VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2003

SPECIALIST MATHEMATICS

Written examination 1
(Facts, skills and applications)

Monday 3 November 2003

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART I
MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
30	30	30

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 17 pages, with a detachable sheet of miscellaneous formulas in the centrefold and two blank pages for rough working.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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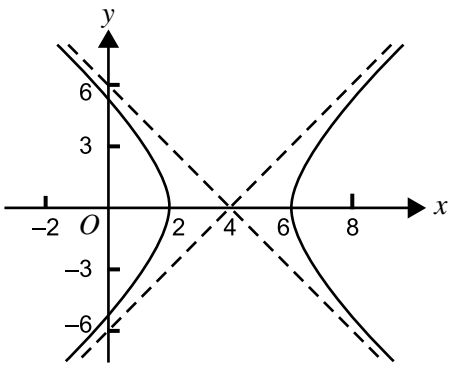
Instructions for Part I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1



The graph shown above could have the equation

A. $\frac{(x-4)^2}{16} - \frac{y^2}{9} = 1$

B. $\frac{(x-4)^2}{4} - \frac{y^2}{36} = 1$

C. $\frac{(x-4)^2}{16} - \frac{y^2}{36} = 1$

D. $\frac{(x-4)^2}{4} - \frac{y^2}{9} = 1$

E. $\frac{(x-4)^2}{9} - \frac{y^2}{4} = 1$

Question 2

If $\sin(x) = -\frac{1}{5}$, $\pi \le x \le \frac{3\pi}{2}$, then $\cot(x)$ is equal to

A. $-2\sqrt{6}$

B. $2\sqrt{6}$

C. $-\frac{1}{2\sqrt{6}}$

D. $\frac{1}{2\sqrt{6}}$

E. $-\frac{5}{2\sqrt{6}}$

Question 3

The number of solutions of $\sin^2(2x) = \frac{3}{4}$, for $0 \le x \le 2\pi$, is

A. 1

B. 2

C. 4

D. 6

E. 8

Question 4

If $y = \text{Sin}^{-1}\left(\frac{4}{x}\right)$, $x > 4$, then $\frac{dy}{dx}$ is equal to

A. $-\frac{4}{x\sqrt{x^2-16}}$

B. $\frac{x}{\sqrt{x^2-16}}$

C. $-\frac{4}{x\sqrt{x^2-4}}$

D. $-\frac{4}{\sqrt{x^2-16}}$

E. $\frac{4}{x\sqrt{x^2-16}}$

Question 5

Which one of the following is a polar form of $-\sqrt{3} - i$?

A. $2\text{cis}\left(-\frac{2\pi}{3}\right)$

B. $2\text{cis}\left(\frac{\pi}{6}\right)$

C. $2\text{cis}\left(\frac{\pi}{3}\right)$

D. $2\text{cis}\left(\frac{5\pi}{6}\right)$

E. $2\text{cis}\left(\frac{7\pi}{6}\right)$

Question 6

If $z^2 = 4\text{cis}\left(\frac{4\pi}{3}\right)$, then z is equal to

A. $1 - \sqrt{3}i$ or $-1 + \sqrt{3}i$

B. $\sqrt{3} + i$ or $-\sqrt{3} - i$

C. $-1 + \sqrt{3}i$ or $-1 - \sqrt{3}i$

D. $\sqrt{3} - i$ or $-\sqrt{3} + i$

E. $1 - \sqrt{3}i$ or $1 + \sqrt{3}i$

Question 7

If $P(z) = z^3 - 2z^2 + 4z - 8, z \in C,$ then a linear factor of $P(z)$ is

A. 2

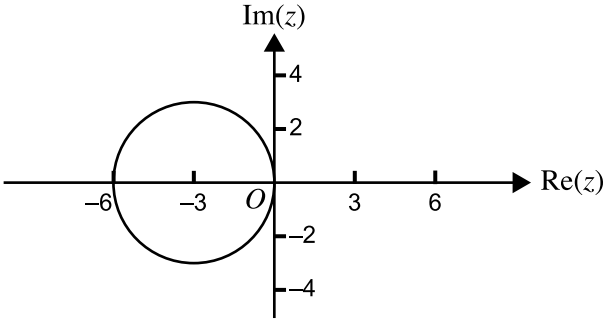
B. $2i$

C. $z + 2$

D. $z + 2i$

E. $z^2 + 4$

Question 8



Which one of the following, where $z \in C$, is the equation of the circle in the diagram above?

A. $(z+3)(\bar{z}+3) = 9$

B. $(z-3)(\bar{z}+3) = 9$

C. $(z+3)(\bar{z}-3) = 9$

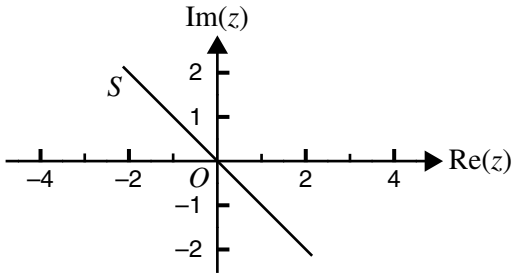
D. $(z-3)(\bar{z}-3) = 9$

E. $(z+3i)(\bar{z}-3i) = 9$

Question 9

Which one of the following shows the region of the complex plane specified by $S = \{z : |z-1| - |z+i| = 0, z \in C\}$?

A.



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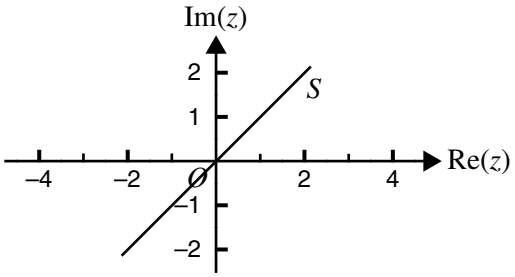
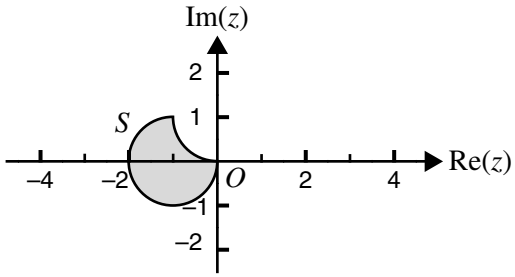
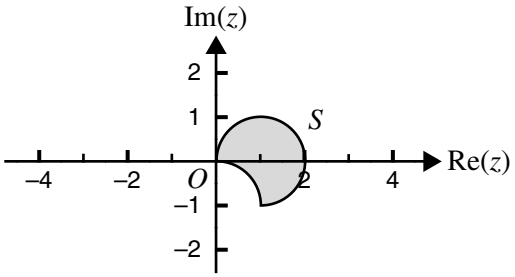


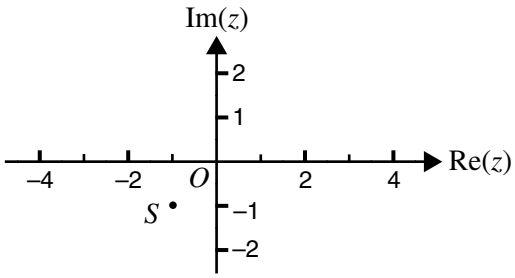
C.



D.



E.



Question 10

With a suitable substitution, $\int_0^{\frac{\pi}{6}} \cos^3(2x)\,dx$ can be expressed as

A. $\frac{1}{2}\int_0^{\frac{1}{2}} (1-u^2)\,du$

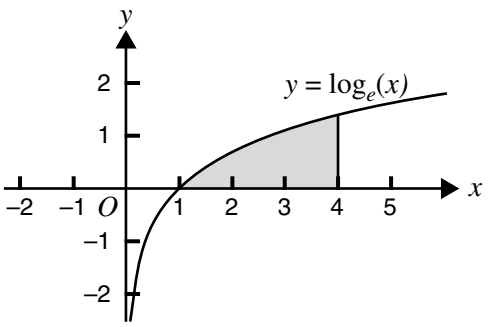
B. $\frac{1}{2}\int_0^{\frac{1}{2}} (u^2-1)\,du$

C. $\frac{1}{2}\int_0^{\frac{\sqrt{3}}{2}} (1-u^2)\,du$

D. $2\int_0^{\frac{1}{2}} (u^2-1)\,du$

E. $2\int_0^{\frac{\sqrt{3}}{2}} (1-u^2)\,du$

Question 11



The shaded region in the diagram above is bounded by the graph of $y = \log_e(x)$, the x-axis, and the line with equation $x = 4$. The trapezium rule with three equal intervals is used to estimate the area of the shaded region. The value obtained is $\log_e(a)$, where a is

A. $\sqrt{17}$

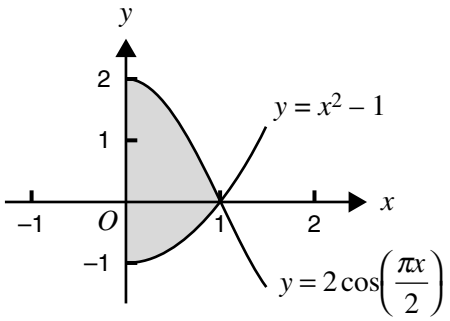
B. $\sqrt{24}$

C. $\sqrt{96}$

D. 12

E. 13

Question 12



The shaded region in the diagram above is bounded by the y-axis, and the curves with equations $y = x^2 - 1$ and $y = 2\cos\left(\frac{\pi x}{2}\right)$.

The **exact** value of the area of the shaded region is

A. $\frac{10}{3}$

B. $\frac{14}{3}$

C. $\frac{4}{\pi} - \frac{4}{3}$

D. $\frac{4}{\pi} - \frac{2}{3}$

E. $\frac{4}{\pi} + \frac{2}{3}$

Question 13

Which one of the following is an antiderivative of $\frac{3}{x(3-x)}$ for $0 < x < 3$?

A. $3(\log_e(x) - \log_e(3-x))$

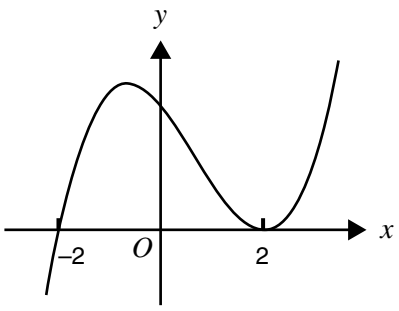
B. $\log_e(x) - \log_e(3-x)$

C. $\log_e(x-3) - \log_e(x)$

D. $3(\log_e(x) + \log_e(3-x))$

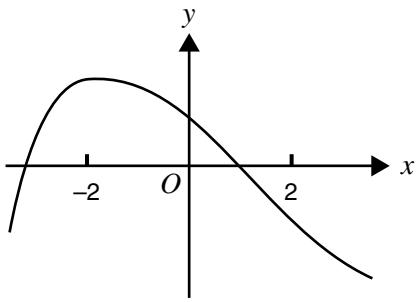
E. $\log_e(x) + \log_e(3-x)$

Question 14

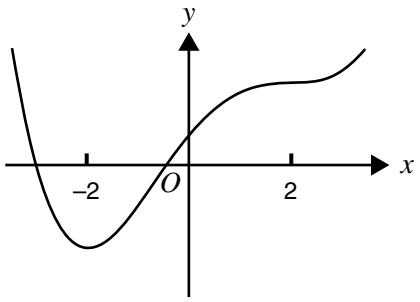


Part of the graph of the derivative of a function f is shown above. Which one of the following could be the graph of the function f?

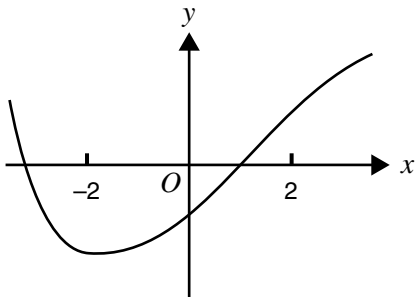
A.



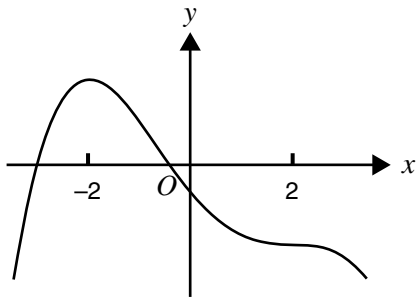
B.



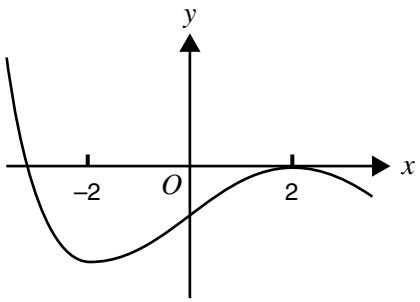
C.



D.



E.



Question 15

If $f'(x) = 2\sin^2\left(\frac{x}{2}\right) - 1$ and $f\left(\frac{\pi}{2}\right) = 0$, then $f(x)$ is equal to

A. $-\sin(x)$

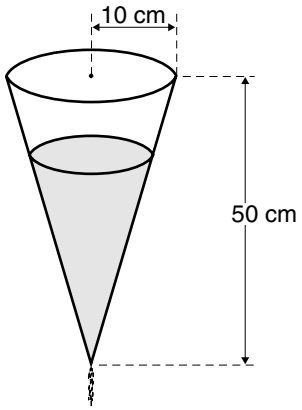
B. $1 - \sin(x)$

C. $\sin(x) + 1$

D. $\sin(x) - 1$

E. $-1 - \sin(x)$

Question 16



Water is draining from a cone-shaped funnel at the constant rate of 600 cm^3/ min.

The cone has height 50 cm and base radius 10 cm.

Let h cm be the depth of water in the funnel at time t min.

The rate of **decrease** of h, in cm/min, is given by

A. 12

B. $\frac{100\pi}{3}$

C. $\frac{15000}{\pi h^2}$

D. $24\pi h^2$

E. $\frac{18}{\pi}$

Question 17

Euler's method, with a step size of 0.2, is used to solve the differential equation $\frac{dy}{dx} = \cos\left(\frac{x}{2}\right)$, with initial condition $y = 2$ when $x = 0$.

The approximation obtained for y when $x = 0.4$ is given by

A. $2 + 0.4\cos(0.1)$

B. $2 + 0.4\cos(0.2)$

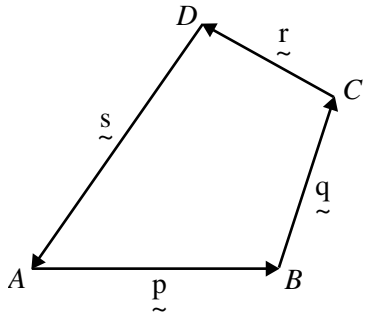
C. $2.2 + 0.2\cos(0.1)$

D. $2.2 + 0.2\cos(0.2)$

E. $2 + 0.2\cos(0.1) + 0.2\cos(0.2)$

Question 18

In the quadrilateral $ABCD$, $\overrightarrow{AB} = \underset{\sim}{p}$, $\overrightarrow{BC} = \underset{\sim}{q}$, $\overrightarrow{CD} = \underset{\sim}{r}$, and $\overrightarrow{DA} = \underset{\sim}{s}$ as shown below.



Which one of the following statements is true?

A. $\underset{\sim}{p} + \underset{\sim}{q} = \underset{\sim}{r} + \underset{\sim}{s}$

B. $\underset{\sim}{p} + \underset{\sim}{r} = \underset{\sim}{q} + \underset{\sim}{s}$

C. $\underset{\sim}{p} - \underset{\sim}{q} = \underset{\sim}{r} - \underset{\sim}{s}$

D. $\underset{\sim}{p} - \underset{\sim}{r} = \underset{\sim}{q} + \underset{\sim}{s}$

E. $\underset{\sim}{p} + \underset{\sim}{q} = -\underset{\sim}{r} - \underset{\sim}{s}$

Question 19

Which one of the following is a unit vector opposite in direction to $\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$?

A. $\frac{1}{\sqrt{14}}(-\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k})$

B. $\frac{1}{\sqrt{14}}(-\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k})$

C. $\frac{1}{\sqrt{6}}(-\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k})$

D. $\frac{1}{\sqrt{6}}(-\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k})$

E. $\frac{1}{\sqrt{2}}(-\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k})$

Question 20

Let O denote the point $(0, 0)$, R the point $(-2, 1)$, and S the point $(-c, 2c)$ where $c \in R$.

The scalar product $\overrightarrow{OS} \cdot \overrightarrow{RS}$ is equal to

A. $4c - 5$

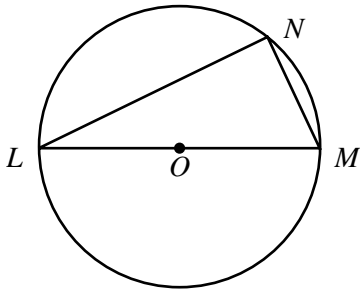
B. $5 - 4c$

C. $4c - 5c^2$

D. $5c^2 - 4c$

E. $(c^2 - 2c)\underset{\sim}{i} + (4c^2 - 2c)\underset{\sim}{j}$

Question 21



In the diagram above, LOM is a diameter of the circle with centre O.
N is a point on the circumference of the circle.
Let $\overrightarrow{OM} = \underset{\sim}{r}$ and $\overrightarrow{LN} = \underset{\sim}{q}$.
Which one of the following must be true?

A. $2\underset{\sim}{r} + \underset{\sim}{q} = \underset{\sim}{0}$

B. $2\underset{\sim}{r} - \underset{\sim}{q} = \underset{\sim}{0}$

C. $\underset{\sim}{r} \cdot \underset{\sim}{q} = \underset{\sim}{q} \cdot \underset{\sim}{q}$

D. $2\underset{\sim}{r} \cdot \underset{\sim}{q} = \underset{\sim}{q} \cdot \underset{\sim}{q}$

E. $2\underset{\sim}{r} \cdot \underset{\sim}{q} = -\underset{\sim}{q} \cdot \underset{\sim}{q}$

Question 22

Let $\underset{\sim}{i}$ and $\underset{\sim}{j}$ be unit vectors in the east and north directions respectively.
At time t, $t \geq 0$, the position vector of particle L is given by $\underset{\sim}{r} = (5t - 8)\underset{\sim}{i} + (t^2 - 5t + 6)\underset{\sim}{j}$, and the position vector of particle M is given by $\underset{\sim}{s} = (t^2 - t)\underset{\sim}{i} + (3 - t)\underset{\sim}{j}$.
Particle L is directly north of particle M at time

A. 0

B. 1

C. 2

D. 3

E. 4

Question 23

The position vector of a particle at time t is given by $\underset{\sim}{r} = 4t\underset{\sim}{i} - e^{2t}\underset{\sim}{j} + 5\underset{\sim}{k}$, $t \geq 0$.
The initial speed of the particle is

A. $\sqrt{12}$

B. $\sqrt{17}$

C. $\sqrt{20}$

D. $\sqrt{26}$

E. $\sqrt{45}$

Question 24

The acceleration, in m/s^2, of a particle at time t s, $t \geq 0$, is given by $\ddot{\underset{\sim}{r}} = \cos(t)\,\underset{\sim}{i} - \sin(t)\,\underset{\sim}{j}$.

The initial velocity of the particle was $(\underset{\sim}{i} + \underset{\sim}{j})$ m/s.

The velocity of the particle, in m/s, at time t s is given by

A. $-\sin(t)\,\underset{\sim}{i} - \cos(t)\,\underset{\sim}{j}$

B. $\sin(t)\,\underset{\sim}{i} + \cos(t)\,\underset{\sim}{j}$

C. $(\sin(t) + 1)\,\underset{\sim}{i} + \cos(t)\,\underset{\sim}{j}$

D. $\sin(t)\,\underset{\sim}{i} + (\cos(t) + 1)\,\underset{\sim}{j}$

E. $(1 - \sin(t))\,\underset{\sim}{i} + (2 - \cos(t))\,\underset{\sim}{j}$

Question 25

A body of mass 12 kg is pulled along rough, horizontal ground by a horizontal force of 66 newtons. The body is moving with an acceleration of 0.5 m/s^2.

The coefficient of sliding friction between the body and the surface, correct to two decimal places, is

A. 0.06

B. 0.51

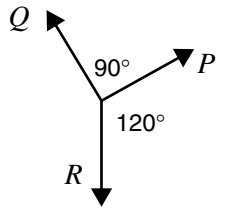
C. 0.56

D. 0.61

E. 1.96

Question 26

The following diagram shows a particle in equilibrium in a plane under the action of three forces of magnitudes P, Q and R.



Which one of the following statements is **not** correct?

A. $P = R\cos(60^\circ)$

B. $Q = R\cos(30^\circ)$

C. $R = Q\cos(30^\circ)$

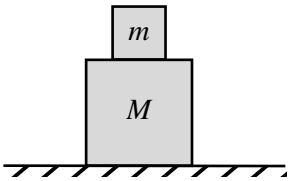
D. $Q\cos(60^\circ) = P\cos(30^\circ)$

E. $P\cos(60^\circ) + Q\cos(30^\circ) = R$

Question 27

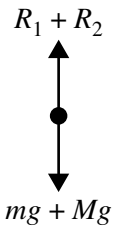
A small mass of m kg sits on top of a larger mass of M kg on rough, level ground. The two masses are at rest.

There is a normal reaction force of magnitude R_1 on the smaller mass due to its contact with the larger mass. There is a normal reaction force of magnitude R_2 on the larger mass due to its contact with the ground. All forces are in newtons.

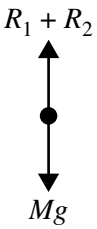


Which one of the following diagrams correctly shows the forces acting on the **larger** mass?

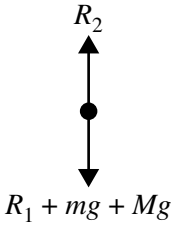
A.



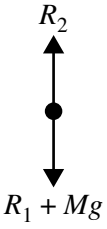
B.



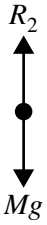
C.



D.



E.



Question 28

A particle moves with constant acceleration in a straight line so that at time t, $t \geq 0$, its velocity is v and its displacement from a fixed point on the line is x.

Which one of the following equations could **not** be true?

A. $t = v - 1$

B. $t = x^2 - 1$

C. $x = t^2 - 1$

D. $x = v^2 - 1$

E. $v = t - 1$

Question 29

At time t s, $t \geq 0$, the velocity, v m/s, of a particle moving in a straight line is given by $v = \cos(t) + \sqrt{3}\sin(t) - 1$.

For what value of t does the particle first attain its maximum **speed** of 3 m/s?

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

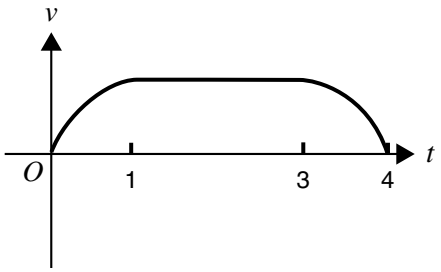
C. $\frac{7\pi}{6}$

D. $\frac{4\pi}{3}$

E. The particle never attains a speed of 3 m/s.

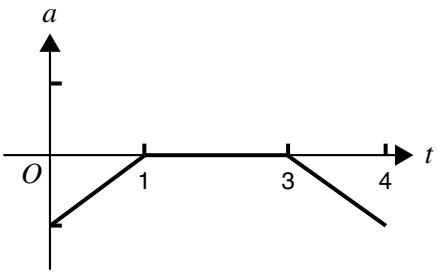
Question 30

The following is the velocity-time graph of a racing car over a short course.

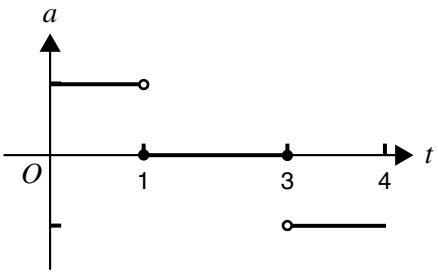


Which one of the following could be the acceleration-time graph of the car's motion?

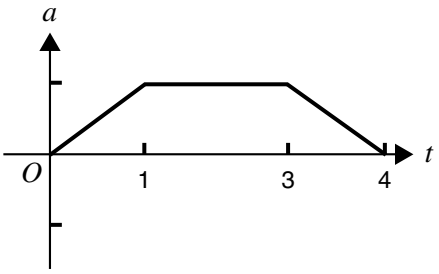
A.



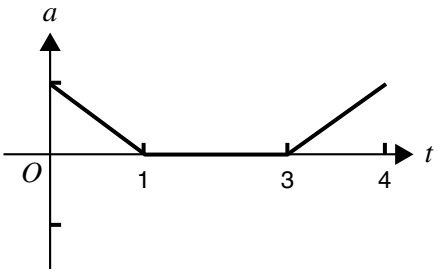
B.



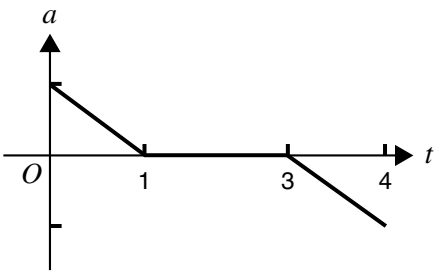
C.



D.



E.



Working space

Working space

END OF PART I MULTIPLE-CHOICE QUESTION BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2003

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

										Letter
Figures										
Words										

SPECIALIST MATHEMATICS
Written examination 1
(Facts, skills and applications)

Monday 3 November 2003

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

PART II
QUESTION AND ANSWER BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions). Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	20

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 7 pages.

Instructions
- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Instructions for Part II

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

Where an **exact** answer is required to a question, appropriate working must be shown.

In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

A particle of mass 5 kg moves in a straight line.

At time t s, $t \geq 0$, it has velocity v m/s, where $v = 4.5t + \cos(2t)$.

a. Find an expression for the acceleration of the particle at time t.

1 mark

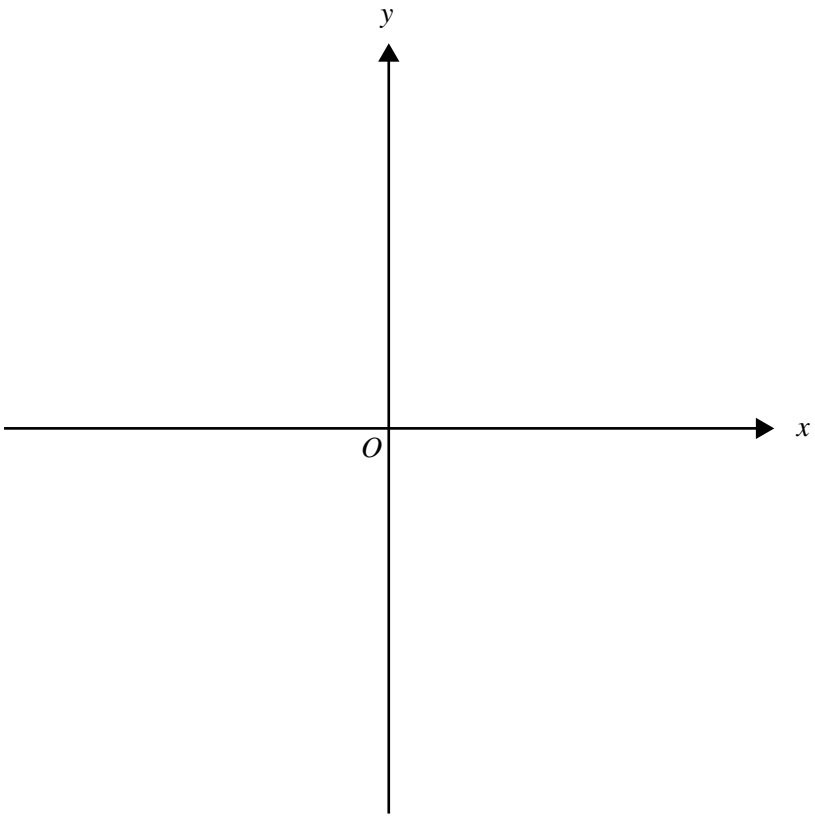
b. Find the minimum resultant force acting on the particle during its motion.

2 marks

Question 2

On the axes below, sketch the graph of $f: R \to R$ where $f(x) = \dfrac{x^2 - 6}{2x}$.

Give the equations of any asymptotes, and the coordinates of any intercepts and any turning points.



4 marks

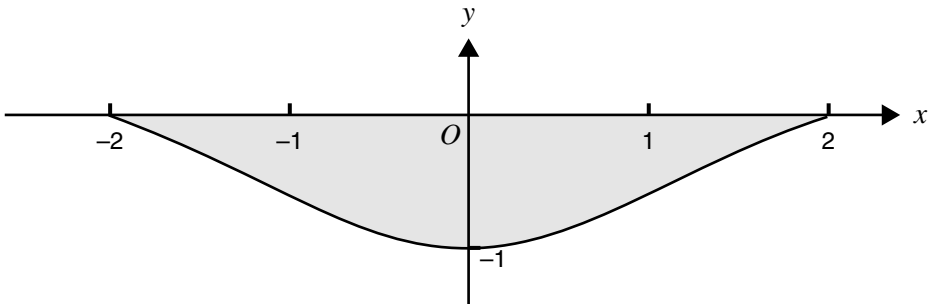
Question 3

$y = xe^{3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + m\frac{dy}{dx} + ny = 0$, where $m,\ n \in R$.
Find the values of m and n.

3 marks

Question 4

The graph of $y = 1 - \dfrac{8}{x^2 + 4}$ is shown below for $-2 \le x \le 2$.



a. Find the **exact** value of the area of the shaded region.

3 marks

b. The shaded region is rotated about the y-axis to form a solid of revolution.

Express the volume of this solid of revolution as a definite integral and hence find the volume correct to three significant figures.

3 marks

Question 5

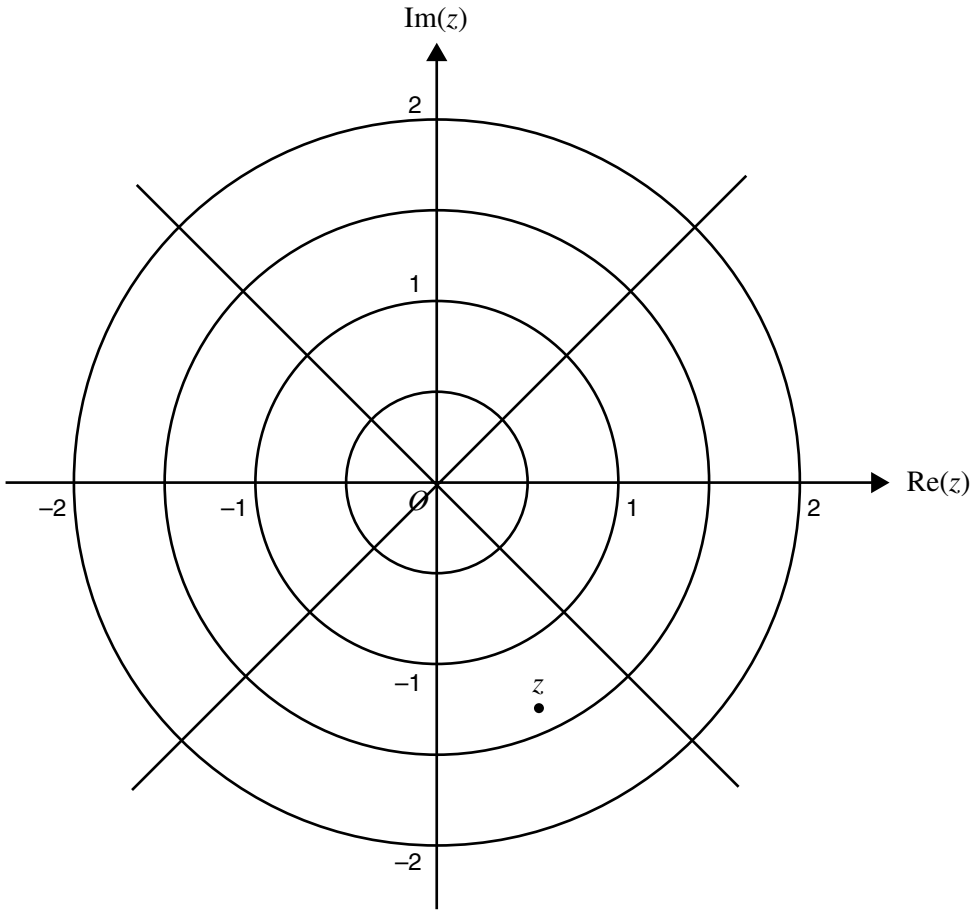
The complex number $z = \sqrt{2}\,\text{cis}(\theta)$ is plotted on the Argand diagram below.

On the same diagram, plot and label clearly the following three complex numbers.

a. $u = z^2$

b. $v = \dfrac{1}{z}$

c. $w = z^2 + \dfrac{1}{z}$



4 marks

END OF PART II QUESTION AND ANSWER BOOK