Specialist Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The number of students who sat for the 2000 examination was 5856, 172 less than the number (6028) in 1999. In 2000, for the first time, the Specialist Mathematics course was fully prescribed so it was no longer necessary to provide students with a choice of questions depending on the optional module (Mechanics or Geometry in recent years) that they had studied. Students had to answer six questions worth a total of 60 marks. However, Question 2 was quite short and the number of questions to be answered on Examination 2 is more likely to be five than six in most years.

About 2.9% of students scored 90% or more of the marks (compared with 2.3% in 1999), with only four students (down from seven in 1999) scoring full marks, although another thirteen only lost 1 mark in 2000. The examination paper was generally more accessible than last year. The early, routine parts of each question were quite well done, with far fewer students scoring less than 10% of the marks than in 1999. However only a small number of students knew how to tackle Question 5d, and students generally had difficulty with the later parts of the other questions (especially Questions 1e, 3d, 6cii and 6e).

In last year's report it was noted that most students would benefit from better 'examination technique' and several general points were emphasised. It was pleasing to observe improvement in some of the areas highlighted and it is hoped that these general points will continue to be given attention. In particular, in Question 4bii many students realised that they had to use a result given earlier (in 4aii), and that they could do so even if they had not been able to complete the earlier part. Similarly, many students who omitted Question 6cii went on to use the result given there to tackle the remaining parts (d. and e.) of the question.

However, more attention needs to be paid to other points. For example, many students lost marks in Questions 4b and 4c by having their calculator in degree, instead of radian, mode. Also, many students lost marks through poor or incorrect notation (in particular, not using vector notation in Question 3e, or omitting the '*dx*' from the integral in 4bii), whilst others penalised themselves through careless work; for example, rewriting the unsimplified answer to Question 2a, $\frac{140}{50g}$ as $\frac{14}{5}g$ and using this incorrect expression in part b.

It was noticeable this year that students lost marks through not reading a question carefully enough. For example, in Question 1d, many students found where the maximum gradient occurred but omitted to evaluate the gradient; in Question 2, many students misinterpreted part b. to be an 'inclined plane' problem; in Question 6a, many students found either the time or the distance but not both; and in Question 6e, many students only drew the graph from the time the parachute was opened. Students should be advised to re-read each question part as they finish it to check that they have answered it in full.

Students seemed to use specific capabilities of their graphics calculators more readily and confidently than on Examination 1. In particular, Questions 1ci and 1ciii were well done (except that some students used the $\log_{10}$ function key instead of the $\log_e$ or ln key), and most students who obtained a reasonably correct expression in 4cii realised that they needed to use the numerical integration facility to evaluate the integral.

Some students used their calculator inappropriately in Question 1cii, ignoring the direction to find ***exact*** coordinates, and similarly some students ignored the direction to ***use calculus*** in 4bii and obtained an answer directly using the numerical integration facility. Teachers should ensure that students are aware of the significance of such directions, and that marks are not awarded for answers obtained in other ways. On the other hand, students should be aware that it is helpful to use a graphics calculator to check answers obtained in the required manner.

Finally, students should be encouraged to not write in pencil and not to erase working (either by rubbing out or using 'liquid paper'). It is quicker to just put a line through work that is repeated; work that is not repeated should be left in case it is worth some marks.

SPECIFIC INFORMATION

Question 1

ai. (Average mark 0.84/Available mark 1)

Answer: $\frac{-6}{x^2} + \frac{3}{x}$

Well done. A small number of students retained a 'log' term in their answer.

aii. (0.78/1)

Answer: $\frac{12}{x^3} - \frac{3}{x^2}$

Well done. Most students who got part i correct also got this part correct.

b. (0.75/1)

Answer: $f'(1) = 6 - 6 + 0 = 0$

Well done. A small number of students tried to do the 'impossible' and solve the equation.

ci. (1.75/2)

Very well done. This question tested a student's ability to use a calculator to assist in accurately sketching a graph. The main features looked for in the graph were: correct shape, asymptotic behaviour at $x = 0$, and reasonably accurate x intercepts. The most common error was to have an 'out-of-range' second x intercept (95.28), apparently caused by using $\log_{10}x$ in the equation instead of $\log_e x$. This confusion probably arose because the corresponding keys on some calculators are labelled 'log' and 'ln' respectively – a fact that of course should be well known to students who are familiar with the use of their calculators.

cii. (1.36/2)

Answer: $(2, 3\log_e 2 - 3)$

Reasonably well done, but many students gave an approximation (e.g. -0.9, -0.92) for the y coordinate instead of finding the exact value as directed.

ciii. (0.65/1)

Answer: (4.92, 0)

Reasonably well done. As in part b., a small number of students tried to do the impossible and solve the equation, but most students realised they needed to use their graphics calculators. Some students only gave the answer to one decimal place (4.9), whilst others had the second decimal place incorrect – presumably because they had used the 'Trace' feature of their calculator to locate the intercept. Students should be aware that the precision of results obtained by 'tracing' is limited by the screen resolution and, accordingly, they should always use the appropriate numerical function (e.g. 'zero' or 'root') instead.

d. (0.83/2)

Answer: 0.375 (at $x = 4$)

About half of the students realised that they had to solve $f''(x) = 0$. Many of those who correctly obtained $x = 4$, then either stopped or evaluated $f(x)$ instead of $f'(x)$.

e. (0.46/2)

Not well done, with many students not attempting this part. This was a new topic in the course ('the relationship of the graph of a function and the graphs of its antiderivatives') and clearly students need more practice at deducing antiderivative graphs from the graph of a function. Some students tried in vain to antidifferentiate $f(x)$ so that they would have a rule to graph, however, this question only required graphical analysis. Many students who deduced correctly that F must have a local maximum at $x = 1$ and a minimum at $x = 4.92$, drew their curve cutting the y-axis (rather than having it as an asymptote).

Question 2

a. (1.52/2)

Answer: $\mu = \frac{14}{5g} = \frac{2}{7}$

Well done. Some students missed out on a mark because they gave $\mu \geq \frac{2}{7}$ as their answer as a result of using the relation $F \leq \mu N$. This relation, rather than the equation $F = \mu N$ is included on the formula sheet to remind students that μN is the *maximum* (or limiting) value of friction, attained only when the body involved is sliding or is on the point of sliding. This *is* a limiting friction situation, since we are told that the box is sliding, so $F = \mu N$ and hence $\mu = \frac{2}{7}$.

b. (1.85/4)

Answer: 135

Many students who answered part a. correctly could not manage this slightly different situation. Most students resolved correctly in the horizontal direction, but many omitted the component of P when resolving in the vertical direction. Some students wrongly let $P = 140$ (from part a.), whilst others incorrectly used F to represent both the pulling force and friction. A large number of students answered this part, often without drawing a diagram, as though the box was being pulled up a plane, inclined at an angle of 15°, by a force parallel to the plane. Presumably these students did not read the question carefully enough and misinterpreted it as a rare opportunity to copy a standard example that they had included in their pre-written notes.

Question 3

a. (0.85; 1)

Well done. The most common error was to plot w as $7 + 2i$.

bi. (0.66/1)

Answer: $v^2 = 36 + 64 = 100$ so $|v| = 10$

Many students omitted this straightforward question.

bii. (0.73/1)

Answer: circle centre O, radius 10

Well done, though some students did not draw the circle through V even after proving $|v| = 10$ in part i.

c. (1.5/2)

Answer: $6 - 8i$

Well done. Most students knew that $\overline{w} = 7 - i$, but not all had good enough algebra skills to arrive at the correct answer.

d. (0.52/2)

Answer: diameter of circle from (–10, 0) to (10, 0)

Not well done, with many students not attempting this part. Most attempts were algebraic with many of these students correctly simplifying $|z - u| = |z - v|$ to $-32y = 0$ (or equivalent), but then seemingly not realising that this represented the real axis. Students should be encouraged to approach problems like this geometrically as it is often easier this way. In this case, *T* is the intersection of the circle *S*, and its interior, with the perpendicular bisector of the line joining the points corresponding to *v* and *u*. Since $u = 6 - 8i = \bar{v}$, the perpendicular bisector is the real axis and so *T* is the diameter of *S* that is coincident with this axis.

e. (1.11/3)

Answer: $\overrightarrow{OW}.\overrightarrow{VW} = \left(7\underset{\sim}{i} + \underset{\sim}{j}\right).\left(\underset{\sim}{i} - 7\underset{\sim}{j}\right) = 7 - 7 = 0 \quad \overrightarrow{OW}$ is perpendicular to $\overrightarrow{VW}$.

Not well done. Most students realised that a scalar product was somehow involved, but relatively few students expressed $\overrightarrow{OW}$ and $\overrightarrow{VW}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$ so that they could legitimately perform one. The most common approach was to use $7 + i$ and $1 - 7i$ in a 'complex scalar product' that miraculously yielded $7 - 7 = 0$. Some students chose to show that $\overrightarrow{OW}^2 - \overrightarrow{VW}^2 = \overrightarrow{OV}^2$ but this is a geometric method, rather than a vector method, and so could not be awarded full marks.

Question 4

ai. (0.83/1)

Answer: $\cos(x) - x\sin(x)$

Well done. Most students realised that it was necessary to apply the product rule.

aii. (0.54/1)

Answer:

$$x\cos(x) = \int \cos(x)dx - \int (x\sin(x))dx$$

$$\int (x\sin(x))dx = \sin x - x\cos(x) + c$$

Not as well done as part i. In such questions, the onus is on the student to clearly set out their work so that the assessor is convinced that the student has shown the given result, in the manner prescribed, and not 'fudged' a proof.

bi. (0.6/1)

Answer: 1.5 m

It is disappointing that 40% of students could not get this straightforward evaluation correct. The most common error was to use degrees instead of radians (and hence obtain 1.6 m).

bii. (2.14/4)

Answer: $289m^2$

It was pleasing to see that most students were able to formulate the correct integral for the area, namely $\int_0^{60} \frac{x}{50}(8 + \sin x)dx$. Further, most students recalled that an antiderivative of $x\sin(x)$ was given in part aii. and were able to use this to help evaluate the integral. Others, however, invented their own rules for doing this antidifferentiation, whilst some students ignored the direction to '*use calculus*' altogether and resorted to using the numerical integration feature of their graphics calculator to evaluate the integral (and so received no further marks). Some students failed only at the final hurdle, using degree mode instead of radian mode in their evaluations.

ci. (0.47/1)

Answer: 87

Reasonably well done given the amount of interpretation needed to get this far, but many had trouble obtaining the correct factor $\left(\frac{150}{500} = 0.3\right)$ with which to multiply their answer for the area.

cii. (0.6/2)

Answer: $0.3\int_0^{60} \frac{x^2}{1600}(8 + \sin x)dx = 109$

Many students omitted this part. Some of the students who obtained a reasonably correct expression apparently did not realise that they needed to use the numerical integration feature of their calculator to evaluate the integral involved.

Question 5

a. (0.66/1)

Answer: $z(4) = 2 + 19.5(4) - 5(4^2) = 2 + 78 - 80 = 0$

Reasonably well done. Many students calculated $\underset{\sim}{r}(4)$, but failed to indicate (or even realise) that the significant fact was that $z(4) = 0$.

b. (1.02/2)

Answer: 78.03 m

Many students did not realise that they had to find $\left|\underset{\sim}{r}(4)\right|$ and either gave $x(4)$ or $x(4)+y(4)$ as their answer. Some students confused units and gave answers such as 78 cm and 78.0 m.

c. (1.08/2)

Answer: $19.5\underset{\sim}{i} + \frac{p}{2}\underset{\sim}{j} - 20.5\underset{\sim}{k}$

Relatively few students 'dropped' $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ when differentiating $\left|\dot{\underset{\sim}{r}}(4)\right|$ but many, having found the correct velocity vector, $\underset{\sim}{r}'(4)$, went on to calculate the speed, $\left|\dot{\underset{\sim}{r}}(4)\right|$ and gave that as the velocity. More attention needs to be given to ensuring that students appreciate the distinction between velocity (a vector) and speed (its magnitude).

d. (0.27/3)

Answer: 46.3 (or 133.7)

This was possibly the most difficult question part on the paper and was omitted by many students. The key to answering the question is to realise that the direction of the tip of the javelin is given by its velocity, and hence at *P* its direction is given by

$\dot{\tilde{r}}(4)$. Of those students who realised this, only a small minority were able to go on and use a correct scalar product – either $\dot{\tilde{r}}(4).\left(195\tilde{i}+\frac{p}{2}\tilde{j}\right)$ or $\dot{\tilde{r}}(4).\tilde{k}$ – to find the required angle; many incorrectly used $\dot{\tilde{r}}(4).\tilde{r}(4)$.

Question 6

a. (1.32/2)

Answer: 2.04 s, 20.4 m

Well done, except that many students apparently failed to read the question carefully enough and only answered one of the two parts, i.e. they found the time or the distance, but not both. The concept of significant figures seemed well understood, with nearly all students giving their answer/s to the specified level of accuracy (three significant figures).

b. (0.61/2)

Answer: $mg - 0.04mgv^2 = ma = m\frac{dv}{dt}$ $\quad \frac{dv}{dt} = -0.04g\left(v^2 - 25\right)$

Many students apparently did not know what was meant by 'equation of motion'. Others did not involve *m* at any stage, starting with a variation of $\frac{dv}{dt} = g - 0.04gv^2$ that may have come from just re-writing the given result.

ci. (0.96/3)

Answer: $t = -\frac{1}{0.4g}\log_e \frac{5(v-5)}{3(v+5)}$

There were relatively few examples of $\int \frac{1}{v^2-25}dv = \log_e\left(v^2-25\right)$, with most students realising that it was necessary to express the integrand in partial fraction form. A common, though not too frequent, error was to use $t = 0$, $v = 0$ as the initial condition instead of $t = 0$, $v = 20$. Note that, rather than first find the general solution and then evaluate the arbitrary constant, the solution can be obtained directly from

$$\int_{20}^{0} \frac{1}{v^2-25}dv = \int_{0}^{t} (-0.04g)dt.$$

cii. (0.46/2)

Of those students who obtained the correct answer to part i., only about half were able to sustain the algebra necessary to deduce the given expression for *v* without 'fudging'. On the other hand, some of the students who obtained an incorrect answer to part i. were able to 'obtain' the correct expression for *v* with some very creative algebra.

d. (0.56/2)

Answer: 5 m/s

Most students who attempted this part gave the correct terminal velocity of 5 m/s; however, some gave no reason (or no valid reason). Most students deduced the answer from the expression for *v* given in part cii., noting that $e^{-0.4gt} \to 0$ as $t \to \infty$, but a substantial percentage took the alternative approach of letting $\frac{dv}{dt} = 0$ in the differential equation given in part b.

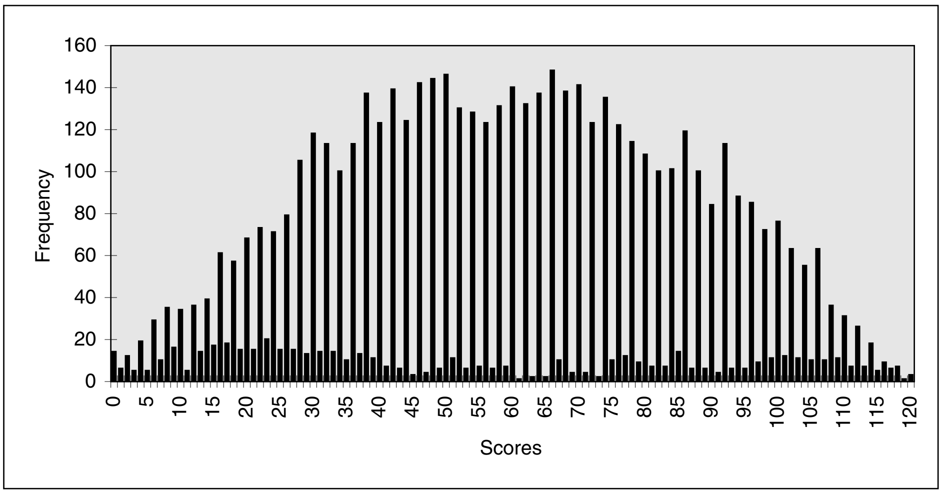
e. (0.59/3)

Few students showed all three distinct stages of the parachutist's motion: constant acceleration (i.e. a straight line) from rest initially to 20 m/s at $t = 2.04$, followed by rapid exponential decay asymptotic to 5 m/s (a graphics calculator can help with the shape here), and finally a discontinuous drop to 0 m/s at $t = 8$. Other common omissions were failure to indicate axes scales, or to indicate coordinates of critical points.

GA MA093 MATHEMATICS: Specialist Mathematics WRITTEN EXAMINATION 2

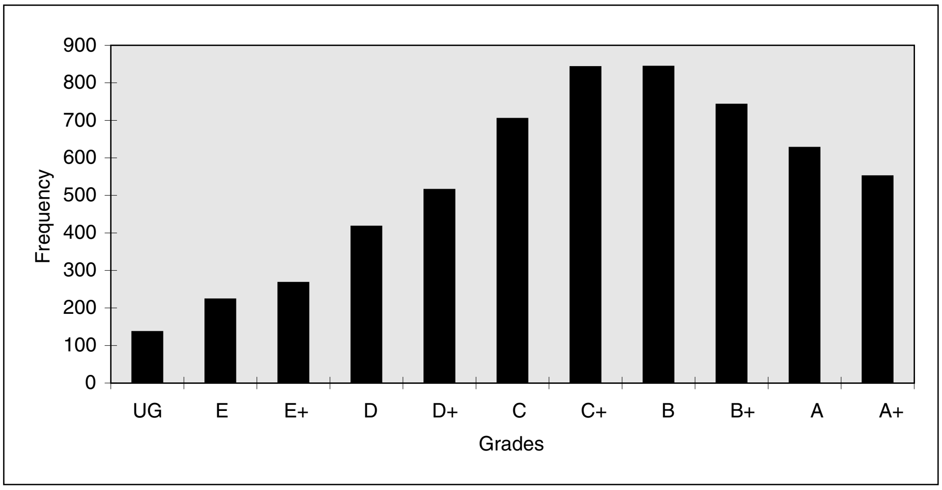
HISTOGRAM OF TOTAL SCORES 2000

Count 5856 Mean 58.87 Standard Deviation 27.23 NA Result 113



HISTOGRAM OF TOTAL GRADES 2000

Count 5856 Mean 6.07 Standard Deviation 2.59 NA Result 113



ENROLMENTS		%
Female	2150	36.0
Male	3819	64.0
Total	5969	

GLOSSARY OF TERMS

Count Number of students undertaking the assessment. This excludes those for whom NA was the result.

Mean This is the 'average' score; that is all scores totalled then divided by the 'Count'.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).