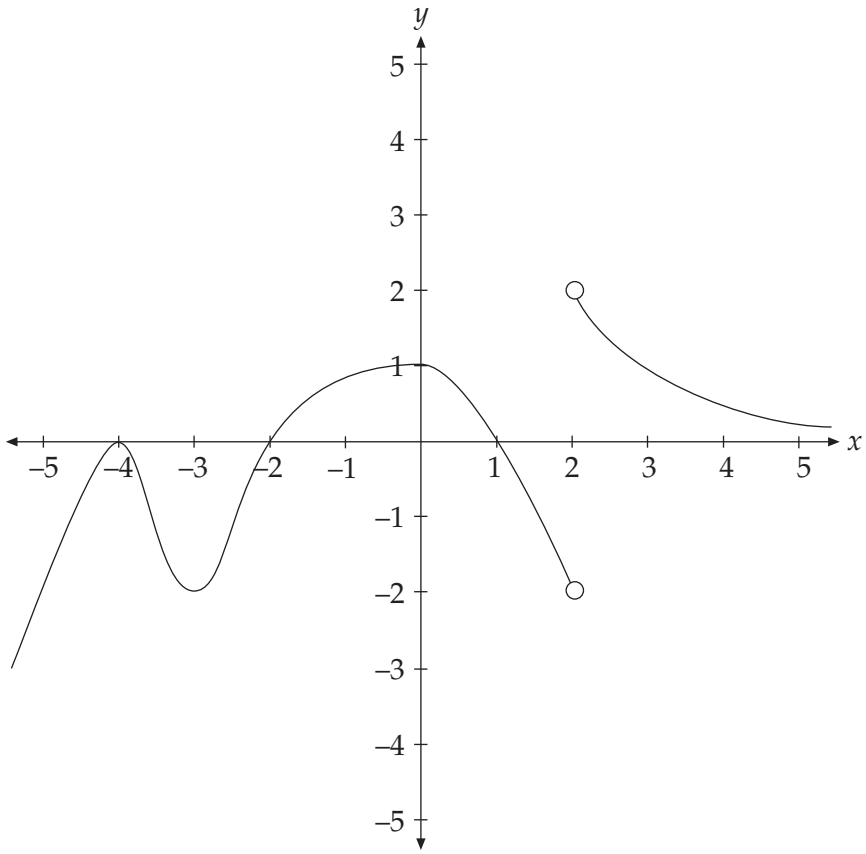
Question 1

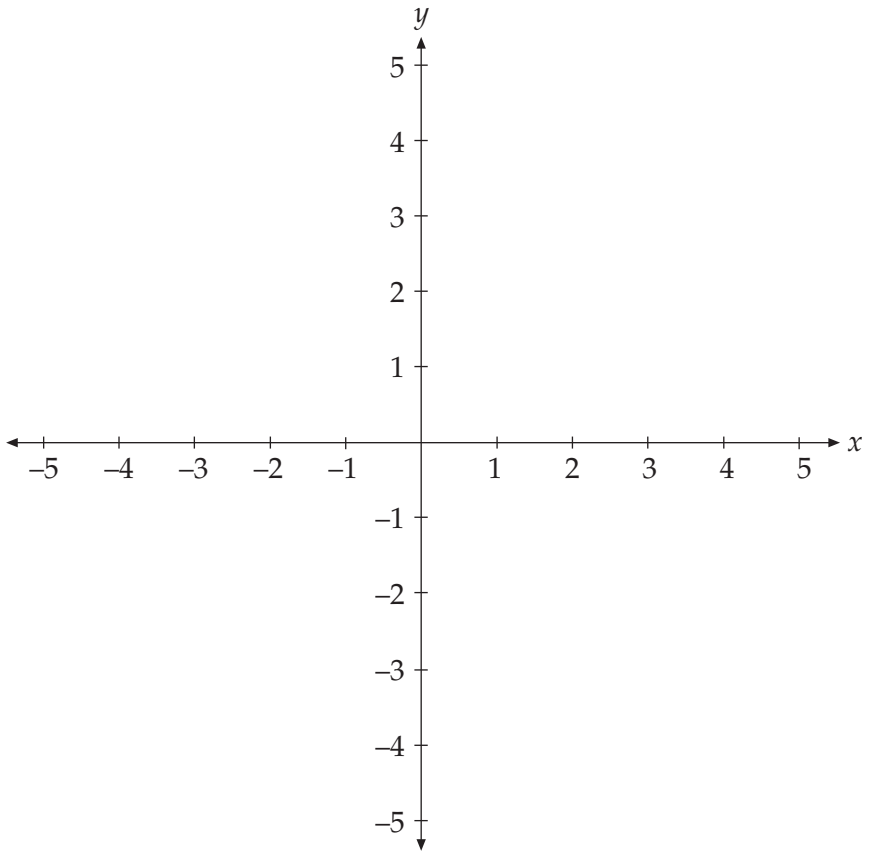
Below is a sketch graph of $y = f'(x)$ where $f'(x)$ is the derivative of the function $y = f(x)$. The curve $y = f'(x)$ has an asymptote at $y = 0$



a. State the x values for each of the turning points of the curve $y = f(x)$ and by using $f''(x)$ classify them.

3 marks

b. On the axes below sketch the curve $y = f(x)$ given $f(2) = -1$, $f(0) = 0$, and the graph of $f(x)$ is continuous.



3 marks

Total 6 marks

Question 2

Two sail boats Ragin' ($\underset{\sim}{r}$) and Starin' ($\underset{\sim}{s}$) are about to commence a race. Both are heading towards the start line with velocity $\underset{\sim}{\dot{r}}(t) = -2t\,\underset{\sim}{i} + 3\,\underset{\sim}{j}$ and $\underset{\sim}{\dot{s}}(t) = \left(4t^3 - 9t^2 + 2\right)\underset{\sim}{i} + 2t\,\underset{\sim}{j}$ (Knots) respectively. As the gun is fired to indicate the start of the race $(t = 0)$ their position vectors are $\underset{\sim}{s} = 2\,\underset{\sim}{i}$ and $\underset{\sim}{r} = 2\,\underset{\sim}{i} - 2\,\underset{\sim}{j}$.

a i. Find the position vector $\underset{\sim}{s}(t)$ at time t of Starin'?

2 marks

ii. Find the position vector $\underset{\sim}{r}(t)$ at time t of Ragin'?

2 marks

b i. Show that Ragin' and Starin' will collide under the conditions described.

5 marks

ii. Find the coordinates of the point of the collision.

1 mark

If at the point of collision the angle between the boats is less than 30° and both boats are travelling at a speed of no more than 5 knots they will be able to continue their race.

c What is the angle between the paths of the sailing boats at collision? Give your answer to the nearest degree.

4 marks

Total 14 marks

Question 3

a i. Show that the derivative of $\text{Cos}^{-1}\left(\frac{x-1}{x+1}\right)$ is $\frac{-1}{\sqrt{x}(x+1)}$

3 marks

ii. Hence, find the **exact** value of $\int_1^3 \frac{1}{\sqrt{x^3}+\sqrt{x}}\,dx$

2 marks

b i. For the functions $u = \text{Cos}^{-1}\left(\frac{x-1}{x+1}\right)$ and $v = \text{Tan}^{-1}\sqrt{x}$, find the value of a where

$\frac{du}{dx} + a\frac{dv}{dx} = 0$

3 marks

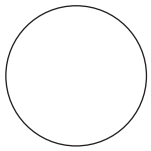
ii. Hence, show that $u + av = \pi$ for all values of x, where $\{x: x > 0\}$

2 marks

Total 10 marks

Question 4

A spherical ball is projected vertically upwards. It has an initial velocity of 10 m/s, and a mass of 2 kg. The air resistance is equal to $\frac{2v^2}{g}$ newtons.



a. On the above diagram show all the forces acting on the ball 1 mark

b. Show that the acceleration of the sphere, until it reaches its highest point, is $a = -g - \frac{v^2}{g}$.

1 mark

c. Solve a suitable differential equation to express v in terms of x, where x represents the distance of the ball from its initial position

5 marks

d. Find the maximum height reached by the sphere correct to nearest metre.

3 marks

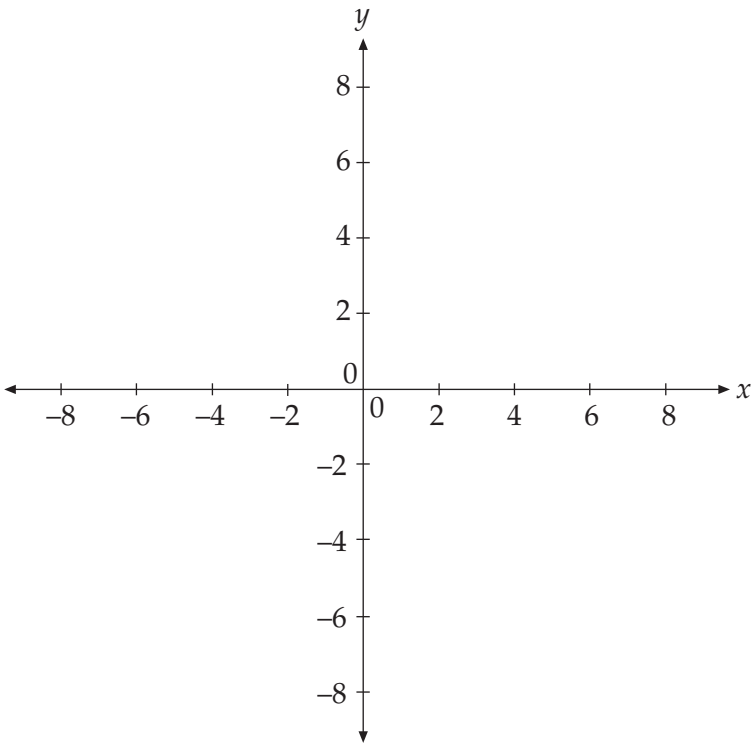
e. How long does it take for the sphere to reach its maximum height? Give your answer to the nearest tenth of a second.

3 marks

Total 13 marks

Question 5

a i. On the axes below sketch the graphs of $|z-6|<2$ and $|z+2|\geq|z-14|$ and hence shade $\{z:|z-6|<2\}\cap\{z:|z+2|\geq|z-14|\}$.



3 marks

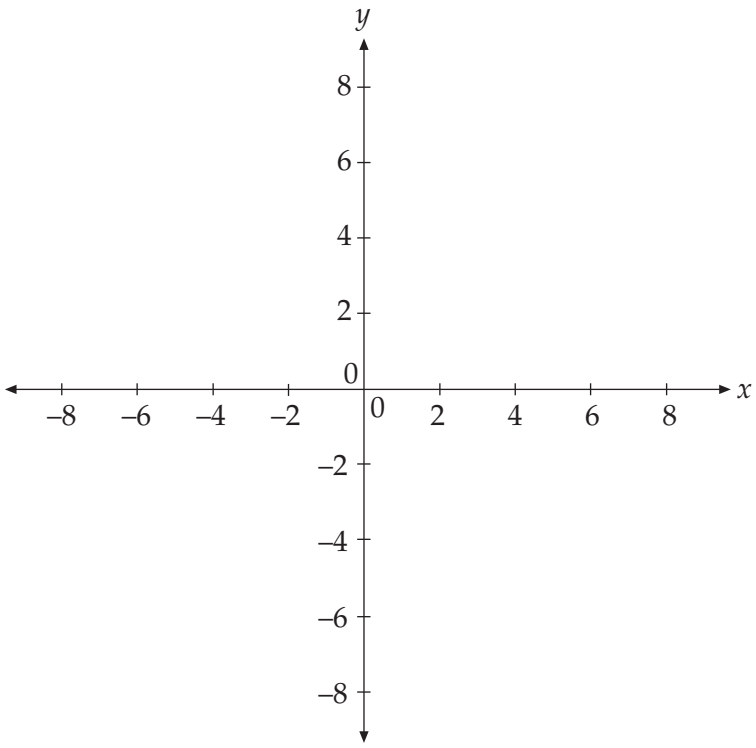
ii. Find the coordinates of the points of intersection between $\{z:|z-6|<2\}$ and $\{z:|z+2|\geq|z-14|\}$.

2 marks

b i. Express $u = 3 + 3\sqrt{3}i$ in the form $u = rcis\theta$.

2 marks

ii. On the axes below sketch the loci $|z - u| = k$ and $|z - 6| = 2$ for the smallest value of k such that the loci intersect at exactly one point.



2 marks

b iii. Write down the value of k and the coordinates of the point of intersection.

2 marks

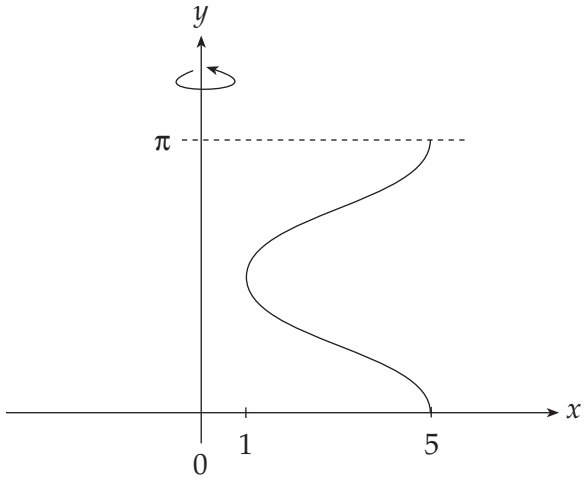
Total 11 marks

Question 6

a. Find the exact value of the definite integral $\int_0^{2\pi} \sin^2\left(\frac{1}{2}x\right)\cos^2\left(\frac{1}{2}x\right)dx$, showing working.

3 marks

The diagram below shows the graph of the relation $x = 2\cos(2y) + 3,\ \ 0 \le y \le \pi$



By rotating the curve about the y-axis, a hollow solid of revolution is produced, having the shape of a vase. Measurements are in centimetres.

b. Find the exact value of the maximum volume of water that this vase can hold.

4 marks

Total 7 marks