TSSM
Creating VCE Success

2024 Trial Examination

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STUDENT NUMBER | | | | | | | | | Letter | |

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	20	20	20
2	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator and if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 20 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 – Multiple-choice questions

Instructions for Section 1
A correct answer scores 1, an incorrect answer scores 0. Marks are not deducted for incorrect answers. If more than 1 answer is completed for any question, no mark will be given.

Question 1

The period and range of $y = 3 - 2\tan\left(3x - \frac{\pi}{2}\right)$ are:

A. $\frac{\pi}{3}, [1, 5]$

B. $\frac{2\pi}{3}, [1, 5]$

C. $\frac{2\pi}{3}, [-1, 5]$

D. $\frac{\pi}{3}, R$

Question 2

Consider the system of simultaneous linear equations below containing the parameter p.

$$px + 2y = 7$$
$$3x + (p-1)y = 2p$$

The value(s) of p for which the system of equations has a unique solution are:

A. $p \in \{-2\}$

B. $p \in \{-2, 3\}$

C. $p \in R\backslash\{-2, 3\}$

D. $p \in [-2, 3]$

Question 3

Suppose that $\int_a^c f(x)\,dx = 10$ and $\int_b^c f(x)\,dx = 2$, where $a < b < c$. $\int_b^a f(x)\,dx$ is equal to:

A. 8

B. 12

C. -8

D. -12

SECTION 1- continued

Question 4

The direct distance between two successive stationary points of the graph $y = a\sin(bx)$ is:

A. $2\sqrt{a^2 + \left(\frac{\pi}{b}\right)^2}$

B. $\sqrt{a^2 + 4\left(\frac{\pi}{b}\right)^2}$

C. $\sqrt{4a^2 + \left(\frac{\pi}{b}\right)^2}$

D. $\sqrt{2a^2 + 2\left(\frac{\pi}{b}\right)^2}$

Question 5

Consider the functions $f(x) = \sqrt{x+5}$ and $g(x) = \frac{1}{x-2}$. The maximal domain of $f + g$ is:

A. $[-5, \infty)$

B. $R\backslash\{2\}$

C. $(-5, 2)$

D. $[-5, 2) \cup (2, \infty)$

Question 6

The maximum number of points of inflection of the graph of $y = x^4 + ax^3 + 3x^2 + bx + c$, where $a, b, c \in R$ is:

A. 0

B. 1

C. 2

D. 3

SECTION 1 - continued
TURN OVER

Question 7

A bag contains a identical gold coins and b identical silver coins. Two coins are selected at random, one at time, without replacement. The probability they are identical is:

A. $\frac{ab}{a+b}$

B. $\frac{a(a-1)+b(b-1)}{(a+b-1)^2}$

C. $\frac{a+b}{(a+b)(a+b-1)}$

D. $1-\frac{2ab}{(a+b)(a+b-1)}$

Question 8

Which one of the following is an even function?

A. $f(x) = 4x^3$

B. $f(x) = \sin\left(x+\frac{\pi}{2}\right)$

C. $f(x) = 2(x-1)^2$

D. $f(x) = \cos\left(x-\frac{3\pi}{2}\right)$

Question 9

The tangent to $y = x^2 + 5x + 1$ passes through the origin for what value(s) of x?

A. $x = 1$ and $x = -1$

B. $x = 1$ only

C. $x = -1$ only

D. $x = -1, 0$ and 1

Question 10

Suppose that the survival rate in a given population of a particular disease is 70%. A sample of 80 people are randomly selected from the population. The probability that at least 60 survive given that at least 50 survive is closest to:

A. 0.2102

B. 0.1978

C. 0.3687

D. 0.3917

SECTION 1- continued

Question 11

Consider the function:

$$f(x) = \begin{cases} kx^2 + 3, & 0 \le x \le 5 \\ 0, & \text{otherwise} \end{cases}$$

In order for $f(x)$ to be a probability density function k must be:

A. -6

B. $-\frac{1}{3}$

C. $-\frac{1}{15}$

D. $-\frac{42}{125}$

Question 12

The probability density function for a discrete random variable is shown below.

X	0	1	2	3
$\Pr(X = x)$	a	a^2	$2a$	$0.4 - a^2$

The expectation of X is

A. 0.2

B. 1.92

C. 2.12

D. 2.08

Question 13

It is known that 65% of the population carries a particular genetic marker. A sample of size n of the population is taken. What is the minimum sample size required such that standard deviation of the sample proportion is below 0.1?

A. 23

B. 24

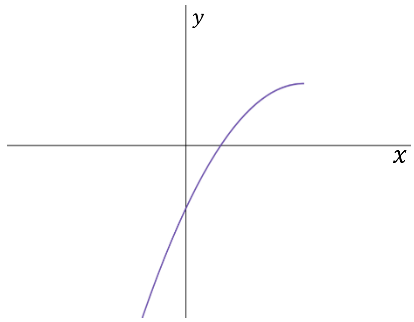
C. 25

D. 26

SECTION 1 - continued
TURN OVER

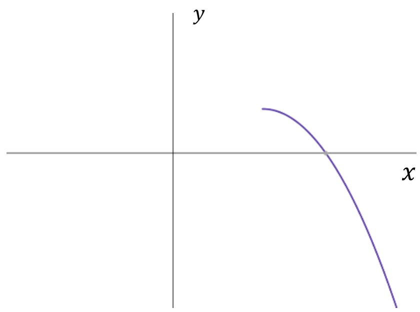
Question 14

Consider the graph below of $y = f(x)$

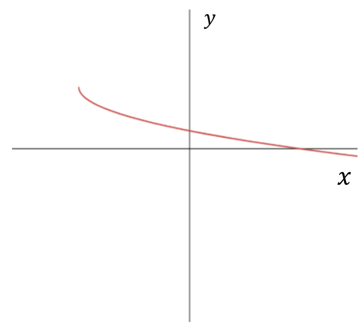


Which of the following could be the graph of $y = f^{-1}(x)$?

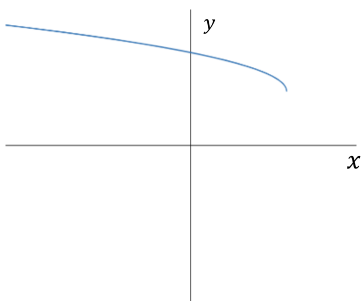
A.



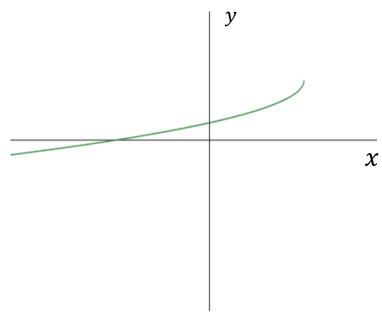
B.



C.



D.



SECTION 1- continued

Question 15

The average rate of change of $y = 2^{x+3} - 5$ between $x = 0$ and $x = 4$ is:

A. 30

B. 120

C. $\frac{1}{30}$

D. $\frac{1}{120}$

Question 16

The average value of $y = x + \sqrt{x}$ between $x = 1$ and $x = 5$ is:

A. $\frac{3+\sqrt{5}}{4}$

B. $3 + \sqrt{5}$

C. $5\sqrt{5} + 17$

D. $\frac{5\sqrt{5}+17}{6}$

Question 17

The function f is given by:

$$f(x) = \begin{cases} x^3 - 27, & x \le 2 \\ bx^2 - 15x - 1, & x > 2 \end{cases}$$

The value of b for which f is continuous over the entire domain:

A. 1

B. 2

C. 3

D. 4

SECTION 1 - continued
TURN OVER

Question 18

Consider the function $f: (0, n] \rightarrow R, f(x) = \frac{m}{\sqrt{nx}}$ where $n \in R^+$ and $m \in R^+$. The range of f is:

A. $\left[\frac{m}{n}, \infty\right)$

B. $\left(0, \frac{m}{n}\right]$

C. $\left[\frac{m}{\sqrt{n}}, \infty\right)$

D. $(0, n]$

Question 19

Consider the following algorithm for Newton's method using a loop with 5 iterations

Inputs:
f(x), a function of x
df(x), the derivative of f(x)
x0, the initial estimate

Define newmet(f(x), df(x), x0)
For i **from** 1 **to** 5
If df(x0) = 0 **Then**
Return "Error. Division by zero"
Else
x0←x0 – f(x0) / df(x0)
End For
Return x0

The result for the function newmet($x^2 - 8x + 2, 2x - 8, 5$) is closest to:

A. 8.8683

B. 7.8363

C. 7.7428

D. 7.7417

SECTION 1- continued

Question 20

Consider the function $f(x) = x^3 - 2x^2 - x + 2$. An approximation can be found for the area bounded between $f(x)$ and the x axis using trapeziums of width 0.5. This approximation is closest to what percentage of the actual area?

A. 90%

B. 91%

C. 92%

D. 93%

END OF SECTION 1
TURN OVER

SECTION 2- Extended response questions

Instructions for Section 2
A decimal approximation will not be accepted if the question specifically asks for an **exact** answer. In questions worth more than one mark, appropriate working **must** be shown. Marks are given as specified for each question. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (13 marks)

Consider the function f with a rule given by: $f(x) = x^4 e^{-x}$

a. Find $f'(x)$

1 mark

b. Find the coordinates of, and classify, the stationary point(s) of $f(x)$.

2 marks

c. Find the coordinates of the point(s) of inflection of $f(x)$.

2 marks

SECTION 2- continued

d. Find $\{x: f'(x) < 0\}$

1 mark

e. State the values of c for which the graph of $y = f(x) + c$ has exactly one x intercept.

1 mark

f. Find the equation of the tangent to $f(x)$ at $x = 3$

1 mark

g. Determine the area, to three decimal places, bounded between $f(x)$ and the tangent found in **part f**.

2 marks

h. Let $P(x, y)$ be a point on the tangent found in **part f**, where $x \in [0, 3]$. Find the coordinates of P, correct to two decimal places, such that the vertical distance between P and the $f(x)$ is a maximum.

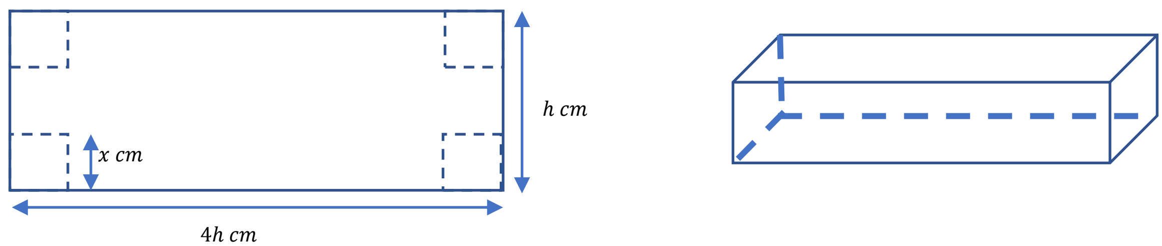
3 marks

SECTION 2 – continued

TURN OVER

Question 2 (10 marks)

A rectangular sheet of cardboard has a width of h cm and a length four times its width. Squares of side length x, where $x > 0$ are cut from each corner so that the four sides can be folded up to create an open box as seen in the diagrams below.



a. Find a rule for V, the volume of the box, in terms of x and h.

2 marks

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b. State the domain of x in terms of h.

2 marks

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c. Find $\frac{dV}{dx}$ in terms of h.

1 mark

SECTION 2 - continued

d. In terms of h, find the maximum volume of the box, and dimensions of the box for which this maximum volume occurs.

3 marks

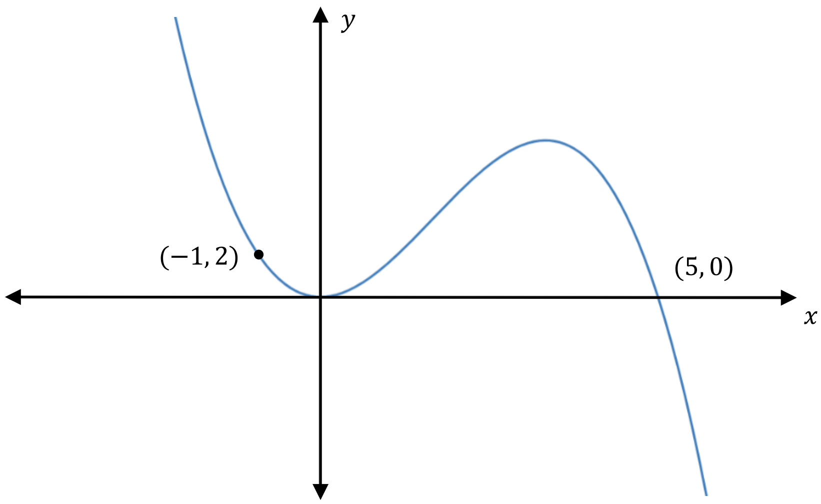
e. Graph $y = V(x)$ on the axes below, labelling all intercepts and stationary points.

2 marks

SECTION 2 - continued
TURN OVER

Question 3 (16 marks)

Part of the graph of $y = g(x)$ is given below where $g(x) = ax^2(b - x),\ a, b \in R$



a. Find the values of a and b.

2 marks

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b. Find $g'(x)$

1 mark

---

c. State the value(s) of x for which $g(x)$ is strictly decreasing.

2 marks

SECTION 2 - continued

d. Using trapeziums of width one unit, find an approximation for the area bound between $g(x)$ and the x axis.

2 marks

e. Find the exact area bound between $g(x)$ and the x axis.

2 marks

Let the point $P(c, g(c))$ be the local maximum of $g(x)$.
A new function $h(x)$ is created such that:

$$h(x) = \begin{cases} g(x), & x \leq c \\ mx(n - x), & x > c \end{cases}$$

where $m, n \in R$.

It is known that $h(x)$ is continuous at $x = c$ and passes through the point $\left(4, \frac{50}{9}\right)$

f. Find the exact values of m and n.

2 marks

SECTION 2 - continued
TURN OVER

g. Find the area bound between $h(x)$ and the x axis, correct to one decimal place.

2 marks

h. Find the co-ordinates of $h(x)$ where $(c < x < n)$ such that the gradient of $h(x)$ is equal to the average gradient of $h(x)$ between $x = c$ and $x = n$.

3 marks

SECTION 2 - continued

Question 4 (21 marks)

For a particular brand of trading cards, it is known that the value of any particular card is normally distributed with a mean of μ and a standard deviation of σ.
It is known that 1.5% of cards have a value less than \$2.90 and 9.1% of cards have a value greater than \$5.00.
Let the value of a trading card be the normal random variable X.

a. Find the mean and standard deviation of X correct to two decimal places.

3 marks

b. Find the probability that a randomly selected card has a value greater than \$3.50, correct to four decimal places.

1 mark

c. Find the probability that a randomly selected card has a value greater than \$3.50, given it is known to have a value less than $E(X)$. Give your answer correct to four decimal places.

2 marks

SECTION 2 - continued
TURN OVER

d. If it is known that $\Pr(n_1 < X < n_2) = 0.4$, find the values of n_1 and n_2 correct to two decimal places. (Assume n_1 and n_2 are symmetrical about the mean.)

2 marks

A particular collector of these cards refuses to trade a card if he knows the value is greater than \$5.50. He is also known to place any cards he owns worth more than \$6.00 in sealed bags.

e. Of the cards this collector refuses to trade, what percentage, to the nearest whole percent, does he keep in sealed bags?

2 marks

Currently the collector has 2800 trading cards.

f. What is the expected number of cards worth more than \$5.50? Give your answer correct to two decimal places.

1 mark

g. What is the minimum number of trading cards the collector will need in order to ensure the probability of having at least 100 cards with a value of greater than $5.50 is greater than 99%?

2 marks

The collector also likes to collect coins. The coins he collects can either be gold or silver. Of his collection of 500 coins, 220 of these are gold.

h. Construct a 95% confidence interval, correct to four decimal places, for p the proportion of gold coins being circulated, and interpret this interval for this context.

2 marks

i. If the confidence interval were changed to be a 99% confidence interval, by what factor, correct to two decimal places, would the width of the interval found in **part h.** be increased or decreased?

2 marks

SECTION 2 - continued
TURN OVER

A particular coin is known to have a value that is distributed with the following probability distribution.

$$P(v) = \begin{cases} kv^3 e^{-v}, & v \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Where v is the value of the coin in dollars.

j. Find the value of k such that $P(v)$ is a probability density function.

2 marks

k. Find the expected value of the coin.

2 marks

END OF QUESTION AND ANSWER BOOK