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Student Name..

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

2000

Reading Time: 15 minutes
Writing Time: 90 minutes

Instructions to Students

This exam consists of Part I and Part II.
Part I consists of 30 multiple-choice questions and should be answered on the detachable answer sheet on page 17 of this exam. This section of the paper is worth 30 marks.
Part II consists of 6 short-answer questions, all of which should be answered in the spaces provided. This section of the paper is worth 20 marks.
There is a total of 50 marks available.
Students may bring up to two A4 pages of pre-written notes into the exam.

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PART I

Part I consists of 30 questions. Answer **all** questions in Part I.

Question 1

The asymptotes of the graph of the function with rule $y = \frac{1}{ax^2 + x}$ where a is a constant, are given by

A. $x = a$
B. $x = 0,\ y = 0$
C. $x = -\frac{1}{a},\ x = 0$
D. $x = a,\ y = 0$
E. $x = -\frac{1}{a},\ x = 0,\ y = 0$

Question 2

If $\cot x = -\frac{1}{2},\ -\frac{\pi}{2} \le x \le 0$ then the exact value of $\cos x$ is

A. $-\frac{\pi}{3}$
B. $-\sqrt{5}$
C. $\sqrt{5}$
D. $-\frac{1}{\sqrt{5}}$
E. $\frac{1}{\sqrt{5}}$

Question 3

If $y = \text{Tan}^{-1}(2x) + \cot(2x)$ then $\frac{dy}{dx}$ is equal to

A. $\frac{2}{2 + x^2} + \frac{\sec(2x)}{\tan^2(2x)}$
B. $\frac{2}{1 + 4x^2} - \frac{2\sec^2(2x)}{\tan^2(2x)}$
C. $\frac{2}{4 + x^2} + \frac{1}{\tan^2(2x)}$
D. $\frac{4}{1 + 2x^2} - \frac{\sec^2(2x)}{\tan(2x)}$
E. $\frac{4}{2 + x^2} - \frac{1}{\tan^2(2x)}$

Question 4

If $y = \sec(2x)$ then $\dfrac{d^2y}{dx^2}$ is equal to

A. $\dfrac{2\sin(2x)}{\cos^2(2x)}$

B. $4(1 - 2\sin^2(2x))$

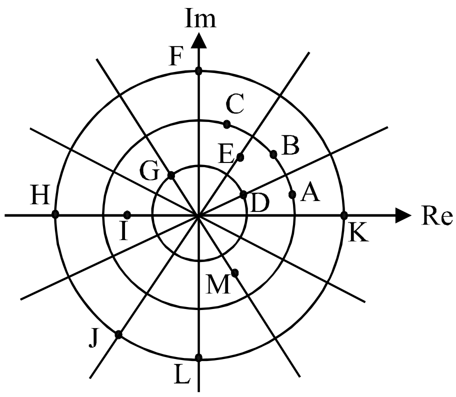
C. $4\cos(2x) + 8\sin(2x)\tan(2x)$

D. $4\sec(2x) + 8\sec(2x)\tan^2(2x)$

E. $8\sec(2x)\cos(2x) - 4\sin(2x)\tan(2x)$

Question 5

Which three points shown on the Argand diagram below could represent the 3 cube roots of a complex number?



A. A, B and C

B. F, H and L

C. E, I and M

D. K,G and J

E. D,G and J

Question 6

If $z = 2\text{cis}(\frac{2\pi}{3})$ then z^5 would equal

A. $7\text{cis}(\frac{5\pi}{3})$

B. $10\text{cis}(\frac{5\pi}{3})$

C. $10\text{cis}(\frac{10\pi}{3})$

D. $32\text{cis}(\frac{10\pi}{3})$

E. $32\text{cis}(\frac{32\pi}{3})$

Question 7

If $z - 5i$ is a factor of $P(z)$ where $P(z) = z^3 - 5iz^2 - 4z + 20i$ then which one of the following is **not** correct?

A. $z + 5i$ is a factor of $P(z)$
B. $5i$ is a solution to the equation $P(z) = 0$
C. $P(5i) = 0$
D. $P(z)$ has 2 real roots
E. $P(z)$ has 3 roots

Question 8

The angle between the vectors $\underset{\sim}{i} - 2\underset{\sim}{j} + 5\underset{\sim}{k}$ and $3\underset{\sim}{i} + 4\underset{\sim}{j} - \underset{\sim}{k}$,to the nearest minute, is

A. 1° 12'
B. 1° 34'
C. 1° 56'
D. 90°
E. 110° 59'

Question 9

If $\underset{\sim}{v}_1 = \underset{\sim}{i}$, $\underset{\sim}{v}_2 = \underset{\sim}{i} - 2\underset{\sim}{j}$, $\underset{\sim}{v}_3 = -2\underset{\sim}{j}$ and $\underset{\sim}{v}_4 = -2\underset{\sim}{i} + 4\underset{\sim}{j}$ then the two linearly dependent vectors are

A. $\underset{\sim}{v}_1$ and $\underset{\sim}{v}_2$
B. $\underset{\sim}{v}_1$ and $\underset{\sim}{v}_3$
C. $\underset{\sim}{v}_1$ and $\underset{\sim}{v}_4$
D. $\underset{\sim}{v}_2$ and $\underset{\sim}{v}_3$
E. $\underset{\sim}{v}_2$ and $\underset{\sim}{v}_4$

Question 10

If $\underset{\sim}{v} = \underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{u} = 3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ then the vector resolute of $\underset{\sim}{v}$ perpendicular to $\underset{\sim}{u}$ is

A. $\frac{1}{6}(27\underset{\sim}{i} - 5\underset{\sim}{j} + 10\underset{\sim}{k})$

B. $\frac{1}{\sqrt{14}}(\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k})$

C. $\frac{1}{3}(10\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k})$

D. $\frac{1}{2}(5\underset{\sim}{i} - 3\underset{\sim}{j} + 6\underset{\sim}{k})$

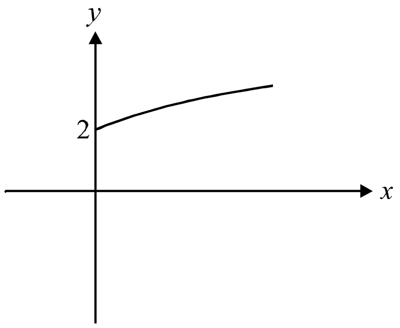
E. $-2\underset{\sim}{i} + 5\underset{\sim}{j} + 7\underset{\sim}{k}$

Question 11

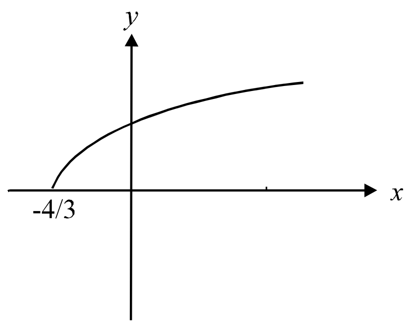
The position of a particle at time t is given by $\underset{\sim}{r}(t) = \frac{t-4}{3}\underset{\sim}{i} + \sqrt{t}\,\underset{\sim}{j},\ t \geq 0$

Which one of the following graphs shows the path of the particle?

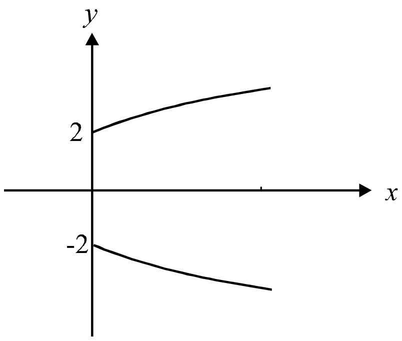
A.



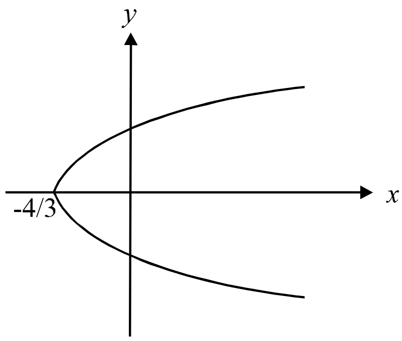
B.



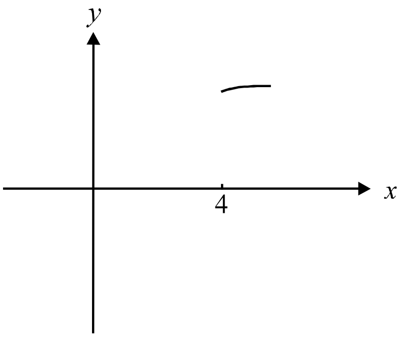
C.



D.



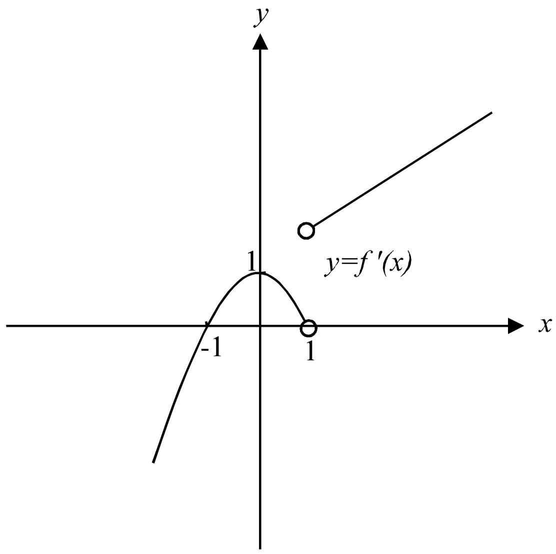
E.



Question 12

The graph of the function $f'(x)$ is shown.

The graph of $f(x)$ would have



A. 4 points of discontinuity

B. 2 points of discontinuity

C. 1 point of inflection

D. 2 points of inflection

E. 2 stationary points

Question 13

For a function *f*, it is known that $f''(2) = 0$.
From this, we know that on the graph of $y = f(x)$, at the point where $x = 2$, we could **not** have

A. a point of inflection

B. a stationary point of inflection

C. a minimum turning point and a point of inflection

D. a point of discontinuity

E. a stationary point

Question 14

An antiderivative of $5x\sqrt{1-2x^2}$ would be

A. $\dfrac{-8}{15\sqrt{1-2x^2}}$

B. $\dfrac{-8(1-2x^2)^{\frac{3}{2}}}{15}$

C. $\dfrac{-5(1-2x^2)^{\frac{3}{2}}}{6}$

D. $\dfrac{5x^2}{3}(1-2x^2)^{\frac{3}{2}}$

E. $\dfrac{5x^2}{4\sqrt{1-2x^2}}$

Question 15

$\int 2x\sqrt{1-\frac{x}{2}}\,dx$ is equal to

A. $\frac{2x^2}{3}(1-\frac{x}{2})^{\frac{3}{2}}+c$

B. $-\frac{2x^2}{3}(1-\frac{x}{2})^{\frac{3}{2}}+c$

C. $\frac{8}{3}(1-\frac{x}{2})^{\frac{3}{2}}-\frac{8}{5}(1-\frac{x}{2})^{\frac{5}{2}}+c$

D. $\frac{8}{3}(1-\frac{x}{2})^{\frac{3}{2}}+\frac{8}{5}(1-\frac{x}{2})^{\frac{5}{2}}+c$

E. $\frac{-16}{3}(1-\frac{x}{2})^{\frac{3}{2}}+\frac{16}{5}(1-\frac{x}{2})^{\frac{5}{2}}+c$

Question 16

$\int_0^2 \frac{3}{\sqrt{4-x^2}}\,dx$ is equal to

A. $\frac{-3\pi}{2}$

B. $\frac{3\pi}{2}$

C. 6

D. 12

E. $\text{Sin}^{-1}6$

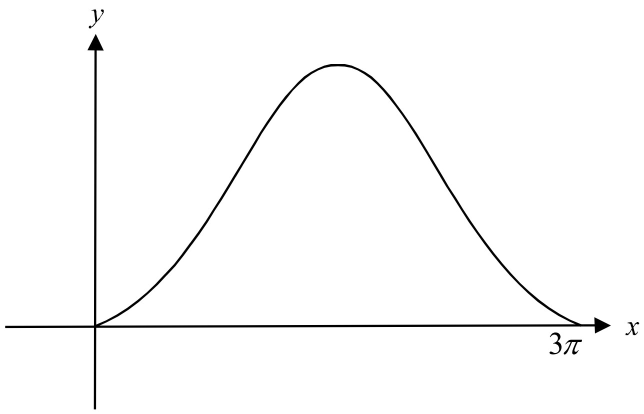
Question 17

The midpoint rule with 2 strips is used to approximate $\int_3^5 (\frac{1}{x^3-2x}+3)\,dx$

That approximation correct to 1 decimal place would be

A. 4.8
B. 5.0
C. 5.7
D. 6.0
E. 6.1

Question 18



The graph of the function $f:[0,3\pi]\rightarrow R$ where $f(x)=\sin^2(\frac{x}{3})$ is shown above.

The area between the graph of *f* and the *x*-axis is given by

A. $\frac{1}{2}\left[x-\frac{3}{2}\sin(\frac{2x}{3})\right]_0^{3\pi}$

B. $\left[x+\frac{3}{2}\sin(\frac{2x}{3})\right]_0^{3\pi}$

C. $-\frac{1}{3}\left[\cos^3(\frac{x}{3})\right]_0^{3\pi}$

D. $3\left[\cos^3(\frac{x}{3})\right]_0^{3\pi}$

E. $\left[x+\frac{3}{2}\sin(\frac{x}{3})\right]_0^{3\pi}$

Question 19

The function $g:\left[\frac{1}{2},5\right]\rightarrow R$ where $g(x)=\log_e(2x)$ is rotated about the *x*-axis. The volume of the solid of revolution formed is given by

A. $\int_{0.5}^{5}\log_e(2x)dx$

B. $\int_{0.5}^{2.3}\log_e(2x)dx$

C. $\int_{0.5}^{5}(\log_e(2x))^2dx$

D. $\pi\int_{0.5}^{5}(\log_e(2x))^2dx$

E. $-\pi\int_{0.5}^{5}(\log_e(2x))^2dx$

Question 20

If $y = \log_e(2t)$ and $x = t^2(t+1)$ then $\frac{\text{dy}}{\text{dx}}$, in terms of *t*, is given by

A. $\frac{1}{2t^2}$

B. $t^2(3t+2)$

C. $\frac{1}{t^2(3t+2)}$

D. $\log_e(3t^2+2t)$

E. $\frac{t(t+1)-(3t^2+2t)\log_e(2t)}{t^2(t+1)}$

Question 21

The solution to the differential equation $\frac{dy}{dx} = \frac{e^{2x}}{2-e^{2x}}$ where $y = 0$ when $x = 0$ is

A. $y = \frac{e^{2x}}{2} - \frac{5}{4}$

B. $y = \frac{1}{2}\log_e(e^{2x}) - \frac{1}{2}$

C. $y = \frac{1}{2}\log_e(\frac{e^{2x}}{2}) - \frac{5}{4}$

D. $y = -\frac{1}{2}\log_e(2-e^{2x})$

E. $y = -\frac{1}{2}\log_e(2-e^{2x}) + \frac{1}{2}$

Question 22

The gradient of the normal to a curve at any point (x, y) is equal to twice the x coordinate at that point.

The rule for this curve *y = f (x),* satisfies the differential equation given by

A. $\frac{dy}{dx} = -\frac{1}{2x}$

B. $\frac{dy}{dx} = \frac{1}{2x}$

C. $\frac{dy}{dx} = \frac{x}{2}$

D. $\frac{dy}{dx} = 2x$

E. $\frac{d}{dx} = x + 2$

Question 23

The position, x, in metres, of a particle which is moving in a straight line relative to a fixed origin at time t seconds is given by $x = \text{Sin}^{-1}\left(\frac{1}{t^2 - 3t}\right),\ t \geq 0$

The particle is at rest for an instant when

A. $t = 0$

B. $t = \frac{1}{3}$

C. $t = \frac{2}{3}$

D. $t = 1$

E. $t = \frac{3}{2}$

Question 24

A particle moves in a straight line relative to a fixed origin such that its acceleration $a\,(\text{m/s}^2)$ is given by $a = 3x^4 - x^2$ where x represents the displacement of the particle at time t seconds, $t \geq 0$
Given that the particle started at rest from the origin, the velocity of the particle when $x = 1$ could be

A. $-\sqrt{\frac{8}{15}}$

B. $-\sqrt{\frac{20}{3}}$

C. $\frac{1}{15}$

D. $\frac{2}{5}$

E. $\frac{10}{3}$

Question 25

The inverse function of the function $f : [-5, 5] \to R$ where $f(x) = \text{Cos}^{-1}\left(\frac{x}{5}\right)$

is given by

A. $f^{-1} : [0, \pi] \to R$ where $f^{-1}(x) = 5\text{Cos}\left(\frac{x}{5}\right)$

B. $f^{-1} : [0, \pi] \to R$ where $f^{-1}(x) = 5\text{Cos}\,x$

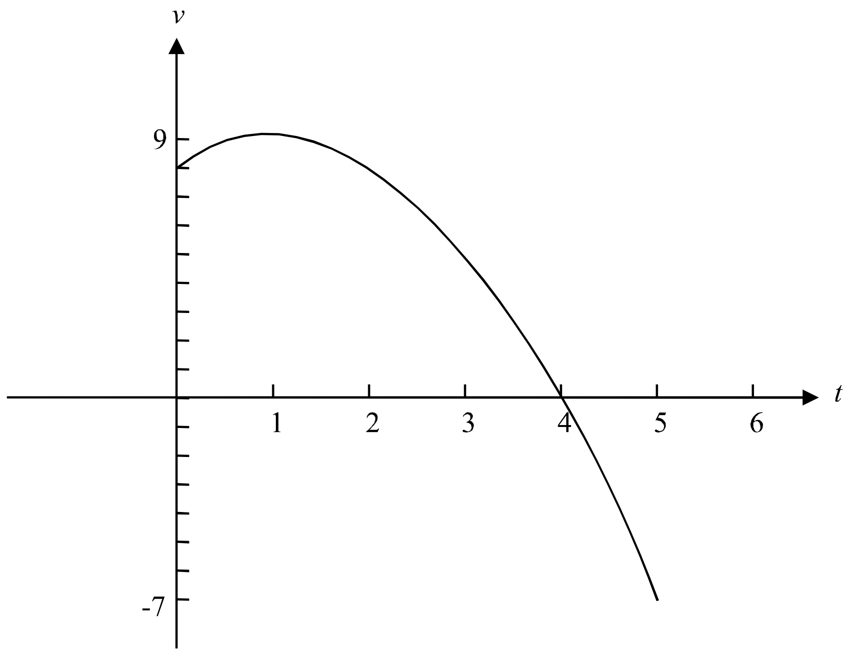
C. $f^{-1} : [-\pi, \pi] \to R$ where $f^{-1}(x) = 5\text{Cos}\left(\frac{x}{5}\right)$

D. $f^{-1} : [-5, 5] \to R$ where $f^{-1}(x) = \text{Cos}\left(\frac{x}{5}\right)$

E. $f^{-1} : [-5, 5] \to R$ where $f^{-1}(x) = 5\text{Cos}\,x$

Question 26

A particle moves with velocity *v* metres per second where $v(t) = -t^2 + 2t + 8,\ t \in [0,5]$
The velocity time graph for this particle is shown below.



The distance covered by this particle for $t \in [0,\ 5]$ is given by

A. $-\int_0^5 (-t^2 + 2t + 8)dt$

B. $\int_0^5 (-t^2 + 2t + 8)dt$

C. $\int_{-7}^9 (-t^2 + 2t + 8)dt$

D. $\int_0^4 (-t^2 + 2t + 8)dt - \int_4^5 (-t^2 + 2t + 8)dt$

E. $\int_4^5 (-t^2 + 2t + 8)dt - \int_0^4 (-t^2 + 2t + 8)dt$

Question 27

A force of $3\underset{\sim}{i} - 7\underset{\sim}{j}$ and a second force of $-\underset{\sim}{i} + 5\underset{\sim}{j}$ act on a body of mass 2 kg.

The acceleration of the body in m/s^2 is

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $4\sqrt{2}$

D. $\underset{\sim}{i} - \underset{\sim}{j}$

E. $\underset{\sim}{i} + \underset{\sim}{j}$

Question 28

A boy of mass 15 kg slides down a smooth straight slide which is inclined at an angle of $30°$ to the horizontal. The acceleration due to gravity is g m/s^2 where $g = 9.8$.
The normal force, in newtons, of the slide acting on the boy is given by

A. 0

B. $\dfrac{g}{2}$

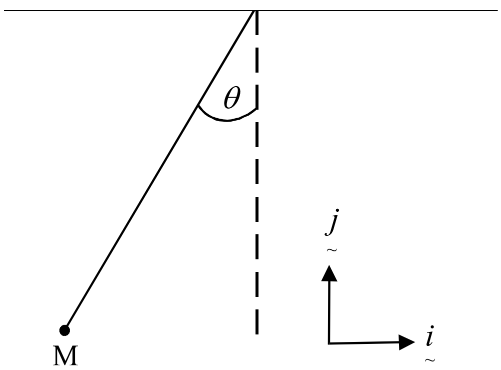
C. $\dfrac{\sqrt{3}}{2}$

D. $15g$

E. $\dfrac{15\sqrt{3}}{2}g$

Question 29

A mass M of 5 kg hangs at the end of a taut string which is attached to a ceiling. A force of 20 newtons acts horizontally on M causing the string to move away to be on an angle of θ from the vertical. If the tension in the string is T and the acceleration due to gravity is g m/s^2 where $g = 9.8$, then the resultant force acting on M is given by



A. $\underset{\sim}{R} = -20\underset{\sim}{i} + (T-5)\underset{\sim}{j}$

B. $\underset{\sim}{R} = -20\underset{\sim}{i} + (T-5g)\underset{\sim}{j}$

C. $\underset{\sim}{R} = (T\cos\theta - 20)\underset{\sim}{i} + (T\sin\theta - 5)\underset{\sim}{j}$

D. $\underset{\sim}{R} = (T\cos\theta - 20)\underset{\sim}{i} + (T\sin\theta - 5g)\underset{\sim}{j}$

E. $\underset{\sim}{R} = (T\sin\theta - 20)\underset{\sim}{i} + (T\cos\theta - 5g)\underset{\sim}{j}$

Question 30

A mass of 5 kg sits at rest on a rough plane which is inclined at an angle of $45°$ to the horizontal and has a coefficient of friction of $\sqrt{2}$. The acceleration due to gravity is g m/s^2 where $g = 9.8$.
The mass is not on the point of slipping down the plane.
How many more newton would the frictional force need to be in order for the mass to be on the point of slipping?

A. $5g$

B. $5\sqrt{2}g$

C. $\dfrac{5}{g} - \sqrt{2}g$

D. $5g - \dfrac{5g}{\sqrt{2}}$

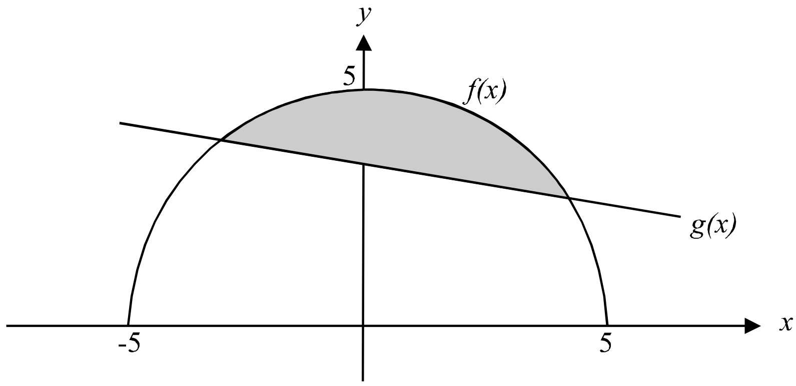
E. $\dfrac{5\sqrt{2}}{g}$

PART II

Part II consists of 6 questions. Answer **all** questions in Part II.

Question 1

The graph below shows the graph of $f(x) = \sqrt{25 - x^2}$ and $g(x) = \dfrac{25 - x}{7}$



Find the shaded area using appropriate technology and express your answer correct to 3 decimal places.

3 marks

Question 2

If ai is a solution to the equation $z^4 - 6z^3 + 14z^2 - 24z + 40 = 0$, find the real values of a and hence find all the solutions to the equation.

4 marks

Question 3

Use calculus to evaluate $\int_0^{\frac{\pi}{4}} \sin(2x)\cos^2 x\, dx$

3 marks

Question 4

XYZ is a right angled triangle where XZ is the hypotenuse and M is the midpoint of XZ.
Let $\overrightarrow{XM} = \overrightarrow{MZ} = \underset{\sim}{a}$ and $\overrightarrow{YM} = \underset{\sim}{b}$

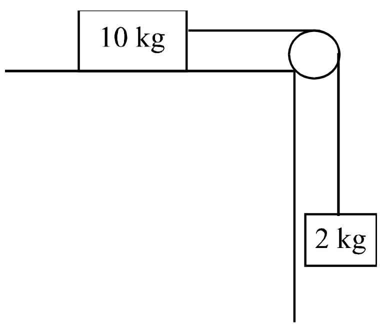
a. Express $\overrightarrow{XY}$ and $\overrightarrow{YZ}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$

b. Hence prove that M is equidistant from the 3 vertices of triangle XYZ.

4 marks

Question 5
A 10 kg weight sits on a table and is connected by a light string passing over a smooth pulley to a 2 kg weight. The coefficient of friction of the table is μ, the normal force of the table on the 10 kg weight is N, the tension in the string is T and the acceleration due to gravity has magnitude g m/s^2 where g = 9.8

a. On the following diagram show the forces acting on each of the weights.



b. Given that the weights are on the point of moving, find the value of μ

3 marks

Question 6

Use calculus to find the value of a given that $\int_3^a \frac{x^2 - x}{x^2 - x - 2} dx = 2 + \frac{2}{3}\log_e 2$

3 marks

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SPECIALIST MATHEMATICS

TRIAL EXAM 1

2000

PART I

MULTIPLE-CHOICE ANSWER SHEET

STUDENT NAME ....

INSTRUCTIONS
Fill in the letter that corresponds to your choice. Example: : A ■ C D E
The answer selected is B. Only one answer should be selected.

1. A B C D E	11. A B C D E	21. A B C D E
2. A B C D E	12. A B C D E	22. A B C D E
3. A B C D E	13. A B C D E	23. A B C D E
4. A B C D E	14. A B C D E	24. A B C D E
5. A B C D E	15. A B C D E	25. A B C D E
6. A B C D E	16. A B C D E	26. A B C D E
7. A B C D E	17. A B C D E	27. A B C D E
8. A B C D E	18. A B C D E	28. A B C D E
9. A B C D E	19. A B C D E	29. A B C D E
10. A B C D E	20. A B C D E	30. A B C D E