Year 2003

VCE

Specialist Mathematics

Trial Examination 2

KILBAHA PTY LTD (Publishers in Education) **ABN 47 065 111 373** **PO BOX 2227 KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** www.chemas.com

STUDENT NUMBER

Figures

Words

VICTORIAN CERTIFICATE OF EDUCATION 2003

SPECIALIST MATHEMATICS

Trial Written Examination 2
(Analysis Task)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Directions to students

Materials
Question and answer booklet consists of 16 pages. There is a detachable formulae sheet of miscellaneous formulas at the end of the booklet.
You may bring into the examination up to (two A4 sheets) of pre-written notes.
You may use approved scientific and or graphics calculators, protractor, set square and aids for curve sketching.

The task.
Write your student number in the space provided above.
This examination consists of 6 questions, there is a total of 60 marks available.
You should attempt all questions.
You need not give numerical answers as decimals unless instructed to do so. A decimal approximation, no matter how accurate will not be accepted if an exact answer is required to a question.
Calculus must be used to evaluate derivatives and definite integrals. A decimal value no matter how accurate, will not be rewarded unless the appropriate working is shown.
Unless otherwise stated, diagrams in this book are not drawn to scale.
All responses should be in English.

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of triangle: $\frac{1}{2}bc\sin A$

sine rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

cosine rule: $c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2 x+\sin^2 x=1$

$1+\tan^2 x=\sec^2 x$ $\quad\quad \cot^2+1=\operatorname{cosec}^2 x$

$\sin(x+y)=\sin x\cos y+\cos x\sin y$ $\quad\quad \sin(x-y)=\sin x\cos y-\cos x\sin y$

$\cos(x+y)=\cos x\cos y-\sin x\sin y$ $\quad\quad \cos(x-y)=\cos x\cos y+\sin x\sin y$

$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$ $\quad\quad \tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$

$\cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x$

$\sin 2x=2\sin x\cos x$ $\quad\quad \tan 2x=\frac{2\tan x}{1-\tan^2 x}$

Function	Sin^{-1}	Cos^{-1}	Tan^{-1}
Domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\mathrm{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < Arg\,z \leq \pi$

$z_1 z_2 = r_1 r_2 \,\mathrm{cis}(\theta_1 + \theta_2)$ $\qquad \frac{z_1}{z_2} = \frac{r_1}{r_2}\,\mathrm{cis}(\theta_1 - \theta_2)$

$z^n = r^n \mathrm{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$|\bar{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \bar{r}_1.\bar{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\bar{r}} = \frac{d\bar{r}}{dt} = \frac{dx}{dt}\bar{i} + \frac{dy}{dt}\bar{j} + \frac{dz}{dt}\bar{k}$

Mechanics

momentum: $\bar{p} = m\bar{v}$

equation of motion: $\bar{R} = m\bar{a}$

sliding friction: $F \leq \mu N$

constant (uniform) acceleration:

$v = u + at$ $\qquad s = ut + \frac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \frac{1}{2}(u + v)t$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e x + c,\ for\ x > 0$$

$$\frac{d}{dx}(\sin ax) = a\cos ax \qquad \int \sin ax dx = -\frac{1}{a}\cos ax + c$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax \qquad \int \cos ax dx = \frac{1}{a}\sin ax + c$$

$$\frac{d}{dx}(\tan ax) = a\sec^2 ax \qquad \int \sec^2 ax dx = \frac{1}{a}\tan ax + c$$

$$\frac{d}{dx}(\mathrm{Sin}^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{dx}{\sqrt{a^2 - x^2}} = \mathrm{Sin}^{-1}\frac{x}{a} + c,\ a > 0$$

$$\frac{d}{dx}(\mathrm{Cos}^{-1}x) = \frac{-1}{\sqrt{1-x^2}} \qquad \int \frac{-1}{\sqrt{a^2 - x^2}} dx = \mathrm{Cos}^{-1}\frac{x}{a} + c,\ a > 0$$

$$\frac{d}{dx}(\mathrm{Tan}^{-1}x) = \frac{1}{1+x^2} \qquad \int \frac{a}{a^2 + x^2} dx = \mathrm{Tan}^{-1}\frac{x}{a} + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}(\frac{u}{v}) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)dx \approx (b-a)f(\frac{a+b}{2})$

trapezoidal rule: $\int_a^b f(x)dx \approx \frac{1}{2}(b-a)(f(a) + f(b))$

Euler's method

If $\frac{dy}{dx} = f(x), x_0 = a$ and $y_0 = b,$ then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

Question 1

Let $u = \frac{1}{4}\left(\sqrt{3} - i\right)$ and $v = \sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)$

i. Find uv working in Cartesian form giving your answer in exact $a + bi$ form.

2 marks

ii. Find uv working in polar form giving your answer in exact $r\,\text{cis}\,\theta$ form.

2 marks

iii. Hence deduce the exact value of $\sin\left(\frac{\pi}{12}\right)$

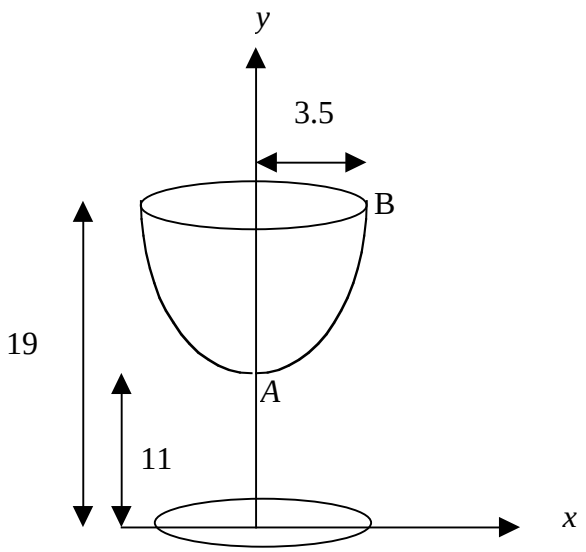
2 marks

iv. Using the formula $\sin(x-y)$ verify your exact value for $\sin\left(\frac{\pi}{12}\right)$

2 marks
Total 8 marks

Question 2

The diagram shows a wine glass with dimensions shown in centimeters, with the x and y axes as shown.



i. Write down the coordinates of the points A and B

_____

_____

1 mark

ii. If the arc AB can be modeled by a curve of the form $y = ax^2 + c$ show that $a = \frac{32}{49}$ and $c = 11$

_____

_____

_____

_____

1 mark

iii. If the glass is filled to the top by rotating the curve $y = ax^2 + c$ about the y axis find using calculus, the exact the volume of the glass.

3 marks

iv. If the arc AB can be modeled by a curve of the form $y = A + e^{kx}$ show that $A = 10$ and $k = \frac{2}{7}\log_e 9$

1 mark

v. If the glass is filled to the top by rotating the curve $y = A + e^{kx}$ about the y axis set up an integral for the volume of the glass, and find the volume of the glass correct to two decimal places.

2 marks

vi. If the glass is filled to a height h (above the x axis) where $11 \leq h \leq 19$ by rotating the curve $y = ax^2 + c$ about the y axis show that the volume of wine in the cup is given by $\frac{49\pi}{64}(h-11)^2$

1 mark

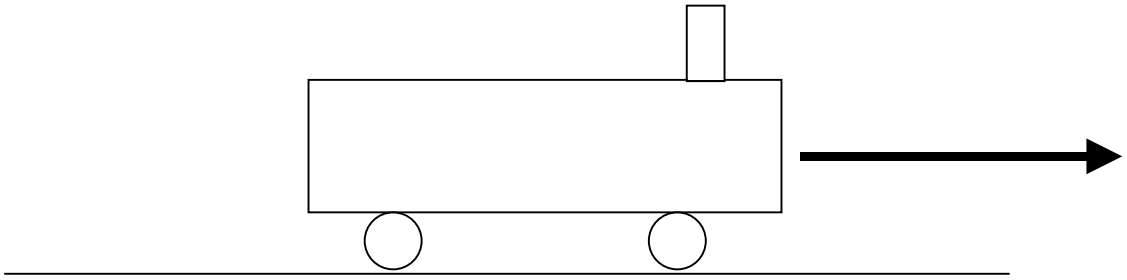
vii. Unfortunately one of the cups has a small crack in the base at the point A and the wine leaks out at a rate proportional to the square root of the remaining depth of wine. If initially the glass is full and after three minutes the height of wine is 16 centimetres above the x axis, find how long before the glass is empty. Give your answer to the nearest tenth of a second.

4 marks

Total 13 marks

Question 3

A train is moving along a straight line track at a speed of 57.6 km/hr when the driver brakes.



a. Assuming a constant retardation of 3 m/s^2 find how long(in seconds) and the distance traveled (in metres) before the speed of the train is reduced to 14.4 km/hr.

2 marks

b. The train has a mass 4096 kg and is moving along a straight line track at a speed of 57.6 km/hr when the driver brakes. The total resistance force is $24v^3$ Newtons where v m/sec is the speed of the train at a time t seconds, and x is its distance from the point where the driver applied the brakes.

i. By choosing an appropriate form for the acceleration, show that a differential equation relating v to x is $512\frac{dv}{dx} = -3v^2$

2 marks

ii. Find the exact distance traveled in metres before the speed of the train is 14.4 km/hr.

3 marks

iii. Find the exact time (in seconds) to reduce the speed of the train from 57.6 km/hr to 14.4 km/hr.

3 marks

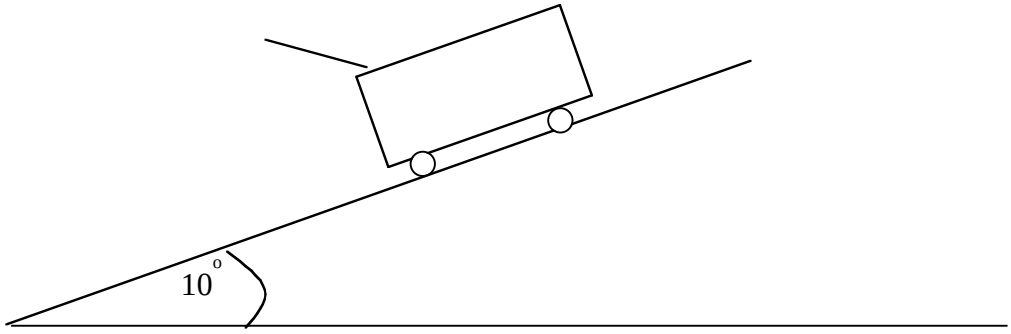
iv. Express the velocity v in terms of the time t, and sketch the velocity time graph.

2 marks
Total 12 marks

Question 4

a. A naughty child has let a supermarket trolley slide down a slope. The slope is inclined at an angle of 10^o to the horizontal and the coefficient of friction between the trolley and the ground is 0.1. The combined mass of the trolley and shopping bags is 32 kg.

i. On the diagram below, mark in all the forces acting on the trolley as it slides down the slope.



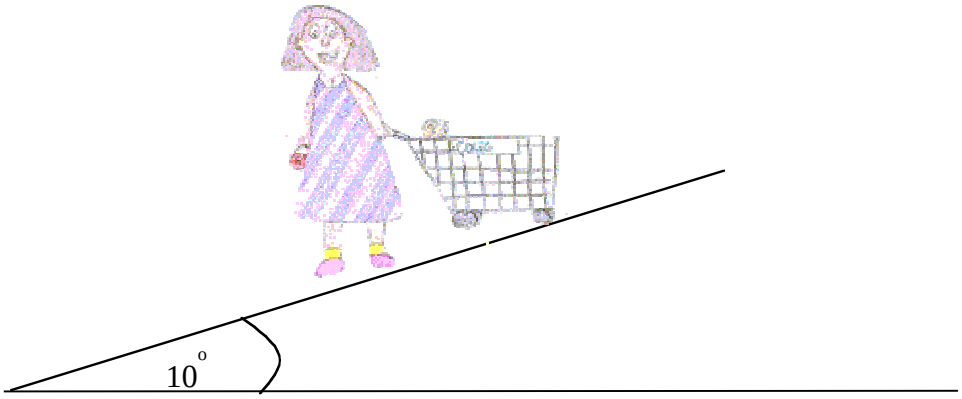
1 mark

ii. By resolving the forces, find the acceleration of the trolley as it moves down the slope, giving your answer correct to two decimal places.

3 marks

b. Unfortunately the lady has not finished her shopping and she must now push the trolley with the shopping bags back up the slope. If her arms make an angle of 30^o with the slope and the coefficient of friction between the trolley and the ground is still 0.1. The combined mass of the trolley and shopping bags is still 32 kg.

i. On the diagram below mark in all the forces on the trolley, as it moves up the slope.



1 mark

ii. If the trolley moves up the slope with constant speed, find the pushing tension in the lady's arms in Newtons, giving your answer correct to two decimal places.

4 marks

Total 9 marks

Question 5

A golf ball is hit on level ground so that its position vector at a time *t* seconds is given by $\bar{r}(t) = 8t\bar{i} + 50t\bar{j} + 12\sin\left(\frac{\pi t}{3}\right)\bar{k}$ for $0 \le t \le T$ where $\bar{i}$ $\bar{j}$ and $\bar{k}$ are unit vectors of magnitude one metre in the directions of east north and vertically upwards respectively,

i. Show that the time taken T for the golf ball to hit ground level is three seconds

1 mark

ii. Find the initial speed of projection correct to two decimal places.

2 marks

iii. Find how far from the initial point of projection the golf ball strikes the ground, giving your answer correct to the nearest metre.

2 marks

iv. Find the when the golf ball reaches its maximum height and give its position vector at this time.

2 marks
Total 7 marks

Question 6

i. State the maximal domain of the function $f(x) = \frac{x}{\sqrt{5x-4}}$

1 mark

ii. If $y = \frac{x}{\sqrt{5x-4}}$ then find the gradient function $\frac{dy}{dx}$ in simplest form.

2 marks

iii. Sketch the graph of $y = \dfrac{x}{\sqrt{5x-4}}$ stating the coordinates of any turning points, equations of any asymptotes and the range.

2 marks

iv. Find the exact equation of the tangent to the curve at the point $x = 4$

2 marks

v. Find an approximation using the trapezoidal rule to the area bounded by the curve $y = \frac{x}{\sqrt{5x-4}}$ the x axis and the lines $x = 1$ and $x = 8$ using each trapezium to have a width of one unit, give your answer correct to three decimal places.

1 mark

vi. Find using calculus the exact area bounded by the curve $y = \frac{x}{\sqrt{5x-4}}$ the x axis and the lines $x = 1$ and $x = 8$.

3 marks

Total 11 marks

End of 2003 Specialist Mathematics Trial Examination 2
Question and Answer Book

KILBAHA PTY LTD (Publishers in Education) **ABN 47 065 111 373** **PO BOX 2227 KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** www.chemas.com