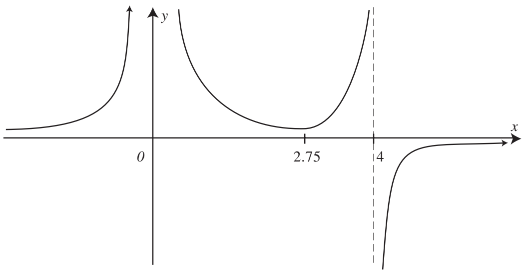
Trial Examination 2 Solutions

Question 1

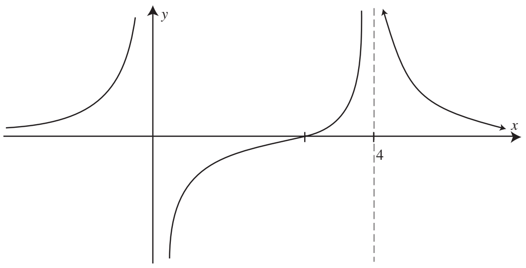
a. $y_2 = \dfrac{1}{f(x)}$

Asymptotes at $x = 0$ and $x = 4$ **[A1]**
Correct shape **[A1]**
Local minimum at $x \approx 2.75$ **[A1]**



b. $A \approx \frac{1}{2}(1 + 2)(1.75) + \frac{1}{2}(2 + 1)(0.75)$ **[M1]**
≈ 3.75 square units **[A1]**

c. Correct shape **[A1]**
Correct asymptotes **[A1]**
Correct position of point of inflexion **[A1]**



Total 8 marks

Question 2

a. $P(-1+\sqrt{3}i) = (-1+\sqrt{3}i)^3 + 5(-1+\sqrt{3}i)^2 + 10(-1+\sqrt{3}i) + 12$ **[M1]**

$= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i + 5 - 10\sqrt{3}i - 15 - 10 + 10\sqrt{3}i + 12$ **[M1]**

$= 0$ (by collecting like terms) **[A1]**

b. $(z+1+\sqrt{3}i)(z+1-\sqrt{3}i)(z+a) = z^3 + 5z^2 + 10z + 12$

Let $z = 0$

$(1+\sqrt{3}i)(1-\sqrt{3}i)(a) = 12$ **[M1]**

$4a = 12$

$a = 3$

$(z+1+\sqrt{3}i)(z+1-\sqrt{3}i)(z+3)$ **[M1]**

c. $\theta = \text{Tan}^{-1}\dfrac{\sqrt{3}}{1} = \dfrac{\pi}{3}$

$r = \sqrt{1+3} = 2$

$z_1 = 2\text{cis}\dfrac{2\pi}{3}$

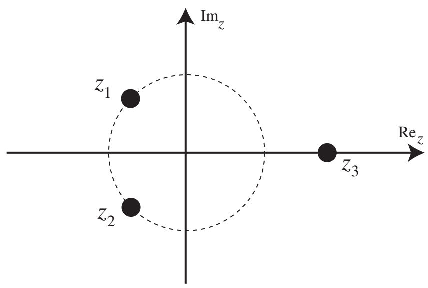
$z_2 = 2\text{cis}\dfrac{-2\pi}{3}$

$z_3 = 2\text{cis}$

[A2] for all three

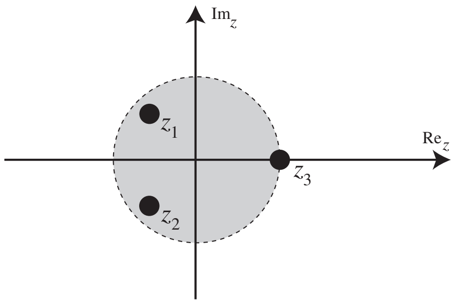
[A1] for z_1 or z_2

d.



[A1]

e.

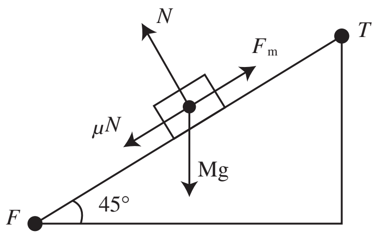


Correct region (shaded disc of radius 3) **[A1]**
Solutions which lie within this radius
are z = 2cis $\frac{2\pi}{3}$ and 2cis $\frac{-2\pi}{3}$ **[A1]**

Total 10 marks

Question 3

a.



[A1]

b. $N = \frac{250g}{\sqrt{2}} = 125\sqrt{2}g$

$\mu N = 12.5\sqrt{2}g$

Resultant force = ma

$F_r = 250 \times -2$

$F_r = -500$

$Fm - \mu N - 250g\sin 45° = -500$ **[M1]**

$Fm = -500 + \frac{25g}{\sqrt{2}} + \frac{250g}{\sqrt{2}}$

$\approx 1406N$ **[A1]**

c. $v^2 = u^2 + 2as$

$u = 17$

$a = -2$

$s = 4$

$v^2 = 17^2 + 2 \times -2 \times 4$ **[M1]**

$= 273$

$v = 16.5$ m/s up the ramp **[A1]**

d. $\ddot{r}(t) = -g\underset{\sim}{j}$

$\dot{r}(t) = -gt\underset{\sim}{j} + c$

$\dot{x}(o) = v\cos\theta$

$= 16.5 \times \frac{1}{\sqrt{2}}$ **[M1]**

$\dot{y}(o) = v\sin\theta$

$= 16.5 \times \frac{1}{\sqrt{2}}$

$c = \frac{16.5}{\sqrt{2}}\underset{\sim}{i} + \frac{16.5}{\sqrt{2}}\underset{\sim}{j}$ **[M1]**

$\dot{r}(t) = \frac{16.5}{\sqrt{2}}\underset{\sim}{i} + \left(\frac{16.5}{\sqrt{2}} - gt\right)\underset{\sim}{j}$ **[A1]**

e. $\dot{r}(t) = \frac{16.5}{\sqrt{2}}\underset{\sim}{i} + \left(\frac{16.5}{\sqrt{2}} - gt\right)\underset{\sim}{j}$

$r(o) = 2\sqrt{2}\underset{\sim}{j}$

$r(t) = \frac{16.5}{\sqrt{2}}t\underset{\sim}{i} + \left(\frac{16.5}{\sqrt{2}}t - \frac{gt^2}{2} + 2\sqrt{2}\right)\underset{\sim}{j}$ **[A1]**

$x = \frac{16.5}{\sqrt{2}}t$ (1)

$t = \frac{\sqrt{2}x}{16.5}$ (2)

substitute vertical component of $r(t)$ for y

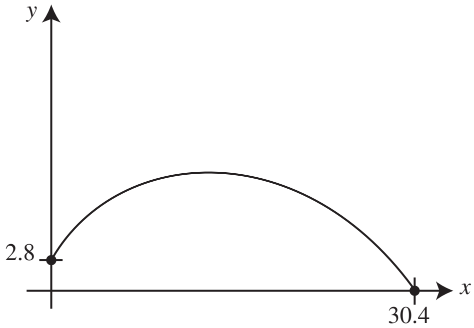
$y = \frac{16.5}{\sqrt{2}}t - \frac{gt^2}{2} + 2\sqrt{2}$

substitute (2)

$y = \frac{16.5}{\sqrt{2}} \times \frac{\sqrt{2}}{16.5}x - \frac{y}{2}\left(\frac{\sqrt{2}x}{16.5}\right)^2 + 2\sqrt{2}$ **[M1]**

$= x - \frac{y}{2} \times \frac{2x^2}{(16.5)^2} + 2\sqrt{2}$

$= x - \frac{gx^2}{272.25} + 2\sqrt{2}$ **[A1]**



f. **[A2]**

g. 30.4 m when height is zero **[A1]**

Total 15 marks

Question 4

a. $x = \tan\theta$

$\theta = \text{Tan}^{-1}x$ ①

[A1]

b. $\dfrac{d\theta}{dt} = \dfrac{d\theta}{dx} \times \dfrac{dx}{dt}$

$\dfrac{d\theta}{dx} = \dfrac{1}{1+x^2}$ from ①

$\dfrac{d\theta}{dt} = \dfrac{2x}{1+x^2}$ **[A1]**

at $x = 1$

$\dfrac{d\theta}{dt} = \dfrac{2}{1+1^2}$ **[A1]**

$= 1$

c. $\dfrac{d\theta}{dt} = \dfrac{2x}{1+x^2}$

$x = \tan\theta$

$\dfrac{d\theta}{dt} = \dfrac{2\tan\theta}{1+\tan^2\theta}$ **[M1]**

$= \dfrac{2\tan\theta}{\sec^2\theta}$ **[M1]**

$= 2\sin\theta\cos\theta$

$= \sin 2\theta$ **[A1]**

d. i. $l = \dfrac{1}{\cos\theta}$ **[A1]**

ii. $\dfrac{dl}{dt} = \dfrac{dl}{d\theta} \times \dfrac{d\theta}{dt}$

$\dfrac{dl}{d\theta} = \dfrac{\sin\theta}{\cos^2\theta}$

$\dfrac{d\theta}{dt} = 2\sin\theta\cos\theta$ **[M1]**

$\dfrac{dl}{dt} = \dfrac{\sin\theta}{\cos^2\theta} \times 2\sin\theta\cos\theta$

$= \dfrac{2\sin^2\theta}{\cos\theta}$ **[A1]**

iii. $\dfrac{dl}{d\theta} = \dfrac{2\sin^2\theta}{\cos\theta}$

$\cos\theta = \dfrac{1}{l}$

$\dfrac{dl}{dt} = \dfrac{2(1-\cos^2\theta)}{\cos\theta}$ **[M1]**

$= \dfrac{2\left(1-\dfrac{1}{l^2}\right)}{\dfrac{1}{l}}$

$= 2\left(\dfrac{l^2-1}{l^2}\right) \times \dfrac{l}{1}$ **[M1]**

$= \dfrac{2(l^2-1)}{l}$

iv. $\dfrac{dl}{dt} = \dfrac{2(l^2-1)}{l} \quad l > 1$

$\dfrac{dt}{dl} = \dfrac{l}{2\left(l^2-1\right)}$

$t = \dfrac{1}{2}\displaystyle\int \dfrac{l}{\left(l^2-1\right)}\,dl$

$t = \dfrac{1}{4}\displaystyle\int \dfrac{2l}{\left(l^2-1\right)}\,dl$

$= \dfrac{1}{4}\log_e\left(l^2-1\right) + c$ **[M1]**

$l = \sqrt{2}$, when $t = 0$

$0 = \dfrac{1}{4}\log_e(2-1) + c$

$c = 0$

$t = \dfrac{1}{4}\log_e\left(l^2-1\right)$ **[M1]**

$4t = \log_e\left(l^2-1\right)$

$e^{4t} = l^2 - 1$

$l = \sqrt{e^{4t}+1}$

Positive answer only because length can't be negative

[A1]

Total 14 marks

Question 5

a. i. WE $= \sqrt{100^2 + 250^2}$

$= 269.26$

EH $= 100$

WE + EH = 369 metres **[A1]**

ii. $t = \dfrac{\sqrt{100^2+250^2}}{5} + \dfrac{100}{3}$ **[M1]**

= 1 minute 27 seconds **[A1]**

b. i. WH $= \sqrt{200^2 + 250^2}$

= 320 metres **[A1]**

ii. The direct route will lead to half the total distance being travelled in the golf course and half in the forest.

$t = \dfrac{\frac{1}{2}\sqrt{200^2+250^2}}{5} + \dfrac{\frac{1}{2}\sqrt{200^2+250^2}}{3}$ **[M1]**

$= \dfrac{\sqrt{200^2+250^2}}{10} + \dfrac{\sqrt{200^2+250^2}}{6}$

= 1 min 25 seconds **[A1]**

c. i. $d = \sqrt{x^2+100^2} + \sqrt{100^2+(250-x)^2}$ **[A1]**

ii. $t = \dfrac{\sqrt{x^2+100^2}}{5} + \dfrac{\sqrt{100^2+(250-x)^2}}{3}$ **[M1]**

domain: $0 \le x \le 250$ **[A1]**

d. i. Find minimum on graphics calculator graph.

t = 82 seconds **[A2]**

ii. From graph, minimum t occurs when $x = 187.5873$

Substitute $x = 187.5873$ into

$d = \sqrt{x^2+100^2} + \sqrt{100^2+(250-x)^2}$ **[M1]**

Hence d = 330 metres. **[A1]**

Total 13 marks