MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

2024 Trial Examination

SOLUTIONS

Question 1 (4 marks)

a. $\frac{dy}{dx} = 2xe^{4x} + 4x^2e^{4x}$

1 mark

b. $f'(x) = \frac{(x^2+e^x)(\cos(x))-(2x+e^x)\sin(x)}{(x^2+e^x)^2}$

2 marks

$$f'(0) = \frac{(0+1)\cos(0) - (0+1)\sin(0)}{(0+1)^2}$$
$$= \frac{1-0}{1} = 1$$

1 mark

Question 2 (4 marks)

a. $f(x) = \frac{3(x+1)-1}{x+1}$

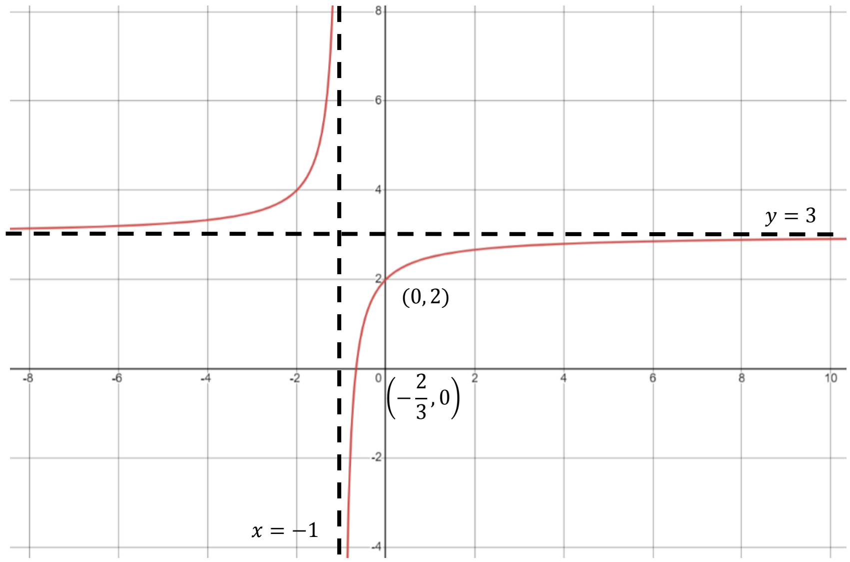
$$= 3 - \frac{1}{x+1} = 3 + \frac{-1}{x+1}$$

1 mark

b. Reflection in the y-axis (or x-axis) followed by a translation of 3 units up and 1 unit to the left.

2 marks

c.



2 marks

Question 3 (4 marks)

a. $\tan(4x) = 1$

$\theta_R = \dfrac{\pi}{4}$

$4x = \dfrac{\pi}{4} + n\pi,\ \ n \in Z$

$x = \dfrac{\pi}{16} + \dfrac{1}{4}n\pi,\ \ n \in Z$

$= \dfrac{\pi}{16}(1 + 4n), \qquad n \in Z$

2 marks

b. $\frac{\pi}{16} + \frac{5\pi}{16} + \frac{9\pi}{16} + \frac{13\pi}{16}$

$= \dfrac{28\pi}{16} = \dfrac{7\pi}{4}$

2 marks

Question 4 (3 marks)

No solutions means that the linear equations have the same gradient but different y-intercepts.

Same gradient: $\frac{2}{m} = \frac{m+1}{3}$

$$m^2 + m - 6 = 0$$
$$(m+3)(m-2) = 0$$
$$m = -3, 2$$

2 marks

For $m = -3$. $\frac{2}{-3} = -\frac{2}{3}$ $\quad \frac{2}{3n} \neq -\frac{2}{3}$, $\quad n \neq -1$

For $m = 2, \frac{2}{2} = 1, \frac{2}{3n} \neq 1,\ n \neq \frac{2}{3}$

So $m = -3$ and $n \in R\backslash\{-1\}$ or $m = 2$ and $n \in R\backslash\left\{\frac{2}{3}\right\}$

1 mark

Question 5 (4 marks)

a. $\frac{\mathrm{d}}{\mathrm{dx}}(x^2 \log_e(x)) = (2x)(\log_e(x)) + (x^2)\left(\frac{1}{x}\right)$

$= 2xlog_e(x) + x$

1 mark

b. $\int (2xlog_e(x) + x)dx = x^2 \log_e(x)$

$$\int xlog_e(x)\, dx = \frac{1}{2}x^2 \log_e(x) - \frac{1}{4}x^2$$

1 mark

So $\int_1^e xlog_e(x)\, dx = \left[\frac{1}{2}x^2 \log_e(x) - \frac{1}{4}x^2\right]_1^e$

$= \left(\frac{1}{2}e^2 \log_e(e) - \frac{1}{4}e^2\right) - \left(\frac{1}{2}\log_e(1) - \frac{1}{4}\right)$

$= \frac{1}{2}e^2 - \frac{1}{4}e^2 + \frac{1}{4}$

$= \frac{1}{4}(e^2 + 1)$

2 marks

Question 6 (4 marks)

a. $g \circ h(x) = \left(\sqrt{4-x}\right)^2 + 3 = 7 - x$

$dom\ g \circ h = dom\ h = (-\infty, 4]$

2 marks

b. Need to restrict $ran\ g^*$ from $[3, \infty)$ to $[3, 4]$

$x^2 + 3 \leq 4$

$x^2 - 1 \leq 0$

$x \in [-1, 1]$

2 marks

Question 7 (4 marks)

a. $\Pr(\$2) = \Pr(L \cap \$2) + \Pr(R \cap \$2)$

$= \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

2 marks

b. $\Pr(L|\$2) = \frac{\Pr(L\cap\$2)}{\Pr(\$2)}$

$= \frac{\frac{1}{8}}{\frac{3}{8}} = \frac{1}{3}$

2 marks

Question 8 (8 marks)

a. $5-4x^2=0,\ x=\pm\frac{\sqrt{5}}{2}$

$$AV=\frac{1}{\frac{\sqrt{5}}{2}--\frac{\sqrt{5}}{2}}\int_{-\frac{\sqrt{5}}{2}}^{\frac{\sqrt{5}}{2}}(5-4x^2)\,dx$$

$$=\frac{1}{\sqrt{5}}\left[5x-\frac{4}{3}x^3\right]_{-\frac{\sqrt{5}}{2}}^{\frac{\sqrt{5}}{2}}$$

$$=\frac{1}{\sqrt{5}}\left(\left(\frac{5\sqrt{5}}{2}-\frac{4}{3}\times\frac{5\sqrt{5}}{8}\right)-\left(-\frac{5\sqrt{5}}{2}-\frac{4}{3}\times-\frac{5\sqrt{5}}{8}\right)\right)$$

$$=\frac{1}{\sqrt{5}}\left(\frac{5\sqrt{5}}{2}-\frac{5\sqrt{5}}{6}+\frac{5\sqrt{5}}{2}-\frac{5\sqrt{5}}{6}\right)=\frac{1}{\sqrt{5}}\left(5\sqrt{5}-\frac{5\sqrt{5}}{3}\right)$$

$$=\frac{1}{\sqrt{5}}\left(\frac{10\sqrt{5}}{3}\right)=\frac{10}{3}$$

2 marks

b. $Area=\frac{\frac{\sqrt{5}}{2}--\frac{\sqrt{5}}{2}}{2\times 4}\left(y\left(-\frac{\sqrt{5}}{2}\right)+2y\left(-\frac{\sqrt{5}}{4}\right)+2y(0)+2y\left(\frac{\sqrt{5}}{4}\right)+y\left(\frac{\sqrt{5}}{2}\right)\right)$

$$=\frac{\sqrt{5}}{8}\left(0+\frac{15}{2}+10+\frac{15}{2}+0\right)=\frac{25\sqrt{5}}{8}$$

3 marks

c. $x_1=x_0-\frac{y(x_0)}{y'(x_0)}$

$y(1)=1,\ y'(x)=-8x, so\ y'(1)=-8$

$$x_1=1-\frac{1}{-8}=\frac{9}{8}$$

$$x_2=\frac{9}{8}-\frac{y\left(\frac{9}{8}\right)}{y'\left(\frac{9}{8}\right)}=\frac{9}{8}-\frac{5-4\left(\frac{9}{8}\right)^2=5-\frac{81}{16}=\frac{-1}{16}}{-9}=\frac{9}{8}-\frac{1}{9\times 16}=\frac{161}{144}$$

3 marks

Question 9 (4 marks)

a. $y''(x) = 2mx + 8 = 0$

$x = -\frac{4}{m}$

$y\left(-\frac{4}{m}\right) = n + \frac{128}{3m^2}$

So, POI=$\left(-\frac{4}{m}, n + \frac{128}{3m^2}\right)$

2 marks

b. $-\frac{4}{m} > 0$ and $n + \frac{128}{3m^2} > 0$

$m < 0, n > -\frac{128}{3m^2}$

2 marks