SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures										Letter
Words										

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2001

SPECIALIST MATHEMATICS

Written examination 2
(Analysis task)

Wednesday 7 November 2001

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	60

Materials
- Question and answer book of 15 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.
- Up to four pages (two A4 sheets) of pre-written notes (typed or handwritten).
- An approved scientific and/or graphics calculator, ruler, protractor, set square and aids for curve sketching.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided on the cover of this book.
- All written responses must be in English.

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Instructions

Answer **all** questions.

A decimal approximation will not be accepted if an exact answer is required to a question.

Where an exact answer is required to a question, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

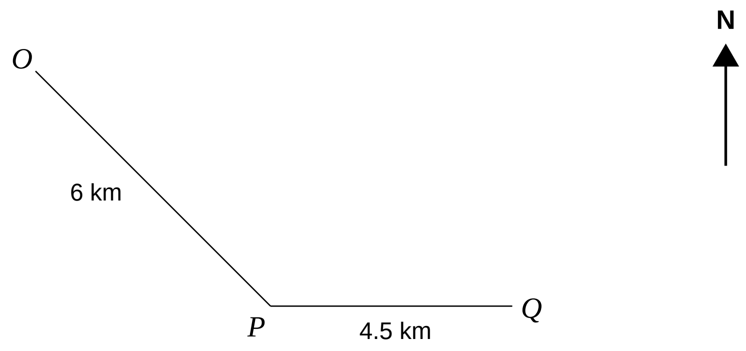
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Working space

Question 1

Chris sets out on a hike from *O* in a national park. She begins by walking 6 km southeast to *P* and then 4.5 km east to *Q*, all the time on horizontal ground.



a. Use the cosine rule to find the distance *OQ* km, correct to two decimal places.

2 marks

For the remainder of Question 1, take $\underset{\sim}{i}$ as a unit vector in the east direction, $\underset{\sim}{j}$ as a unit vector in the north direction, and $\underset{\sim}{k}$ as a unit vector vertically up.

b. **i.** Find $\overrightarrow{OQ}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$.

2 marks

ii. Using a suitable scalar product, find the bearing of Q from O in degrees, to the nearest tenth of a degree.

3 marks

From Q, Chris continues to head east. However, she now walks up a steep slope inclined at a constant angle of $\text{Sin}^{-1}(0.28)$ to the horizontal.

c. **i.** Chris reaches a point R which is 0.5 km up the slope from Q. Find $\overrightarrow{OR}$ in terms of $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$.

2 marks

ii. At R, Chris falls and twists an ankle. She has an emergency radio transmitter with a maximum range of 10 km. Determine whether her transmitter has sufficient range to alert the park ranger at O.

2 marks

Total 11 marks

TURN OVER

Question 2

An oil tanker hits a reef and spills oil into the sea. Initially the oil spills at an increasing rate, but action by the crew and coastal authorities enables the spill to be brought under control some time later. Assume that the oil spills from the tanker at the rate of $\frac{kt}{12+t^4}$ litres/day, where $k = 10^6$ and t is the time in days from when the oil tanker hit the reef.

a. Find the rate, in litres/day, at which the oil spills into the sea after 4 days, correct to three significant figures.

1 mark

b. **i.** If the rate at which oil spills into the sea has a maximum value when $t = a$, find the exact value of a.

3 marks

ii. Hence or otherwise find the maximum rate at which oil spills into the sea, in litres/day, correct to three significant figures.

1 mark

Question 2 – continued

c. **i.** Use the substitution $u = t^2$ to find an antiderivative of $\frac{t}{12 + t^4}$.

3 marks

ii. If V litres is the volume of oil spilled into the sea in the first t days, find V in terms of t.

1 mark

iii. Some time after the oil began to spill into the sea, a newspaper report stated that: 'It is expected that eventually 300 000 litres of the tanker's oil will have spilled into the sea'.

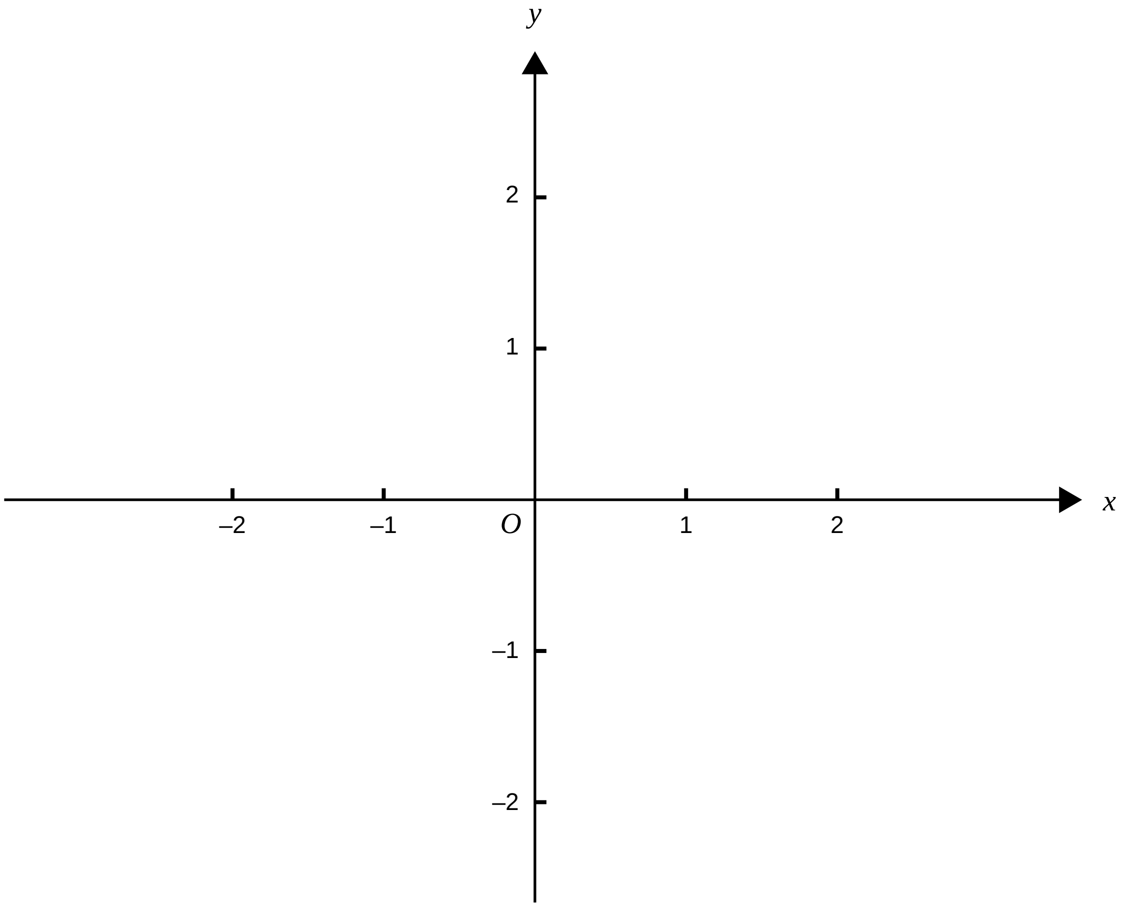
Determine whether the newspaper report statement is in agreement with the model above.

2 marks

Total 11 marks

Question 3

a. On the axes provided, sketch the curve given by the rule $x^2 - 4y^2 = 1$.



2 marks

The part of the curve for $x \geq 1$ and $0 \leq y \leq 1$ is rotated about the y-axis to form a volume of revolution which is to model an ornamental fountain.

b. When the depth of the water in the fountain is h metres, show that the volume of water in the fountain is V cubic metres, where $V = \pi(\frac{4h^3}{3} + h)$.

2 marks

The fountain is initially empty. Water is pumped into it at a rate of 0.003 cubic metres per second. At the same time, water flows out from the bottom of the fountain at a rate of $0.004\sqrt{h}$ cubic metres per second, where h metres is the depth of the water in the fountain.

c. Find the rate, in metres per second, at which the depth is increasing when the depth is 0.25 metres. Give your answer correct to two significant figures.

3 marks

d. Express the time taken for the depth to rise to 0.5 metres as a definite integral and evaluate it correct to the nearest second.

2 marks

e. At what depth does the water ultimately stabilise in the fountain?

2 marks

Total 11 marks

TURN OVER

Question 4

a. Find the roots of $z^2 - 6z + 25 = 0$ where $z \in C$, and **hence** find the sum of the roots and the product of the roots.

3 marks

b. Let u and v be the roots of the equation $z^2 + bz + c = 0$ where $b, c, z \in C$.

i. Show that $u + v = -b$ and $uv = c$.

2 marks

ii. Hence show that if $u = p + qi$ where $p, q \in R$, and u and v are complex conjugates, then b and c are real.

2 marks

Question 4 – continued

c. Find the quadratic equation in z which has roots $2 + \sqrt{5}\,i$ and $-2 + \sqrt{5}\,i$.

2 marks

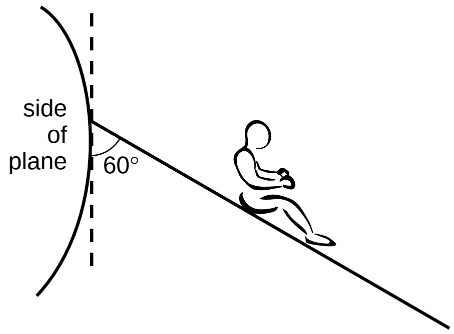
d. A quadratic equation in z has roots u and v. The sum of the roots is –3 and the product of the roots is 4. Find a quadratic equation in z which has roots $(u + v)$ and $(u - v)$.

4 marks

Total 13 marks

Question 5

A plane is forced to make an emergency landing. After landing, the passengers are instructed to exit using an emergency slide, of length 6 metres, which is inclined at an angle of 60° to the vertical.



Passenger Jay has mass 75 kilograms. The coefficient of friction between Jay and the slide is $\frac{1}{5}$.

a. Clearly mark and label on the diagram the forces acting on Jay as he slides down.

1 mark

b. Show that the acceleration a m/s^2 of Jay down the slide is given by $a = \frac{g}{2}\left(1 - \frac{\sqrt{3}}{5}\right)$.

3 marks

c. **i.** Jay starts from rest at the top of the slide. Find the time, in seconds, that he takes to reach the end of the slide. Give your answer correct to two decimal places.

2 marks

ii. Show that Jay reaches the end of the slide with a speed of 6.2 m/s.

1 mark

The plane came to rest so that the end of the slide is 2 metres vertically above level ground.

d. How far horizontally from the end of the slide does Jay land? Give your answer to the nearest tenth of a metre.

4 marks

e. Find the magnitude of Jay's momentum, in kg m/s, when he hits the ground. Give your answer correct to three significant figures.

3 marks

Total 14 marks

Working space

END OF QUESTION AND ANSWER BOOK

Click here for Formula Sheet

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\qquad$ $\cot^2(x)+1=\text{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\qquad$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\qquad$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\qquad$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\qquad$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	Sin^{-1}	Cos^{-1}	Tan^{-1}
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z=x+yi=r(\cos\theta+i\sin\theta)=r\,\text{cis}\,\theta$

$|z|=\sqrt{x^2+y^2}=r$ $\qquad$ $-\pi<\text{Arg}\,z\le\pi$

$z_1z_2=r_1r_2\,\text{cis}(\theta_1+\theta_2)$ $\qquad$ $\frac{z_1}{z_2}=\frac{r_1}{r_2}\text{cis}(\theta_1-\theta_2)$

$z^n=r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$

$$\int \sec^2(ax)dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\text{Sin}^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \text{Sin}^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}\left(\text{Cos}^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \text{Cos}^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}\left(\text{Tan}^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \text{Tan}^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

mid-point rule: $\int_a^b f(x)dx \approx (b-a)f\left(\frac{a+b}{2}\right)$

trapezoidal rule: $\int_a^b f(x)dx \approx \frac{1}{2}(b-a)\left(f(a) + f(b)\right)$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h\,f(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

TURN OVER

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \leq \mu N$

END OF FORMULA SHEET