Trial Examination 1 Answers & Solutions

Part I (Multiple-choice) Answers

1. **D**	2. **A**	3. **C**	4. **D**	5. **A**
6. **E**	7. **B**	8. **C**	9. **B**	10. **B**
11. **D**	12. **B**	13. **D**	14. **E**	15. **A**
16. **A**	17. **D**	18. **D**	19. **B**	20. **E**
21. **B**	22. **C**	23. **D**	24. **C**	25. **D**
26. **A**	27. **E**	28. **C**	29. **E**	30. **D**

Solutions

Question 1 [D]

$$u^2v = 9cis\frac{3\pi}{2} \times 2cis\frac{-\pi}{3}$$

$$= 18cis\left(\frac{3\pi}{2} + \frac{-\pi}{3}\right)$$

$$= 18cis\left(\frac{9\pi - 2\pi}{6}\right)$$

$$= 18cis\left(\frac{7\pi}{6}\right)$$

Question 2 [A]

$$y = \text{Tan}^{-1}u \qquad u = \frac{2}{x} \qquad \frac{dy}{dx} = \frac{1}{1+\frac{4}{x^2}} \times \frac{-2}{x^2}$$

$$\frac{dy}{du} = \frac{1}{1+u^2} \qquad \frac{du}{dx} = \frac{-2}{x^2} \qquad = \frac{-2}{x^2+4}$$

Question 3 [C]

$$w = 1 + i$$

$$= \sqrt{2}cis\frac{\pi}{4}$$

$$w^5 = 4\sqrt{2}cis\frac{5\pi}{4}$$

$$Arg\ w = \frac{-3\pi}{4}$$

Question 4 [D]

$$\hat{\underset{\sim}{b}} = \frac{\underset{\sim}{b}}{\left|\underset{\sim}{b}\right|} = \frac{2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}}{3}$$

$$\underset{\sim}{a} \bullet \underset{\sim}{b} = -6 - 2 + 4 = -4$$

$$\left(\underset{\sim}{a} \bullet \underset{\sim}{b}\right)\hat{\underset{\sim}{b}} = \frac{-8\underset{\sim}{i} + 8\underset{\sim}{j} - 4\underset{\sim}{k}}{3}$$

Question 5 [A]

$$\overrightarrow{AC} = \underset{\sim}{b} - \underset{\sim}{a}$$

$$\overrightarrow{AM} = \frac{1}{2}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)$$

$$\overrightarrow{BM} = \underset{\sim}{a} + \overrightarrow{AM}$$

$$= \frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right)$$

If $\overrightarrow{AC} \bullet \overrightarrow{BM} = 0$ then triangle is isoceles

$$\overrightarrow{AC} \bullet \overrightarrow{BM} = \left(\underset{\sim}{b} - \underset{\sim}{a}\right) \bullet \frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right)$$

Question 6 [E]

Centre of circle 10 + 10i

Radius 10

$$\{z : |z - 10 - 10i| = 10\}$$

Question 7 [B]

$$\int \frac{-16}{\sqrt{1-4x^2}}dx$$

$$= \int \frac{-16}{\sqrt{4\left(\frac{1}{4} - x^2\right)}}dx$$

$$= \int \frac{-8}{\sqrt{\frac{1}{4} - x^2}}dx$$

$$= 8\text{Cos}^{-1}(2x)$$

Note: + c not required, since question asks for an antiderivative.

Question 8 [C]

$$4(x-1)^2 - 9y^2 = 36$$

$$\frac{4(x-1)^2}{36} - \frac{9y^2}{36} = 1$$

$$\frac{(x-1)^2}{9} - \frac{y^2}{4} = 1$$

$$y = \pm\frac{2}{3}(x-1)$$

Question 9 [B]

$$\frac{5x+6}{x^2+6x+9} = \frac{5x+6}{(x+3)^2} = \frac{A}{(x+3)^2} + \frac{B}{x+3}$$

$$= \frac{A+B(x+3)}{(x+3)^2}$$

$5x+6 \equiv A + B(x+3)$

When $x = -3$, $A = -9$

When $x = 0$, $B = 5$

$$\frac{5x+6}{x^2+6x+9} = \frac{-9}{(x+3)^2} + \frac{5}{x+3}$$

Question 10 [B]

$x = 2\cos 2t \qquad y = 3\sin 2t$

$\frac{x}{2} = \cos 2t \qquad \frac{y}{3} = \sin 2t$

$$\frac{x^2}{4} + \frac{y^2}{9} = \cos^2 2t + \sin^2 2t$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$9x^2 + 4y^2 = 36$

Question 11 [D]

$\ddot{\underset{\sim}{r}}(t) = 2\underset{\sim}{i} + 6t\underset{\sim}{j} + \underset{\sim}{k}$

$\dot{\underset{\sim}{r}}(t) = 2t\underset{\sim}{i} + 3t^2\underset{\sim}{j} + t\underset{\sim}{k} + c$

$\dot{\underset{\sim}{r}}(0) = 0 \Rightarrow c = 0$

$\dot{\underset{\sim}{r}}(t) = 2t\underset{\sim}{i} + 3t^2\underset{\sim}{j} + t\underset{\sim}{k}$

$\underset{\sim}{r}(t) = t^2\underset{\sim}{i} + t^3\underset{\sim}{j} + \frac{t^2}{2}\underset{\sim}{k} + d$

$\underset{\sim}{r}(0) = \underset{\sim}{i} + 3\underset{\sim}{j}$

$\underset{\sim}{r}(t) = t^2\underset{\sim}{i} + t^3\underset{\sim}{j} + \frac{t^2}{2}\underset{\sim}{k} + \underset{\sim}{i} + 3\underset{\sim}{j}$

$\underset{\sim}{r}(t) = \left(t^2+1\right)\underset{\sim}{i} + \left(t^3+3\right)\underset{\sim}{j} + \frac{t^2}{2}\underset{\sim}{k}$

Question 12 [B]

$\dot{\underset{\sim}{r}}(t) = \cos\frac{\pi t}{6}\underset{\sim}{i} + 3\sin\frac{\pi t}{6}\underset{\sim}{j}$

$\ddot{\underset{\sim}{r}}(t) = -\frac{\pi}{6}\sin\frac{\pi t}{6}\underset{\sim}{i} + \frac{\pi}{2}\cos\frac{\pi t}{6}\underset{\sim}{j}$

$\ddot{\underset{\sim}{r}}(6) = -\frac{\pi}{6}\sin\frac{6\pi}{6}\underset{\sim}{i} + \frac{\pi}{2}\cos\frac{6\pi}{6}\underset{\sim}{j}$

$= 0 - \frac{\pi}{2}\underset{\sim}{j}$

$\left|\ddot{\underset{\sim}{r}}(6)\right| = \frac{\pi}{2}$

Question 13 [D]

$\underset{\sim}{v} = \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{w} = 2\underset{\sim}{i} - \underset{\sim}{k}$

$\underset{\sim}{v} + 2\underset{\sim}{w} = \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k} + 2(2\underset{\sim}{i} - \underset{\sim}{k})$

$= \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k} + 4\underset{\sim}{i} - 2\underset{\sim}{k}$

$= 5\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$

$\left|\underset{\sim}{v} + 2\underset{\sim}{w}\right| = \sqrt{25+4+1}$

$= \sqrt{30}$

Question 14 [E]

$y = \cos 3x + \sin 3x$

$\frac{dy}{dx} = -3\sin 3x + 3\cos 3x$

$\frac{d^2y}{dx^2} = -9\cos 3x - 9\sin 3x$

$= -9(\cos 3x + \sin 3x)$

$= -9y$

Question 15 [A]

$\int \cos^3 x\,dx$

$= \int \cos x \cos^2 x\,dx$

$= \int \cos x(1 - \sin^2 x)\,dx$

$= \int (\cos x - \cos x \sin^2 x)\,dx$

$= \sin x - \frac{\sin^3 x}{3}$

Note: + c not required, since question asks for an antiderivative.

Question 16 [A]

$y = \tan(\log_e 3x)$

$y = \tan u$, where $u = \log_e 3x$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \quad \text{(chain rule)}$$

$$= \sec^2 u \times \frac{1}{x}$$

$$= \frac{1}{x}\sec^2(\log_e 3x)$$

Question 17 [D]

$$\int_0^{\frac{\pi}{2}} \cos 3x\, e^{\sin 3x}\, \text{dx} \quad \text{let } u = \sin 3x$$

$$\frac{du}{dx} = 3\cos 3x$$

Terminals: $x = \frac{\pi}{2}$, $u = \sin\frac{3\pi}{2} = -1$

$x = 0$, $u = \sin 0 = 0$

$$\int_0^{\frac{\pi}{2}} \cos 3x\, e^{\sin 3x}\, \text{dx} = \int_{u=0}^{u=-1} \frac{1}{3} e^u du$$

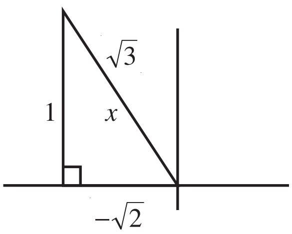
$$= -\frac{1}{3}\int_{-1}^{0} e^u du$$

Question 18 [D]

$\text{cosec} x = \sqrt{3}$

$$\frac{1}{\sin x} = \sqrt{3}$$

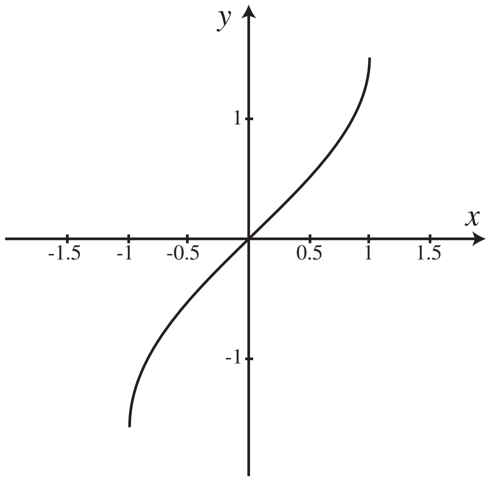
$$\sin x = \frac{1}{\sqrt{3}}$$



(Pythagoras' Theorem)

$$\tan x = \frac{1}{-\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

Question 19 [B]



The graph of $\text{Sin}^{-1}(x)$ is shown above with domain $[-1, 1]$.

$\text{Sin}^{-1}(x - a)$ will be translated 'a' units to the right, hence

$[a - 1, a + 1]$

Question 20 [E]

$$\int \frac{3}{3+4x^2} dx = \int \frac{3}{4(\frac{3}{4} + x^2)} dx$$

$$= \frac{3}{4}\int \frac{1}{\frac{3}{4} + x^2} dx$$

$$= \frac{3}{4}\int \frac{\sqrt{4}}{\sqrt{3}} \frac{\sqrt{\frac{3}{4}}}{(\sqrt{\frac{3}{4}})^2 + x^2} dx$$

$$= \frac{3}{4} \times \frac{2}{\sqrt{3}} \int \frac{\sqrt{\frac{3}{4}}}{(\sqrt{\frac{3}{4}})^2 + x^2} dx$$

$$= \frac{3}{2\sqrt{3}} \text{Tan}^{-1} \frac{x}{\frac{\sqrt{3}}{\sqrt{4}}} + c$$

$$= \frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \text{Tan}^{-1} \frac{2x}{\sqrt{3}} + c$$

$$= \frac{\sqrt{3}}{2} \text{Tan}^{-1} \frac{2x}{\sqrt{3}}$$

Note, + c not required, since question requests an antiderivative.

Question 21 [B]

$$\frac{dv}{dr} = 4\pi r^2$$

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$

$$= \frac{1}{4\pi r^2} \times 4$$

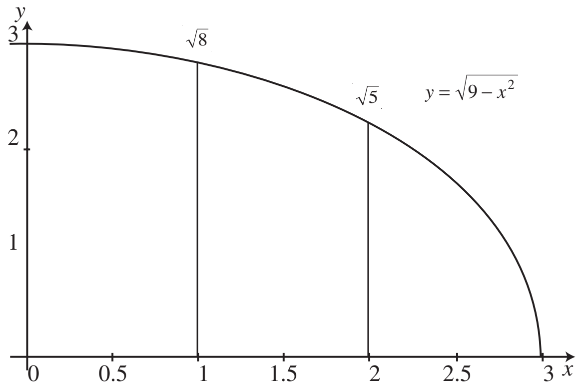
$$= \frac{1}{\pi r^2}$$

Question 22 [C]

$$\frac{dx}{dt} = -8\cos 2t$$

When $t = o,\ \frac{dx}{dt} = -8\cos 0 = -8$

Question 23 [D]



$$A = \frac{1}{2}(3+\sqrt{8})1 + \frac{1}{2}(\sqrt{8}+\sqrt{5})1 + \frac{1}{2}(\sqrt{5}+0)$$

$$= \frac{1}{2}(3 + 2\sqrt{8} + 2\sqrt{5})$$

$$\approx 6.56$$

Question 24 [C]

Magnitude of force $= \sqrt{4^2 + (-3)^2} = 5$ newtons

$$a = \frac{f}{m} = \frac{5}{2} = 2.5$$

Question 25 [D]

$\int_0^1 2x(x+4)^5 dx$ let $u = x + 4$

$$\frac{du}{dx} = 1$$

$$x = u - 4$$

Terminals: $x = 1$, $u = 1 + 4 = 5$
$x = 0$, $u = 0 + 4 = 4$

$$\int_0^1 2x(x+4)^5 dx = \int_{u=4}^{u=5} 2(u-4)u^5 du$$

$$= \int_4^5 (2u^6 - 8u^5) du$$

Question 26 [A]

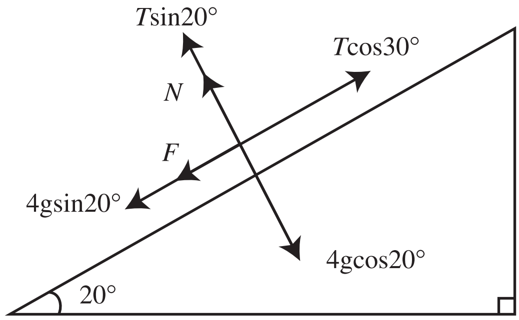
$$V = \int_{y_1}^{y_2} \pi x^2 dy$$

$y = \sin x$

$x = \text{Sin}^{-1} y$

$$V = \int_0^1 \pi(\text{Sin}^{-1} y)^2 dy$$

Question 27 [E]



Forces shown on the diagram have been resolved parallel and perpendicular to the plane.
Considering equilibrium forces parallel to the plane:

$T \cos 30° = F + 4g \sin 20°$ and since

$F = \mu N = 0.3N$

$T \cos 30° = 0.3N + 4g \sin 20°$

Question 28 [C]

$$\underset{\sim}{F_1} + \underset{\sim}{F_2} = 7\underset{\sim}{i} - 2\underset{\sim}{j}$$

Hence $\underset{\sim}{F_3} = -7\underset{\sim}{i} + 2\underset{\sim}{j}$

$$\left|\underset{\sim}{F_3}\right| = \sqrt{(-7)^2 + 2^2} = \sqrt{53}$$

$$\approx 7.28$$

Question 29 [E]

$v = (2x-3)^2$

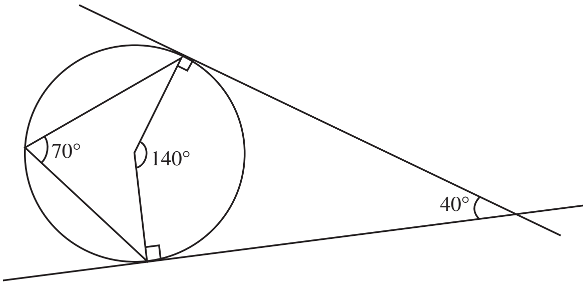
$v\frac{dv}{dx} = v \times 2(2x-3)^1 2$

$= v \times 4(2x-3)$

$= 4(2x-3)^3$

When x = 3, $v\frac{dv}{dx} = 108$ cm/s^2

Question 30 [D]



Part II (Short answer questions)

Question 1

$\frac{dv}{dt} = g - kv$,

$\frac{dt}{dv} = \frac{1}{g-kv}$

$t = \int \frac{1}{g-kv} dt$

$= -\frac{1}{k}\int \frac{-k}{g-kv} dt$ **[M1]**

$= -\frac{1}{k}\log_e(g-kv) + c$

$t = 0, v = 0 \quad \Rightarrow c = \frac{1}{k}\log_e g$

$\therefore t = \frac{1}{k}\log_e g - \frac{1}{k}\log_e(g-kv)$

$t = \frac{1}{k}\log_e \frac{g}{g-kv}$ **[A1]**

$kt = \log_e \frac{g}{g-kv}$

$e^{kt} = \frac{g}{g-kv}$

$ge^{-kt} = g - kv$

$kv = g - ge^{-kt}$

$v = \frac{g}{k}(1 - e^{-kt})$ **[A1]**

Question 2

$\underset{\sim}{r}(t) = (e^t \sin t)\underset{\sim}{i} - (e^t \cos t)\underset{\sim}{j}$.

$\underset{\sim}{r}'(t) = (e^t \cos t + e^t \text{sint})\underset{\sim}{i} - (-e^t \sin t + e^t \cos t)\underset{\sim}{j}$

$= (e^t \cos t + e^t \text{sint})\underset{\sim}{i} + (e^t \sin t - e^t \cos t)\underset{\sim}{j}$ **[M1]**

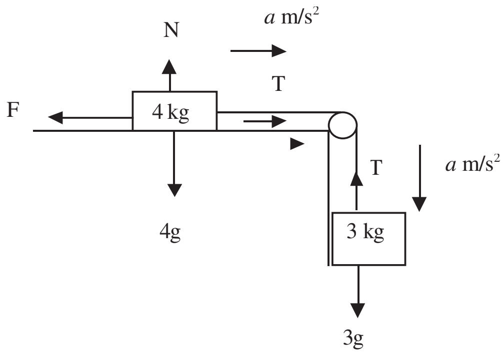
$\left|\underset{\sim}{r}'(t)\right| = \sqrt{(e^t \cos t + e^t \sin t)^2 + (e^t \sin t - e^t \cos t)^2}$ **[M1]**

$= \sqrt{(e^t)^2(\cos^2 t + 2\cos t \sin t + \sin^2 t) + (e^t)^2(\sin^2 t - 2\sin t \cos t + \cos^2 t)}$

$= e^t\sqrt{1 + 2\cos t \sin t - 2\cos t \sin t + 1}$ **[A1]**

$= e^t\sqrt{2}$

Question 3



a. $3g - T = 3a$

$N = 4g$

$F = \mu N = 0.3(4g) = 1.2N$

$3g - T = 3a$ (I)

$T - 1.2g = 4a$ (II)

(I) + (II) **[M1]**

$1.8g = 7a$

$a = \frac{1.8g}{7} = 2.5$ m/s^2 **[A1]**

b. $3g - T = 3a$ **[M1]**

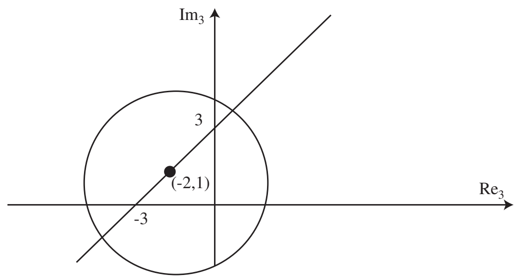
$T = 3g - 3a = 21.8$ newton **[A1]**

Question 4

a. $\{z: \text{Re}(z) - \text{Im}(z) = -3\}$ and $\{z: |z + 2 - i| = 3\sqrt{2}\}$

$\{z: \text{Re}(z) - \text{Im}(z) = -3\}$ $\quad$ $\{z: |z + 2 - i| = 3\sqrt{2}\}$

$y = x + 3$ $\quad$ $(x+2)^2 + (y-1)^2 = 18$



Line through (-3, 0) and (0, 3) **[A1]**

Circle **[M1]**

Centre (-2, 1) and radius $3\sqrt{2}$ **[A1]**

b. $y = x + 3$ ------------------(1)

$(x+2)^2 + (y-1)^2 = 18$ -------(2)

$(x+2)^2 + (x+3-1)^2 = 18$

$(x+2)^2 + (x+2)^2 = 18$

$(x+2)^2 = 9$

$x = 1, -5$

$(x, y) = (1, 4), (-5, -2)$

[A1 for one of the points]
[A1 for the other point]

Question 5

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2(2x)dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2}(\cos 4x + 1)dx$$ **[M1]**

$$= \frac{1}{2}\left[\frac{1}{4}\sin(4x) + x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$ **[M1]**

$$= \frac{1}{2}\left[\left(\frac{1}{4}\sin 2\pi + \frac{\pi}{2}\right) - \left(\frac{1}{4}\sin \pi + \frac{\pi}{4}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{\pi}{4}\right]$$

$$= \frac{\pi}{8}$$ **[A1]**

Question 6

$\int 2x\sqrt{x-2}dx$

let $u = x - 2$

$\Rightarrow x = u + 2$

$f(u) = \int 2(u+2)\sqrt{u}du$

$= \int 2u^{\frac{3}{2}} + 4u^{\frac{1}{2}}du$ **[M1]**

$= \frac{4}{5}u^{\frac{5}{2}} + \frac{8}{3}u^{\frac{3}{2}} + c$

$f(x) = \frac{4}{5}(x-2)^{\frac{5}{2}} + \frac{8}{3}(x-2)^{\frac{3}{2}} + c$ **[M1]**

$f(2) = \frac{4}{5}(2-2)^{\frac{5}{2}} + \frac{8}{3}(2-2)^{\frac{3}{2}} + c = 3$

$c = 3$

$f(x) = \frac{4}{5}(x-2)^{\frac{5}{2}} + \frac{8}{3}(x-2)^{\frac{3}{2}} + 3$ **[A1]**