GENERAL COMMENTS

Six-thousand and eighty-eight students sat for the 2002 examination, 166 more than the number (5922) in 2001, an increase of about 3%. As in 2001, students had to answer five questions worth a total of 60 marks, with each question worth from 9 to 17 marks.

Students found the examination slightly easier than the 2001 paper. The mean and median scores, out of a possible 60, were 26.1 and 25 respectively, compared with 24.2 and 23 in 2001. About 2.7% of students scored at least 90% of the marks, compared with 1.6% last year. The number of students who scored full marks was 9, whereas in 2001 only 2 students scored full marks. About 5% of students only scored a handful of marks.

Less than 10% of students scored full marks on Questions 4diii, 4eiii, 5bii and 5c. On the other hand, about 90% of students obtained full marks on Questions 2a (force diagram), 3ai and 3aii (use of constant acceleration formulas). The average score for Question 1, expressed as a percentage of the marks available (13), was 52.5%. This percentage decreased gradually over the next three questions, reducing to 42.1% for Question 4, then dropped dramatically to 24.9% for the fifth and final question (complex numbers). This supports evidence of recent years, from both examinations, that students tend to find complex number questions more difficult than questions of apparently similar difficulty on other topics.

As is normally the case, in quite a few questions (3aii, 3bii, 4c, 5aii and 5bi) students were asked to *show* (prove) given results. It needs to be emphasised that, in such questions, *all* steps need to be shown in order to gain full credit. On the other hand, students need to be reminded that, even if they cannot establish a given result, they are entitled to use that result in the remainder of the question – so, for example, the given results of 3aii and 3bii could be used to solve 3biii, and the result of 5bi could be used in 5bii.

More students than in previous years were able to make good use of graphics calculator technology when appropriate. Examples included Questions 2c (in which many students used their calculator to find where T is a minimum), 3biii (used for numerical integration), and 4b (used to draw a graph, though the asymptotic behaviour was often missed). However, there was evidence of *inappropriate* use, in particular in Questions 3c and occasionally 4diii. It should also be noted that, while the use of programs for procedures such as Euler's method to solve a differential equation numerically is to be encouraged, particularly in a coursework task, students still need to be familiar with the theory of such procedures and be able to apply it, without the use of a program, for a small number of steps. Many students were unable to write down the defining expression for Euler's method in Question 4ei, yet were able to obtain the correct numerical answer in Question 4eii (presumably they used a program for this part).

The instructions at the beginning of the paper concerning 'exact answers' and the 'use of calculus' must be clearly understood by students. For example, in this year's paper, some students used the numerical integration capability of their graphics calculator to evaluate the integral in Question 3c, rather than using calculus as directed.

SPECIFIC INFORMATION

Question	Marks	%	Response
Question 1	**a**		Answer: $\overrightarrow{CS} = 6\underset{\sim}{i} + 2\underset{\sim}{j}$; 6.3 km
	0/2	10	Quite well done. Arithmetic errors and the switching of $\underset{\sim}{i}$ and $\underset{\sim}{j}$ were occasional mistakes. Some students forgot to find the distance, and some left this answer as a surd. Others ignored 'hence', and got no marks for this part, by using Pythagoras to find the distance without first finding $\overrightarrow{CS}$.
	1/2	27	
	2/2	63	
	(Average mark 1.52)		
	bi		Answer: $8m(7 - 5m)$
	0/2	28	Some students tried to use $\lvert \underset{\sim}{r_1} \rvert \lvert \underset{\sim}{r_2} \rvert \cos\theta$ to find the scalar product, but to no avail. Many students made minor algebraic errors, with the most common incorrect answer, $56m - 32m^2$, resulting from a sign error.
	1/2	27	
	2/2	46	
	(Average mark 1.18)		
	bii		Answer: (8.4, –2.8)
	0/3	50	Most students who tackled part i correctly realised that the easiest way to approach this part was to equate their part i expression to zero to find the relevant value of m. However, some students thought they had to find the value of m that minimised their expression and so differentiated the expression and equated this result to zero.
	1/3	4	
	2/3	6	
	3/3	39	
	(Average mark 1.34)		

	biii 0/1 72 1/1 28 (Average mark 0.28)	Answer: 1.3 km About a third of the students who had part ii correct failed to capitalise by getting this final part correct. Some made arithmetic errors, while a common mistake was to find the distance *OP* instead of the distance *PS*.
	ci 0/1 52 1/1 48 (Average mark 0.48)	Answer: $(15t+2)\underset{\sim}{i}-(5t+6)\underset{\sim}{j}$ Most students anti-differentiated the velocity vector correctly, although some probably just multiplied it by *t*. However, about half of the students either omitted to include and evaluate the constant vector or took it to be the zero vector $\underset{\sim}{0}$.
	cii 0/2 20 1/2 44 2/2 36 (Average mark 1.16)	Answer: $(12t+8)\underset{\sim}{i}-(8t+3\cos(t)+1)\underset{\sim}{j}$ A common error among those students who were otherwise correct was to evaluate the constant vector incorrectly as $8\underset{\sim}{i}-4\underset{\sim}{j}$ by taking cos(0) to be 0.
	d 0/2 29 1/2 50 2/2 21 (Average mark 0.91)	Answer: $\overrightarrow{SC}=-0.2\underset{\sim}{j}$, i.e. the cargo ship is 0.2 km due south of the sailing ship. Most students evaluated the $\underset{\sim}{i}$ components of their two position vectors at *t* = 2, but some could go no further because they were not equal. Many of the students whose $\underset{\sim}{i}$ components were equal to $32\underset{\sim}{i}$, either had incorrect $\underset{\sim}{j}$ components, did not evaluate them for *t* = 2, or evaluated them incorrectly, generally because they had their calculator in degree mode and so obtained a wrong value for cos (2). Some students obtained $\overrightarrow{SC}=-0.2\underset{\sim}{j}$ but failed to draw the required conclusion.
Question 2	**a** 0/1 11 1/1 89 (Average mark 0.89)	Answer: Forces with magnitudes and directions as follows: 80*g* vertically down, *N* vertically up, *F* horizontally left, *T* along rope away from surfer. Very well done.
	b 0/4 25 1/4 8 2/4 14 3/4 17 4/4 36 (Average mark 2.3)	Answer: 520 Most students managed to get substantially correct vertical and horizontal equations of motion. The most common errors were omitting the vertical component of the tension (*T*sin60º) and obtaining *T*cos60º – *F* = 0 by overlooking the acceleration of the surfer. Many students who had the correct equations made algebraic or arithmetic errors when combining them to find *T*. Some students were clearly (and mistakenly) following a worked example from their notes of a particle sliding on an inclined plane.
	ci 0/3 54 1/3 14 2/3 10 3/3 22 (Average mark 0.99)	Answer: 16.7° Most students were unable to make a decent attempt at expressing *T* as a function of θ. Many of the students who got the correct expression ($T=\dfrac{395.2}{\cos\theta+0.3\sin\theta}$) made good use of their graphics calculator, rather than calculus, to find where the minimum value occurs. Some students recognised that this could be done most simply by finding where the denominator is a maximum.
	cii 0/1 80 1/1 20	Answer: 379 N A straightforward mark for those students who got part i fully correct.

	(Average mark 0.20)	

Question 3	**ai** 0/1 1/1 (Average mark 0.89)	 11 89	Answer: 12.5 m/s^2 Very well done.
	aii 0/1 1/1 (Average mark 0.90)	 10 90	Answer: $v = 0 + (12.5)(8) = 100$ Very well done.
	bi 0/1 1/1 (Average mark 0.54)	 46 54	Answer: $400a = -5000 - 0.5v^2$ Surprisingly, only slightly more than half of the students got this correct. Common errors were omitting to multiply *a* by the mass (in kg) of the dragster (400), and having positive signs throughout. Some students seemed not to know what was meant by the term 'equation of motion'.
	bii 0/2 1/2 2/2 (Average mark 0.86)	 52 9 39	Answer: Use $a = v\frac{dv}{dx}$ Most students who had the correct equation of motion substituted the correct form for *a*, but many then had trouble (especially with signs) in deriving the desired result.
	biii 0/3 1/3 2/3 3/3 (Average mark 1.28)	 43 15 13 29	Answer: 277 m Most students started correctly by inverting the part ii equation. The most popular approach then was to anti-differentiate to find an expression for *x*. Common mistakes with this method were failure to evaluate correctly the constant of integration or overlooking it altogether. Those students who, alternatively, expressed the answer as a definite integral ($-\int_{100}^{0} \frac{800v}{10000+v^2}\,dv$), and then used their graphics calculator to evaluate it, were generally more successful. The most common mistake with this method was to have the terminals reversed and so obtain a negative answer.
	c 0/3 1/3 2/3 3/3 (Average mark 0.74)	 66 10 6 17	Answer: 6.3 (2π is exact value) s Only approximately one-third of the students realised that they needed to return to their equation of motion and replace *a* with $\frac{dv}{dt}$. About one-half of these students went on to complete the question successfully. The others had trouble with the anti-differentiation, often producing a $\log_e$ expression as their answer, or ignored the 'use calculus' direction and used the numerical integration capability of their graphics calculator without ever finding the correct anti-derivative.
Question 4	**a** 0/1 1/1 (Average mark 0.69)	 31 69	Answer: $(-2, 2)$ Well done, although it should have been done better. As expected, the most common mistake was to include the endpoints. It is likely that some students who wrote $[-2, 2]$ intended the correct answer, but had the incorrect notation. This was certainly the case for the occasional student who wrote $-2 < D < 2$.
	b 0/3 1/3 2/3 3/3 (Average mark	 6 11 35 48	Answer: local max at $(0, \log_e(4))$, intersects *x*-axis at $\pm\sqrt{3}$, asymptotes with equations $x = \pm 2$, symmetric about *y*-axis. Most students labelled the axes intercepts correctly, though many obviously used their graphics calculator to find these values. Some students omitted the asymptotes altogether, but the most common error was to have the shape of the graph wrong because it did not exhibit asymptotic

	2.25)		behaviour.

	c 0/2 49 1/2 28 2/2 23 (Average mark 0.73)	Answer: Use (area of lower rectangle) < A < (area of upper rectangle) Not done well. Most students apparently did not understand what was meant by 'use the graph'.
	di 0/2 17 1/2 18 2/2 66 (Average mark 1.19)	Answer: $\log_e(4-x^2)-\frac{2x^2}{4-x^2}$ Reasonably well done. The most common mistake was to have the sign of the second term incorrect.
	dii 0/3 72 1/3 6 2/3 7 3/3 15 (Average mark 0.65)	Answer: $-x+\log_e\left(\frac{2+x}{2-x}\right)+c$ Not done well. Most students failed to recognise the need to divide and tried unsuccessfully to express $\frac{x^2}{4-x^2}$ in partial fraction form.
	diii 0/2 73 1/2 18 2/2 9 (Average mark 0.36)	Answer: $-2+3\log_e(3)$ The answers to parts i and ii needed to be used to answer this part. About half of the students who got part ii correct also got this part.
	ei 0/2 56 1/2 18 2/2 26 (Average mark 0.69)	Answer: $y_{20} = y_{19} + 0.05\log_e(4–0.95^2)$ Among those who had an expression of the correct form, a common error was to confuse x_{19} and y_{19} and write $y_{20} = y_{19} + 0.05\log_e(0.95^2)$.
	4eii 0/1 72 1/1 28 (Average mark 0.28)	Answer: 1.3029 Most students who got part i correct also got this part correct. On the other hand, some students who had little or no idea about part i got this part correct – presumably by using a calculator program.
	4eiii 0/1 98 1/1 2 (Average mark 0.02)	Answer: $y_{20} \approx \int_0^{x_{20}} \log_e(4-x^2)\,dx = \int_0^1 \log_e(4-x^2)\,dx = A$ Only a handful of students earned this mark. Most answers concentrated on Euler's method being an approximate method for solving a differential equation (given an initial condition).
Question 5	**ai** 0/1 80 1/1 20 (Average mark 0.20)	Answer: By definition, z is equidistant from the points i (L) and u (N). Not done well. Few students realised what was required. The most common response was to do what was required in part ii.
	aii 0/2 39 1/2 4 2/2 57	Answer: Substitute $z = x + yi$ into $\|z – i\| = \|z – u\|$ and expand both sides. By far the most successfully completed part of this question.

	(Average mark 1.18)		
	bi 0/2 1/2 2/2 (Average mark 0.44)	 76 4 20	Answer: $w = u + yi$, where (u, y) satisfies the equation of part aii (since w lies on the perpendicular bisector of LN) and so $2y = 2u^2 - u^2 + 1$, giving $y = \frac{1}{2}(u^2 + 1)$. Most students failed to realise that they needed to use the relation of part aii to find the imaginary part of w.
	bii 0/2 1/2 2/2 (Average mark 0.36)	 71 22 7	Answer: $y = \frac{1}{2}(x^2 + 1)$, $x > 0$. Curve is $\text{Re}(z) > 0$ portion of the parabola with vertex $0.5i$ and passing through w. (Note also that, from part c, line M is tangent to the curve at w.) About 30% of students found the Cartesian equation of the curve correctly. However, only a few of these students sketched the curve accurately. Common errors were drawing the full parabola and having the vertex at i.
	c 0/3 1/3 2/3 3/3 (Average mark 0.31)	 85 5 2 7	Answer: Gradient of perpendicular bisector of LN is u (from part aii equation). Gradient of curve is given by $\frac{dy}{dx} = x$. But $x = u$ at w, so curve at w and the perpendicular bisector have same gradient. Since the perpendicular bisector of LN passes through w (by definition of w), the desired result follows. *Alternatively*: Since $\frac{dy}{dx} = x$, gradient of tangent at $x = u$ is u. So, using $y - b = m(x - c)$, find equation of tangent is $2y = 2u^2 - u^2 + 1$. From part aii, this is the equation of the perpendicular bisector. Very few students got this far. Approximately half of the students who made a reasonable attempt, by at least finding the gradient of the curve at w, went on to get full marks for this. Others did not seem to realise what else had to be done.