= \dfrac{1}{2} - \dfrac{i}{2}$

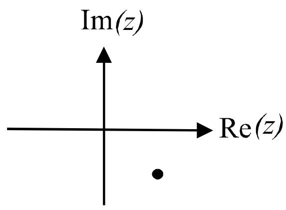
The answer is A.

Question 6

$r = \sqrt{3+1}$

$\;\; = 2$

Note that the question asks for "a" polar form.



Now $\text{Arg}\left(\sqrt{3} - i\right) = \tan^{-1}\left(\dfrac{-1}{\sqrt{3}}\right)$ (in 4th quadrant)

$= -\dfrac{\pi}{6}$

This is the value of Arg*(z)* in the range $(-\pi, \pi]$.

So a polar form of $\sqrt{3} - i$ could be $2\text{cis}\left(\dfrac{-\pi}{6}\right)$ or $2\text{cis}\left(\dfrac{-13\pi}{6}\right)$ and so on.

In this case the only correct option is A.

The answer is A.

Question 7

We have a semicircle.

Now $\{z : |z| \le 2\}$ describes a circle with radius 2 units.

Also, $\{z : \text{Im}(z) \ge 0\}$ describes the top half of the complex plane including the real axis.

Hence $\{z : |z| \le 2\} \cap \{z : \text{Im}(z) \ge 0\}$ describes the semicircle we have. Note that $\{z : |z| \le 2\} \cap \{z : \text{Im}(z) > 0\}$ is close but excludes the Real axis between –2 and 2 which is included in the diagram.

The answer is C.

Question 8

Since $P(z)$ is a cubic polynomial with real coefficients and one of its solutions is $1 + i$ then another of its solutions must be $1 - i$, that is, the conjugate of $1 + i$ (conjugate root theorem).

The other solution must be real.

Note that $z - 1 + i$ is a factor not a solution, as is $z - 2$.

The only feasible answer is 3.

The answer is A.

Question 9

Let $\qquad z = r\text{cis}\theta$

So $\qquad z^3 = \text{cis}\left(\frac{\pi}{2}\right)$

becomes $\quad (r\text{cis}\theta)^3 = \text{cis}\left(\frac{\pi}{2}\right)$

$$r^3\text{cis}(3\theta) = \text{cis}\left(\frac{\pi}{2}\right) \quad \text{(De Moivre's Theorem)}$$

So, $\quad r = 1$ and $3\theta = \frac{\pi}{2} + 2k\pi, \quad k \in J$

$$\theta = \frac{\pi}{6} + \frac{2k\pi}{3}$$

If $k = 0, \qquad \theta = \frac{\pi}{6}$

If $k = -1, \qquad \theta = \frac{\pi - 4\pi}{6}$

$$= \frac{-\pi}{2}$$

If $k = 1, \qquad \theta = \frac{\pi + 4\pi}{6}$

$$= \frac{5\pi}{6}$$

So the three solutions are $\text{cis}\left(\frac{\pi}{6}\right), \text{cis}\left(\frac{-\pi}{2}\right),$ and $\text{cis}\left(\frac{5\pi}{6}\right)$.

The answer is E.

Question 10

$$\int \frac{x}{\sqrt{3-x}}\,dx$$

$$= \int \frac{3-u}{\sqrt{u}} \cdot -1\frac{du}{dx}\,dx$$

let $u = 3 - x$

$\frac{du}{dx} = -1$

$x = 3 - u$

$$= -1\int \left(3u^{\frac{-1}{2}} - u^{\frac{1}{2}} \right) du$$

$$= -1\left(2 \times 3u^{\frac{1}{2}} - \frac{2u^{\frac{3}{2}}}{3} \right) + c$$

$$= \frac{2}{3}u^{\frac{3}{2}} - 6u^{\frac{1}{2}} + c$$

$$= \frac{2}{3}(3-x)^{\frac{3}{2}} - 6(3-x)^{\frac{1}{2}} + c$$

Note "an antiderivative" means c takes on a particular value. In this case $c = 0$.
The answer is E.

Question 11

$$\int \frac{-2}{\sqrt{4-x^2}}\,dx = -2\int \frac{1}{\sqrt{4-x^2}}\,dx$$

$$= -2\,\text{Sin}^{-1}\left(\frac{x}{2}\right) + c$$

OR

$$\int \frac{-2}{\sqrt{4-x^2}}\,dx = 2\int \frac{-1}{\sqrt{4-x^2}}\,dx$$

$$= 2\,\text{Cos}^{-1}\left(\frac{x}{2}\right) + c$$

Only the second answer is offered.
The answer is D.

Question 12

With the integration techniques available to us in this course, we are not able to antidifferentiate $\sqrt{9-x^2}$.
Use a graphics calculator instead.
The answer, correct to 4 decimal places is 8.4633.
The answer is D.

Question 13

$$f'(x) = \sin^2(3x)$$

So, $f(x) = \int \sin^2(3x)\,dx$

$$= \frac{1}{2}\int (1 - \cos 6x)\,dx$$

$$= \frac{1}{2}\left(x - \frac{1}{6}\sin(6x)\right) + c$$

When $x = \frac{\pi}{6}$, $f(x) = \frac{\pi}{12}$

So, $\frac{\pi}{12} = \frac{1}{2}\left(\frac{\pi}{6} - \frac{1}{6}\sin(\pi)\right) + c$

$$\frac{\pi}{12} = \frac{1}{2}\left(\frac{\pi}{6} - 0\right) + c$$

$$\frac{\pi}{12} = \frac{\pi}{12} + c$$

$$c = 0$$

$$f(x) = \frac{x}{2} - \frac{1}{12}\sin(6x)$$

The answer is C.

Question 14

$$y = \log_e\left(e^{2x}\right)$$

$$\frac{dy}{dx} = \frac{2e^{2x}}{e^{2x}}$$

$$= 2$$

$$\frac{d^2y}{dx^2} = 0$$

So $\frac{d^2y}{dx^2} = \frac{dy}{dx} - 2$

Alternatively,

$$y = \log_e(e^{2x})$$

$$= 2x$$

$$\frac{dy}{dx} = 2$$

$$\frac{d^2y}{dx^2} = 0$$

So, $\frac{d^2y}{dx^2} = \frac{dy}{dx} - 2$

The answer is B.

Question 15

$$\text{Total Area} = 3\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(2\cos(2x) - \cos(2x)\right)dx \qquad \text{by symmetry}$$

$$= 3\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(2x)dx$$

$$= 6\int_{0}^{\frac{\pi}{4}} \cos(2x) \qquad \text{again by symmetry}$$

Note that in option D, the second term should be negative and similarly in option E.
The answer is B.

Question 16

On the graph of $y = f'(x), f'(-2) = 0$ and $f'(1) = 0$. So at $x = -2$ and at $x = 1$ on the graph of $y = f(x)$ we have a stationary point.
For $x < -2, f'(x) > 0$ and for $-2 < x < 1,\ f'(x) > 0$ so at $x = -2$ on the graph of $y = f(x)$, we must have a stationary point of inflection.
For $-2 < x < 1,\ f'(x) > 0$ and for $x > 1, f'(x) < 0$ so at $x = 1$ on the graph of $y = f(x)$, we must have a local maximum.
There cannot be a stationary point of inflection at $x = 0$ since $f'(0) \neq 0$.
The answer is C.

Question 17

Using the formula sheet, we have, if $\frac{dy}{dx} = f(x),\ \ x_0 = a$ and $y_0 = b$,
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
Now $x_0 = 0$ and $y_0 = 0$

and $\frac{dy}{dx} = f(x)$

$= \log_e(2x+1)$

Also, $h = 0.1$

So, $x_1 = 0 + 0\cdot 1$ and $y_1 = 0 + 0\cdot 1 \times f(0)$

$= 0\cdot 1 \qquad\qquad = 0$

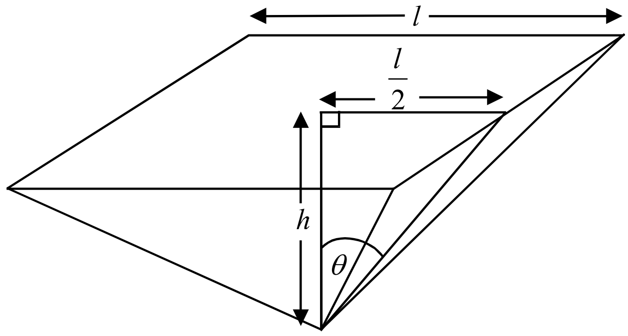
$x_2 = 0\cdot 1 + 0\cdot 1 \qquad y_2 = 0 + 0\cdot 1 \times f(0\cdot 1)$

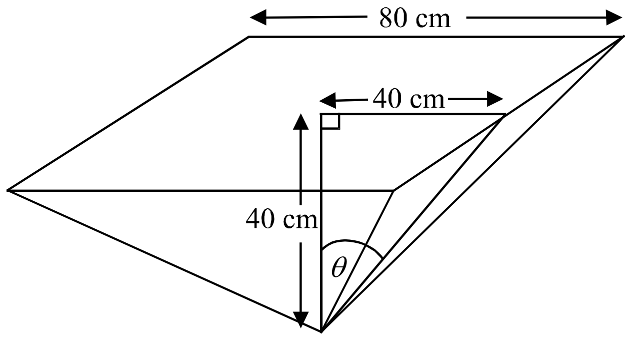
$= 0\cdot 2 \qquad\qquad = 0\cdot 1\log_e(1\cdot 2)$

So, when $x = 0\cdot 2,\ y = \frac{\log_e(1\cdot 2)}{10}$

The answer is B.

Question 18





$V = \frac{1}{3}Ah$

$= \frac{1}{3} \times l^2 h$ where l is the sidelength of the square

In the triangle drawn, $\tan\theta = 1$

so, $\frac{l}{2} = h, \quad l = 2h$

So $V = \frac{1}{3} \times 4h^2 \times h$

$= \frac{4h^3}{3}$

$\frac{dV}{dh} = 4h^2$

Now, $\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$

$= \frac{1}{4h^2} \cdot -10$

$= \frac{-5}{2h^2}$

The answer is C.

Question 19

$$a = v(2v+1)$$
$$v\frac{dv}{dx} = v(2v+1)$$
$$\frac{dv}{dx} = 2v+1$$
$$\frac{dx}{dv} = \frac{1}{2v+1}$$
$$x = \int \frac{1}{2v+1}\,dv$$
$$x = \frac{1}{2}\log_e(2v+1) + c$$

When $x = 0,\ v = 1$

$$0 = \frac{1}{2}\log_e(3) + c$$
$$x = \frac{1}{2}\log_e(2v+1) - \frac{1}{2}\log_e(3)$$
$$x = \frac{1}{2}\log_e\left(\frac{2v+1}{3}\right)$$

The answer is D.

Question 20

$$\overrightarrow{OP} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$$

$$\overrightarrow{OQ} = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$$

$$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{PO} + \overrightarrow{OQ}\\ &= -3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k} + 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\\ &= -\underset{\sim}{i} - \underset{\sim}{j}\end{aligned}$$

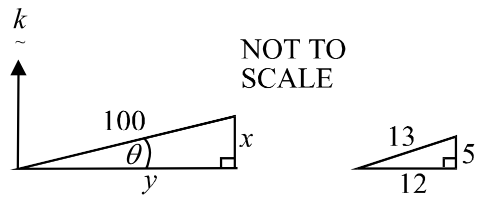
$$\begin{aligned}\left|\overrightarrow{PQ}\right| &= \sqrt{1+1}\\ &= \sqrt{2}\end{aligned}$$

The answer is B.

Question 21

Note that since the surveyor is travelling up at a gradient of $\frac{5}{12}$ he won't travel 100 m in the $\underset{\sim}{i} - \underset{\sim}{j}$ plane.

Consider the vertical component.



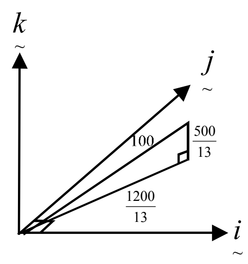
From the diagram we have $\frac{x}{5} = \frac{100}{13}$ since the triangles are similar.

$$x = \frac{500}{13}$$

Also note from the diagram that $\frac{y}{12} = \frac{100}{13}$

$$y = \frac{1200}{13}$$

This is the distance travelled in the $\underset{\sim}{i} - \underset{\sim}{j}$ plane.

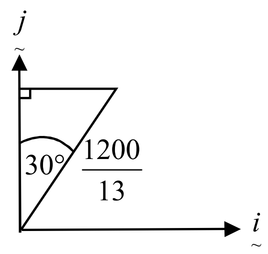


In the $\underset{\sim}{i} - \underset{\sim}{j}$ plane,

$$\sin 30^\circ = \text{opp} \div \frac{1200}{13} \quad \text{and} \quad \cos 30^\circ = \text{adj} \div \frac{1200}{13}$$

$$\frac{1}{2} = \text{opp} \times \frac{13}{1200} \qquad \frac{\sqrt{3}}{2} = \text{adj} \times \frac{13}{1200}$$

$$\text{opp} = \frac{600}{13} \qquad \text{adj} = \frac{600\sqrt{3}}{13}$$



So $\overrightarrow{OP} = \frac{600}{13}\underset{\sim}{i} + \frac{600\sqrt{3}}{13}\underset{\sim}{j} + \frac{500}{13}\underset{\sim}{k}$

The answer is D.

Question 22

$\underset{\sim}{u} = n\,\underset{\sim}{i}, \quad \underset{\sim}{v} = \underset{\sim}{i} - \underset{\sim}{j}$

$\underset{\sim}{u} \bullet \underset{\sim}{v} = n$

$\underset{\sim}{u} \bullet \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta$ where θ is the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$

$= \sqrt{n^2}\sqrt{1+1}\cos\theta$

So $n = \sqrt{2}n\cos\theta$

$\frac{1}{\sqrt{2}} = \cos\theta$

$\theta = \frac{\pi}{4}^c$

The answer is C.

Question 23

$\underset{\sim}{a} \bullet \underset{\sim}{b} = 0$ since $\angle RPQ = 90°$.

So option A is correct.

Option B is correct. (triangle rule for addition of vectors)

Consider $\underset{\sim}{a} \bullet \underset{\sim}{a} + \underset{\sim}{b} \bullet \underset{\sim}{b} = \underset{\sim}{c} \bullet \underset{\sim}{c}$

$$
\begin{aligned}
LS &= \underset{\sim}{a} \bullet \underset{\sim}{a} + \underset{\sim}{b} \bullet \underset{\sim}{b} \\
&= |\underset{\sim}{a}||\underset{\sim}{a}|\cos(0^\circ) + |\underset{\sim}{b}||\underset{\sim}{b}|\cos(0^\circ) \\
&= |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 \\
&= |\underset{\sim}{c}|^2 \quad \text{(Pythagoras)} \\
RS &= \underset{\sim}{c} \bullet \underset{\sim}{c} \\
&= |\underset{\sim}{c}||\underset{\sim}{c}|\cos(0^\circ) \\
&= |\underset{\sim}{c}|^2 \\
&= LS
\end{aligned}
$$

So option C is correct.

$$
\begin{aligned}
&\left(\underset{\sim}{a} - \underset{\sim}{b}\right) \bullet \left(\underset{\sim}{a} + \underset{\sim}{b}\right) \\
&= \underset{\sim}{a} \bullet \underset{\sim}{a} - \underset{\sim}{b} \bullet \underset{\sim}{b} \\
&= |\underset{\sim}{a}||\underset{\sim}{a}|\cos(0^\circ) - |\underset{\sim}{b}||\underset{\sim}{b}|\cos(0^\circ) \\
&= |\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2 \\
&= 0 \text{ since } |\underset{\sim}{a}| = |\underset{\sim}{b}|
\end{aligned}
$$

So option D is incorrect. Option E is correct since the sum of side lengths *PR* and *PQ* must be greater than sidelength *QR*. The answer is D.

Question 24

$$\left|2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right|=\sqrt{4+1+1}$$
$$=\sqrt{6}$$
$$\left|-\underset{\sim}{i}+2\underset{\sim}{j}+\underset{\sim}{k}\right|=\sqrt{1+4+1}$$
$$=\sqrt{6}$$

Now the vector resolute of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$ is $\underset{\sim}{a}-(\underset{\sim}{a}\bullet\hat{\underset{\sim}{b}})\hat{\underset{\sim}{b}}$.

So, the vector resolute of $2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}$ perpendicular to $-\underset{\sim}{i}+2\underset{\sim}{j}+\underset{\sim}{k}$ is

$$2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}-\left(\left(2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right)\bullet\frac{1}{\sqrt{6}}\left(-\underset{\sim}{i}+2\underset{\sim}{j}+\underset{\sim}{k}\right)\right)\frac{1}{\sqrt{6}}\left(-\underset{\sim}{i}+2\underset{\sim}{j}+\underset{\sim}{k}\right)$$
$$=2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}-\left(\frac{1}{\sqrt{6}}(-2+2-1)\right)\times\frac{1}{\sqrt{6}}\left(-\underset{\sim}{i}+2\underset{\sim}{j}+\underset{\sim}{k}\right)$$
$$=2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}+\frac{1}{6}\left(-\underset{\sim}{i}+2\underset{\sim}{j}+\underset{\sim}{k}\right)$$
$$=\frac{11}{6}\underset{\sim}{i}+\frac{8}{6}\underset{\sim}{j}-\frac{5}{6}\underset{\sim}{k}$$
$$=\frac{1}{6}\left(11\underset{\sim}{i}+8\underset{\sim}{j}-5\underset{\sim}{k}\right)$$

The answer is D.

Question 25

$$\underset{\sim}{r}(t)=\tan(t)\underset{\sim}{i}+3\underset{\sim}{j}+e^{4t}\underset{\sim}{k}$$
$$\underset{\sim}{v}(t)=\sec^2(t)\underset{\sim}{i}+4e^{4t}\underset{\sim}{k}$$
$$\underset{\sim}{v}(0)=\sec^2(0)\underset{\sim}{i}+4e^{0}\underset{\sim}{k}$$
$$=\frac{1}{\cos^2(0)}\underset{\sim}{i}+4\underset{\sim}{k}$$
$$=\underset{\sim}{i}+4\underset{\sim}{k}$$

The motion of the particle initially is in the direction of $\underset{\sim}{i}+4\underset{\sim}{k}$.

The answer is D.

Question 26

$$\underset{\sim}{a}(t) = 6t^2\,\underset{\sim}{i} + \cos(2t)\,\underset{\sim}{j}\ \ ,\ \ t \geq 0$$

$$\underset{\sim}{v}(t) = 2t^3\,\underset{\sim}{i} + \frac{1}{2}\sin(2t)\,\underset{\sim}{j} + \underset{\sim}{c}$$

When $t = 0,\ \underset{\sim}{v} = \underset{\sim}{0}$

$$\underset{\sim}{0} = 0\,\underset{\sim}{i} + 0\,\underset{\sim}{j} + \underset{\sim}{c}$$

So $\underset{\sim}{v}(t) = 2t^3\,\underset{\sim}{i} + \frac{1}{2}\sin(2t)\,\underset{\sim}{j}$

The momentum of the particle, $m\underset{\sim}{v}$, is given by

$$8t^3\,\underset{\sim}{i} + 2\sin(2t)\,\underset{\sim}{j}$$

Note that momentum is a vector quantity.
The answer is C.

Question 27

$$\underset{\sim}{R} = m\,\underset{\sim}{a}$$

$$\underset{\sim}{F_1} + \underset{\sim}{F_2} = 5\,\underset{\sim}{a}$$

$$5\,\underset{\sim}{i} + (n+5)\,\underset{\sim}{j} = 5(\underset{\sim}{i} + 2\,\underset{\sim}{j})$$

$$5\,\underset{\sim}{i} + (n+5)\,\underset{\sim}{j} = 5\,\underset{\sim}{i} + 10\,\underset{\sim}{j}$$

So $\quad n = 5$
The answer is D.

Question 28

The box is stationary.
Resolving horizontally, we have

$$Fr = 5\cos(30^\circ)$$

$$Fr = \frac{5\sqrt{3}}{2}$$

So options A and C are not correct.
Note, since the box is not on the point of moving across the table,

$$Fr \neq \mu N$$

In fact $Fr < \mu N$.
So option D is correct.
Resolving vertically, we have

$$N + 5\sin(30^\circ) = 12g$$

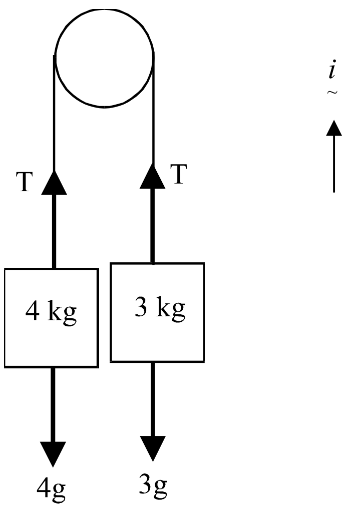
$$N = 12g - \frac{5}{2}$$

So option B and option E are incorrect.
The answer is D.

Question 29

The gravitational forces acting on each particle are $m_A g$ and $m_B g$. This eliminates options A and D.
The friction forces act in the opposite direction to the force *F*. This eliminates option E. The tension force in the connection runs in both directions. This eliminates option C.
Only option B shows all forces correctly.
The answer is B.

Question 30



Around the 3kg box.

$\underset{\sim}{R} = m\,\underset{\sim}{a}$

$(T - 3g)\underset{\sim}{i} = 3a\,\underset{\sim}{i}$

So $a = \dfrac{T - 3g}{3}$

The answer is A.

PART II

Question 1

When George finally passes the stationary red car, each has covered the same distance since George was originally overtaken.
Let the original speed of the red car be v.

So, $v\times 10+\frac{1}{2}\times(30-10)\times v=20\times 30$ **(1 mark)**

$$10v+10v=600$$

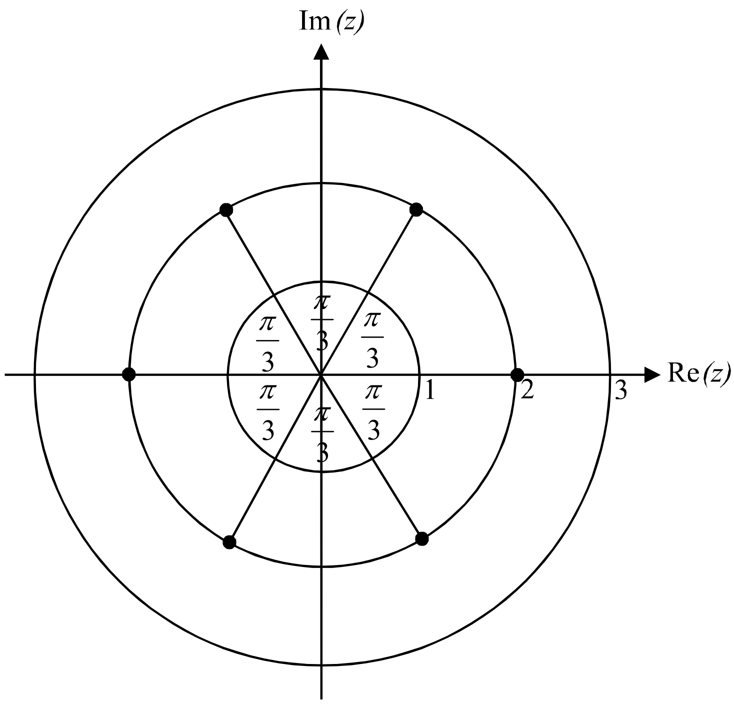
$$v=30$$

The red car was travelling at 30 m/s.

(1 mark)

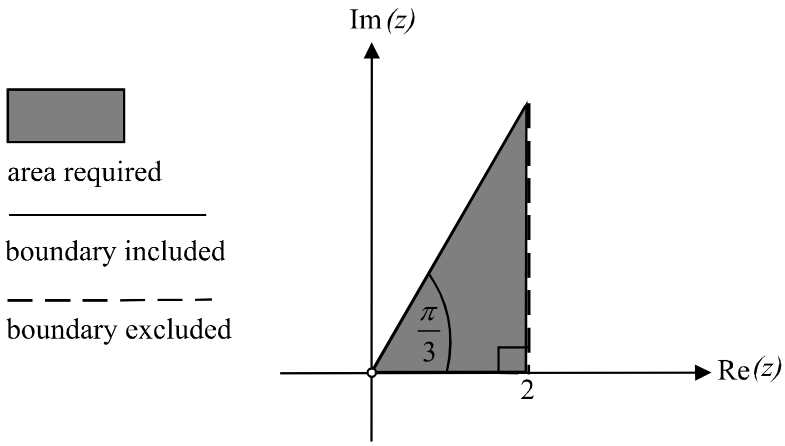
Question 2

a. The 6 solutions to the equation $z^6-64=0$ are spaced evenly around a circle with radius $64^{\frac{1}{6}}=2$. Since one has already been given to us the others must be spaced at intervals of $2\pi\div 6=\frac{\pi}{3}$ apart as indicated in the diagram below.



(1 mark)

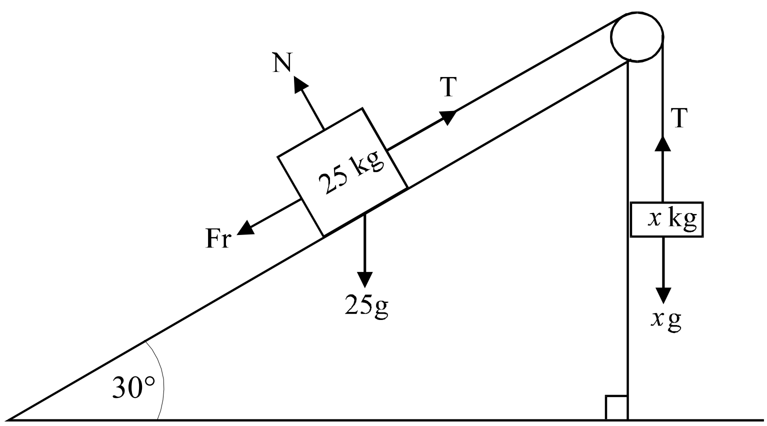
b.



(1 mark) correct area
(1 mark) correct boundaries

Question 3

a.



(1 mark)

b. The system is at the point of moving so $\underset{\sim}{R} = \underset{\sim}{0}$.

Resolving around the 25kg object we have

$T = Fr + 25g\sin(30^\circ)$ and $N = 25g\cos(30^\circ)$

$= \mu N + \dfrac{25g}{2}$ $\quad\quad = \dfrac{25\sqrt{3}g}{2}$

$= 0\cdot 6 \times \dfrac{25\sqrt{3}g}{2} + \dfrac{25g}{2}$ **(1 mark)** for correct resolution

$= 7\cdot 5\sqrt{3}g + 12.5g$

(1 mark)

Resolving around the x kg object we have

$T = xg$

So $x = 7\cdot 5\sqrt{3} + 12.5$

$= 25.49$ (correct to 2 decimal places)

(1 mark)

Question 4

a.
$$\sqrt{2}\sin\left(2x+\frac{\pi}{4}\right)=\cos(2x)$$

$$\sqrt{2}\left(\sin(2x)\cos\left(\frac{\pi}{4}\right)+\cos(2x)\sin\left(\frac{\pi}{4}\right)\right)=\cos(2x)$$ **(1 mark)**

$$\sqrt{2}\left(\sin(2x)\times\frac{1}{\sqrt{2}}+\cos(2x)\times\frac{1}{\sqrt{2}}\right)=\cos(2x)$$

$$\sin(2x)+\cos(2x)=\cos(2x)$$ Domain $0\le x\le 2\pi$

$$\sin(2x)=0$$ so $0\le 2x\le 4\pi$

$$2x=0,\pi,2\pi,3\pi,4\pi$$

$$x=0,\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi$$ **(1 mark)**

b.
$$\int_0^{\frac{\pi}{6}}\sin^2(x)\cos^3(x)\,dx$$

$$=\int_0^{\frac{\pi}{6}}\sin^2(x)\cos^2(x)\cos(x)\,dx$$

$$=\int_0^{\frac{\pi}{6}}\sin^2(x)\left(1-\sin^2(x)\right)\cos(x)\,dx$$

$$=\int_0^{\frac{\pi}{6}}\left(\sin^2(x)-\sin^4(x)\right)\cos(x)\,dx$$ **(1 mark)**

$$=\int_0^{\frac{1}{2}}\left(u^2-u^4\right)\frac{du}{dx}\,dx$$ **(1 mark)** for integrand
(1 mark) for terminals

$u=\sin(x)$

$\frac{du}{dx}=\cos(x)$

$x=\frac{\pi}{6},\ u=\frac{1}{2}$

$x=0,\ u=0$

$$=\left[\frac{u^3}{3}-\frac{u^5}{5}\right]_0^{\frac{1}{2}}$$

$$=\left\{\left(\frac{\left(\frac{1}{2}\right)^3}{3}-\frac{\left(\frac{1}{2}\right)^5}{5}\right)-0\right\}$$

$$=\frac{1}{24}-\frac{1}{160}$$

$$=\frac{20-3}{480}$$

$$=\frac{17}{480}$$ **(1 mark)**

Question 5

a. Area required

$$= \frac{\frac{\pi}{3} - 0}{2 \times 2}\left(f(0) + 2f\left(\frac{\pi}{6}\right) + f\left(\frac{\pi}{3}\right)\right)$$ **(1 mark)**

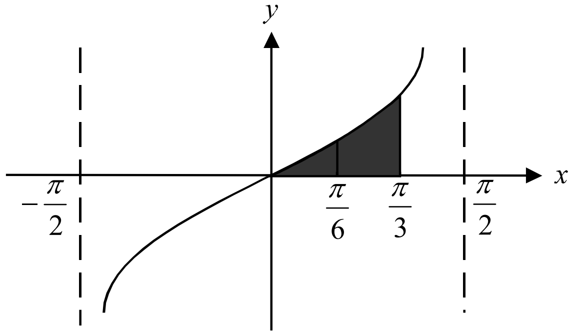
$$= \frac{\pi}{12}\left(\tan(0) + 2\tan\left(\frac{\pi}{6}\right) + \tan\left(\frac{\pi}{3}\right)\right)$$

$$= \frac{\pi}{12}\left(0 + \frac{2}{\sqrt{3}} + \sqrt{3}\right)$$

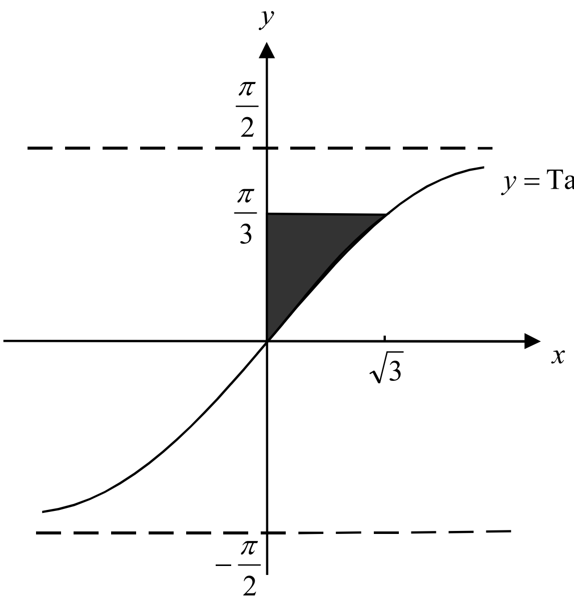
$$= \frac{\pi}{12}\left(\frac{2+3}{\sqrt{3}}\right)$$

$$= \frac{\pi}{12} \times \frac{5}{\sqrt{3}}$$

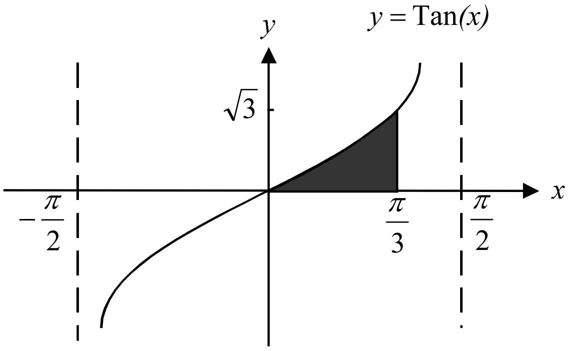
$$= \frac{5\sqrt{3}\pi}{36} \text{ square units}$$ **(1 mark)**



b. Draw a diagram.



The shaded area above is equal to the shaded area shown in the diagram below on the graph of its inverse function, $y = \text{Tan}(x)$.



$$\text{Area required} = \int_0^{\frac{\pi}{3}} \text{Tan}(x)\, dx$$ **(1 mark)**

$$= \int_0^{\frac{\pi}{3}} \tan(x)\, dx, \qquad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$$= \int_0^{\frac{\pi}{3}} \frac{\sin(x)}{\cos(x)} dx$$

let $u = \cos(x)$

$$\frac{du}{dx} = -\sin(x)$$

$x = 0$ so $u = 1$

$x = \frac{\pi}{3}$ so $u = \frac{1}{2}$

(1 mark)

$$= \int_1^{\frac{1}{2}} \frac{1}{u} \cdot -\frac{du}{dx} dx$$

$$= \int_{\frac{1}{2}}^{1} \frac{1}{u} du$$

$$= \left[\log_e(u)\right]_{\frac{1}{2}}^{1}$$

$$= \log_e(1) - \log_e\left(\frac{1}{2}\right)$$

$$= 0 - \log_e\left(\frac{1}{2}\right)$$

$$= \log_e(2)$$

Area required is $\log_e(2)$ square units. **(1 mark)**

(Do NOT express this answer as a decimal approximation since you have been asked for an exact value. Also, if you have time, check your answer using your graphics calculator.)

Total 20 marks

