Specialist Mathematics GA 2: Written examination 1

GENERAL COMMENTS

The number of students who sat for the 2000 examination was 5856, 176 less than the number (6032) in 1999. In 2000, for the first time, the Specialist Mathematics course was fully prescribed so it was no longer necessary to provide students with a choice of questions depending on the optional module (Mechanics or Geometry in recent years) that they had studied. In addition, there was a small change in the relative weighting of the two parts of the examination compared with CAT 2 in previous years: students were required to answer thirty multiple-choice questions in Part I (reduced from thirty-three), with the number of marks allocated to Part II (short-answer questions) increased correspondingly from 17 to 20.

Just over 3% of students scored more than 90% of the available marks (similar to 1999), but only four students scored full marks (compared with eight in 1999 and 19 in 1998) although another twenty lost only 1 mark. It appeared that most students had sufficient time to complete the examination, but only students with a thorough knowledge of all aspects of the course could expect to earn perfect, or near-perfect, scores.

Students' responses to the short-answer questions ranged from no attempt and untidy fragments to efficient, accurate and logically-complete answers. Most students wrote legibly in ink, but in some cases illegibility (poor writing and/or faint pencil writing) presented a problem, often for the students themselves. It is recommended that teachers encourage students to answer written examination questions legibly, and in ink.

Finally, it appears that teachers need to give students more direction and instruction on the effective use of graphics calculators in Specialist Mathematics examinations. Access to an approved graphics calculator is assumed, and students need to be able to recognise when its use is required or likely to be of assistance, and to be able to use it effectively in these cases. Unless it is indicated otherwise – for example, by an instruction to **use calculus** or to find an **exact** value – the full power of a graphics calculator (including relevant 'non-graphics' features such as complex arithmetic capability) can be used to answer a question or at least check an answer to a question. For Examination 1 in 2000, most students would have benefited from using a graphics calculator to help answer Question 1 in Part I, while some students would have benefited from using it in Questions 2b and 6a, and it could have been used to check the answer to Question 3. It appeared, however, that some students either chose not to use their graphics calculator or thought they could not use one (see comments on Question 1 later). Others were unable to enter a reasonably simple expression correctly or were not sufficiently familiar with their calculator's capabilities.

SPECIFIC INFORMATION

Part I: Multiple-choice questions

The correct responses are as follows:

Question	Answer	Percentage correct
1	B	87
2	A	58
3	B	87
4	D	59
5	E	62
6	E	43
7	A	58
8	B	43
9	E	65
10	C	79
11	C	53
12	E	57
13	E	38
14	D	43
15	D	53
16	B	69
17	B	56
18	D	69
19	C	50
20	A	48
21	A	66
22	C	79

Question	Answer	Percentage correct
23	E	30
24	A	49
25	D	59
26	B	55
27	C	16
28	D	45
29	C	40
30	A	36

Four of the questions [Questions 13 (38%), 23 (30%), 27 (16%) and 30 (36%)] were answered correctly by less than 40% of students. In each case, but for no other question, one of the distractors proved more popular than the correct answer.

In Question 13, almost half (46%) of the students gave D as their answer instead of the correct answer E. Although it was anticipated that D would be a strong distractor, its strong support indicates that students need more experience with calculating the volume of the solid formed when an enclosed region is rotated about one of the axes.

Question 23 concerned sufficient conditions for a vector proof that a quadrilateral is a rhombus. Over a third (37%) of students chose D (perpendicular diagonals) which is a necessary condition, but not a sufficient one (for example, the diagonals of a kite are also perpendicular). Similarly, 24% of students chose B (opposite sides equal vectors) which is a property shared with all parallelograms.

The very poor performance on Question 27 was particularly disappointing given that the concept involved (that friction is an adjusting force) has been examined repeatedly in recent years. Nearly half of the students (48%) gave E as their answer, suggesting that they believed that $F = \mu N$ always. It should be impressed on students that this equation gives the maximum (limiting) value of friction and that this is only reached when the body concerned is in motion or is on the point of motion. The correct relation in general is $F \le \mu N$. In this particular case, $\mu N = 50g = 245$ (N), which is greater than the force (100 N) exerted by the worker, so the crate remains at rest because the friction is able to adjust to this value (100 N) – and no further – to prevent motion. A simple force diagram can be used to emphasise that if friction is greater than 100 N, then the crate would move in the direction of the frictional force that, by definition, must oppose motion.

Question 30 was regarded as relatively difficult. Slightly more students (39%) chose graph C, which gave the shape of the corresponding velocity-time graph. This question could be tackled by rearranging the equation of motion, $T - mg = ma$, as $T = m(g + a)$.

Part II: Short-answer questions

Question 1 (Average mark 1.86/Available marks 4)

Answer: asymptotes $y = \frac{x^2}{5}$ and or $x = 0$ or $y = -\frac{5}{x}$;
intercept (2.92, 0); local minimum turning point (–2.32, 3.23)

Almost half of the students scored a maximum of only 1 mark on this question, with 24% scoring zero and 25% scoring 1. Most of the students who scored zero, and many who scored 1, apparently entered the equation in their graphics calculator as $y = (x^3 - 25)/5x$, rather than as $y = (x^3 - 25)/(5x)$ and so obtained the graph of $y = (x^4 - 25x)/5$. This highlights the need for students to have the habit of checking the 'reasonableness' of calculator results, including the shape of graphs. It should have been immediately apparent from the original equation that its graph must have an asymptote at $x = 0$.

Many students who drew the correct graph lost 1 or 2 easy marks because they ignored the direction to give coordinates 'correct to two decimal places', instead either giving exact answers only (e.g. $\sqrt[3]{25}$) or quoting only one decimal place (e.g. 2.9). Other students lost a mark because their answer was correct to only one decimal place, presumably because they had used the 'Trace' feature of their calculator to find the coordinate/s. Students should be aware that the precision of results obtained by 'tracing' is limited by the screen resolution and, accordingly, always use the appropriate numerical function (e.g. 'zero' or 'root') instead.

Although a graphics calculator was a useful aid in tackling this question, it was not possible to obtain full marks by complete reliance on the calculator since it was necessary to identify the parabolic asymptote – by re-expressing the function rule as $f(x) = \frac{x^2}{5} - \frac{5}{x}$. On the other hand, it was also disappointing to find that some students were able to find the coordinates and asymptotes correctly, using algebra and calculus, but were unable to sketch the graph.

Question 2

a. (0.4/2)

Answer: 1.5

Most students (75%) scored zero on this question, despite last year's CAT 3 (Analysis task) *Report for Teachers* highlighting the poor performance on a very similar question (Question 2b). It seems that many students failed to realise that it was necessary to work with the distance expression (or, even better, its square). The most common error amongst those students who did know this was to write $l = r^2 = t - (t - 2)^2$, again like last year. Some students suffered from indifferent algebra skills, with a favourite example being

$$r = \sqrt{t + (t-2)^2} = \sqrt{t + t^2 - 4t + 4} = \sqrt{t} + t - 2\sqrt{t} + 2.$$

b. (0.89/2)

Answer: $y = -x^2 + 2, x \ge 0$

It was expected that students would tackle this question by first finding the Cartesian equation of the path to determine its basic shape, and then plotting a few points to help sketch it carefully. However, many students omitted the first step and so generally failed to draw a sufficiently accurate graph. Others wrote $y = (t - 2)$ instead of $y = -(t - 2)$ and sketched the inverted parabola. A more common error was to draw the whole parabola instead of only the right-hand half, while other common errors included not indicating clearly that the vertex (0, 2) was included and having a very inaccurate x-intercept.

Use of parametric graphing by graphics calculator was an acceptable alternative approach.

Question 3 (1.27/3)

Answer: $\frac{3\sqrt{3}}{4}$

This question was generally poorly done. Most students seemed to have learned an automatic response, namely expressing $\sin^3 x$ as $(\sin^2 x)(\sin x)$ and so gained an easy mark, but many lacked the necessary depth of understanding to use it correctly. The most common error was to proceed to substitute a double angle identity for $\sin^2 x$.

Somewhat unexpectedly, some students appeared to access a formula for $\int(\sin^3 x)dx$ (almost always without success), whilst a few used one for $\int(\sin^3 ax)dx$ (sometimes with success). In future, the examiners will bear in mind that students may have included such formulas on their pre-written notes, and try to ensure that the real intent of a question cannot be avoided by their use.

Many students were very sloppy with their notation in this question, writing '*dx*' instead of '*du*' after substituting for *x* in the integrand, and/or not changing the terminals accordingly. It cannot be assumed that such sloppiness will be acceptable in the future. Of the students who integrated correctly, many failed to earn the final mark because they did not have sufficient skill in handling surds to simplify their answer correctly to a single term.

Question 4

a. (1.37/2)

Answer: $\frac{4}{3}$ m/s^2

This question was generally well done, providing many students with all or most of their marks on Part II of the examination. Most students considered the two blocks separately, but a substantial number arrived quickly at the correct answer by considering the equation of motion of the system as a whole (though often their work in part b. raised doubts, they really knew that this was what they were doing). Some students used 5*g* and 10*g* as the masses of the blocks, whilst others got into difficulty by trying to involve the weights and normal reactions – sometimes even going as far as to regard the pulling force, 20 N, as 20 times a common normal reaction (*N*). Some students gave separate answers for each block 4 m/s^2 and 2 m/s^2, whilst a few included a third answer $\frac{4}{3}$ m/s^2 for good measure.

b. (0.41/1)

Answer: $\frac{20}{3}$ N

Not as well done as part a., especially by those students who considered the system as a whole in part a and so did not need to involve a tension force at that stage. The most common error was to assume that the tension was 20 N.

Question 5

a. (0.45/1)

Answer: Exterior and circumference of circle centre (0, 1), radius 2

Fairly well done. Most students realised that the region involved a circle of radius 2, but some had it centred incorrectly. The most common error was to shade the interior of the circle instead of the exterior.

b. (0.64/2)

Answer: All of the complex plane, except for the origin and the 'sector' bounded by the negative *x*-axis and the ray $y = -x, x < 0$, including $y = -x, x < 0$ but excluding the negative *x*-axis.

Only a small percentage (9%) of students gained both marks. Most students obtained the first mark by drawing the ray $\text{Arg}(z) = \frac{3\pi}{4}$ correctly, but then generally only shaded the region $0 \le \text{Arg}(z) \le \frac{3\pi}{4}$ or, less often, the region $\frac{3\pi}{4} \le \text{Arg}(z) \le \pi$. Some students almost had the correct answer, omitting only to indicate that the negative imaginary axis ($\text{Arg}(z) = \pi$) was excluded. Strictly speaking, the origin also should have been excluded since Arg(0) is regarded as being undefined.

Question 6

a. (0.56/1)

Answer:

$$(2-i)^3 - (2-i)(2-i)^2 + (2-i) - 2 + i$$
$$= \left[(2-i)^3 - (2-i)^3\right] + \left[(2-i) - (2-i)\right] = 0$$

This question was most easily done as indicated above: by substituting (2–*i*) for *z* and noting that the two successive pairs of terms cancel. Students who attempted to expand $(2-i)^3$ or $(2-i)^2$ often got into trouble (though many graphics calculators can perform these expansions), with many incorrectly writing $(2-i)^3 = 8 - i$. Students must make sure it is clear exactly how they are collecting and simplifying terms when answering 'show'… questions like this one.

b. (0.37/2)

Answer: $2 - i, -i, i$

This was not done well. Many students who attempted this question mistakenly assumed that the conjugate root theorem applied, wrote down 2+*i* as a second solution, and then attempted in vain to find a third solution by dividing through by the quadratic term $(z - (2-i))(z - (2+i)) = z^2 - 4z + 5$. Students should know that the conjugate root theorem only applies to polynomials with *real* coefficients – this point was made in the 1998 CAT 3 (Analysis task) *Report for Teachers* (Question 1biii) and is an important concept for teachers to cover carefully with their students.

The solutions could be found easily by taking out common factors as follows:

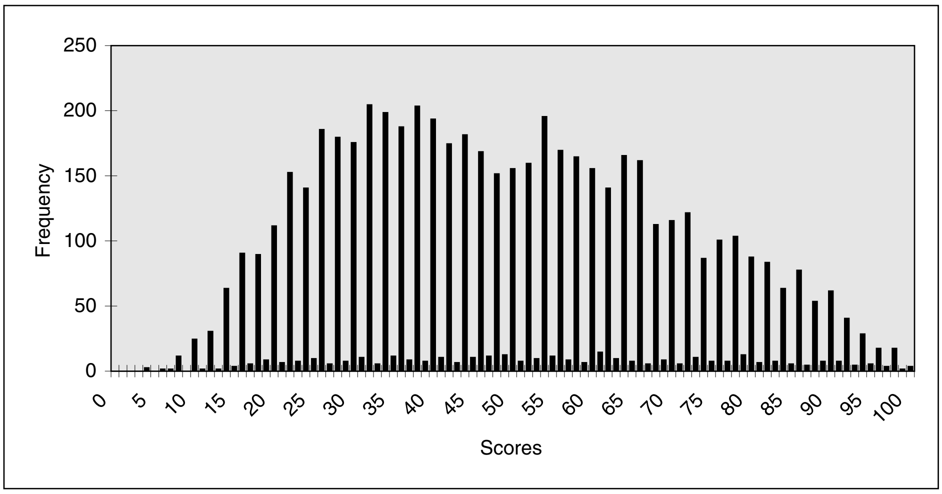
$z^3 - (2-i)z^2 + z - 2 + i = z^2(z - 2 + i) + (z - 2 + i) = (z^2 + 1)(z - 2 + i)$.

Some students lost a mark because they gave the factors as the solutions (zeros), a confusion also noted in previous editions of *Report for Teachers*.

GA MA092 MATHEMATICS: Specialist Mathematics WRITTEN EXAMINATION 1

HISTOGRAM OF TOTAL SCORES 2000

Count 5857 Mean 49.23 Standard Deviation 20.79 NA Result 112



HISTOGRAM OF TOTAL GRADES 2000

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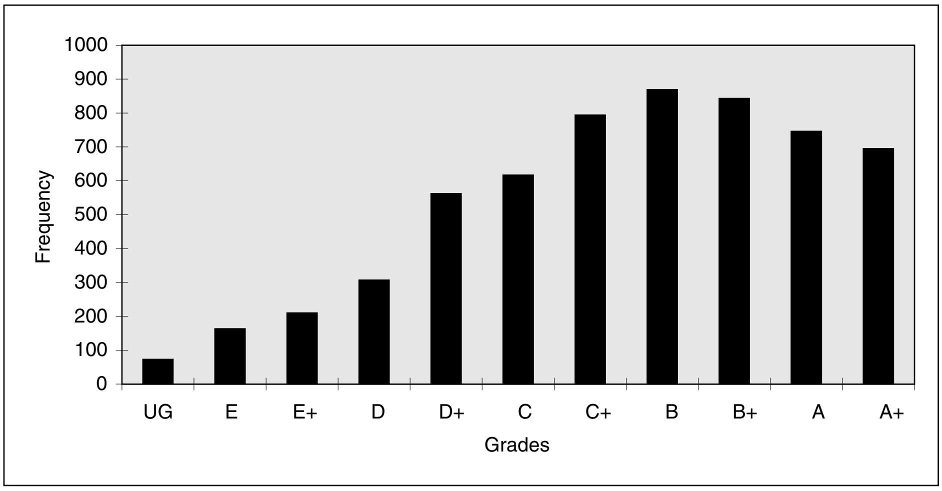

ENROLMENTS		%
Female	2150	36.0
Male	3819	64.0
Total	5969	

GLOSSARY OF TERMS

Count Number of students undertaking the assessment. This excludes those for whom NA was the result.

Mean This is the ‘average’ score; that is all scores totalled then divided by the ‘Count’.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).