2024
VCE
Mathematical Methods
Year 12
Trial Examination 1

Kilbaha Education (Est. 1978) (ABN 47 065 111 373)
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Victorian Certificate of Education
2024

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 16 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Evaluate $f'(2)$, where $f(x)=\log_e\left(\sqrt{x^3+1}\right)$.

2 marks

b. If $\frac{d}{dx}\left(\frac{x}{\sqrt{4x+9}}\right)=\frac{px+q}{(4x+9)^n}$, find the values of p, q and n.

2 marks

Question 2 (3 marks)

a. Solve for x if $3^{x^2+6x}=\frac{1}{243}$

1 mark

b. Solve for x if $\log_2\left(x^2+4\sqrt{2}\right)+\log_2\left(x^2-4\sqrt{2}\right)=5$

2 marks

Question 3 (3 marks)

Find the values of a and b for which the simultaneous linear equations,

$$\begin{aligned} 2ax - 2by &= 5 \\ (1-3b)x + 12y &= 2-4b \end{aligned}$$

have an infinite number of solutions.

Question 4 (3 marks)

For random samples of six year 12 students, $\hat{P}$ represents the proportion of students who have brown eyes. If $\Pr\left(\hat{P}=\frac{1}{3}\right)=\Pr\left(\hat{P}=\frac{1}{2}\right)$ find $\Pr\left(\hat{P}=1\right)$ giving your answer in the form $\left(\frac{a}{b}\right)^n$ where $a,b,n \in N$.

Question 5 (3 marks)

Consider the function defined by $f(x)=\begin{cases}\sqrt{5-x^2}, & x\le 2\\ ax^2+bx, & x>2\end{cases}$ where a and b are real numbers.

If the function has a smooth join at $x=2$, find the values of a and b.

Question 6 (3 marks)

A certain curve has its gradient given by $5\sin\left(\frac{x}{2}\right)+me^{-2x}+4$, if the curve has a turning point at the origin, find the value of m and the equation of the curve.

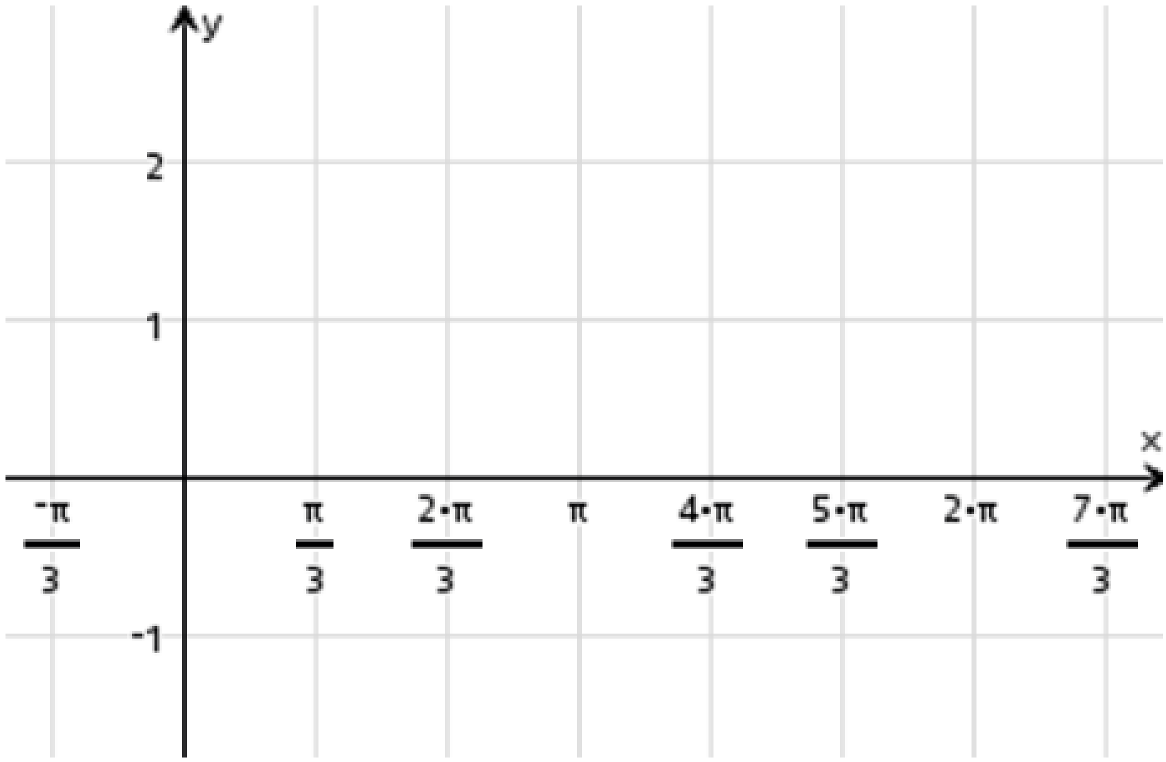
Question 7 (6 marks)

a. Find the general solution of $2\sin^2(2x)+\cos(2x)-1=0$ for $x \in R$.

3 marks

b. Consider the functions $f:[0,2\pi]\to R,\ f(x)=2\sin^2(2x)$ and $g:[0,2\pi]\to R,\ g(x)=1-\cos(2x)$, on the axes below, sketch the graphs of the functions $y=f(x)$ and $y=g(x)$ and determine $2\sin^2(2x)<1-\cos(2x)$ for $x\in[0,2\pi]$.

3 marks



Question 8 (4 marks)

Given the two functions $f(x) = \log_e(x-2)$ and $h(x) = 6 + 3x - x^2$ defined on their maximal domains.

a. Explain why $f \circ h(x)$ does not exist.

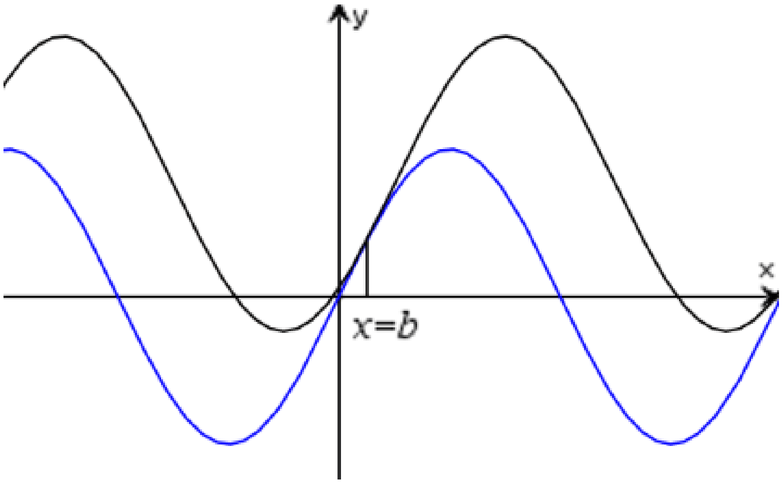
1 mark

b. Consider $g : D \to R$, $g(x) = 6 + 3x - x^2$, find the largest subset D of R, such that $f \circ g(x)$ exists. Find the domain and rule for $f \circ g(x)$.

3 marks

Question 9 (4 marks)

The diagram shows the two curves $y=\sin(x)$ and $y=\sin(x-\alpha)+c$, where $0<\alpha<\frac{\pi}{2}$ and $c>0$.



The two curves have a common tangent at $x=b$ where, $0<b<\alpha<\frac{\pi}{2}$, show that $\sin(b)=\sin(\alpha-b)$ and express c in terms of α.

Question 10 (7 marks)

a. The random variable X has a probability density function f given by

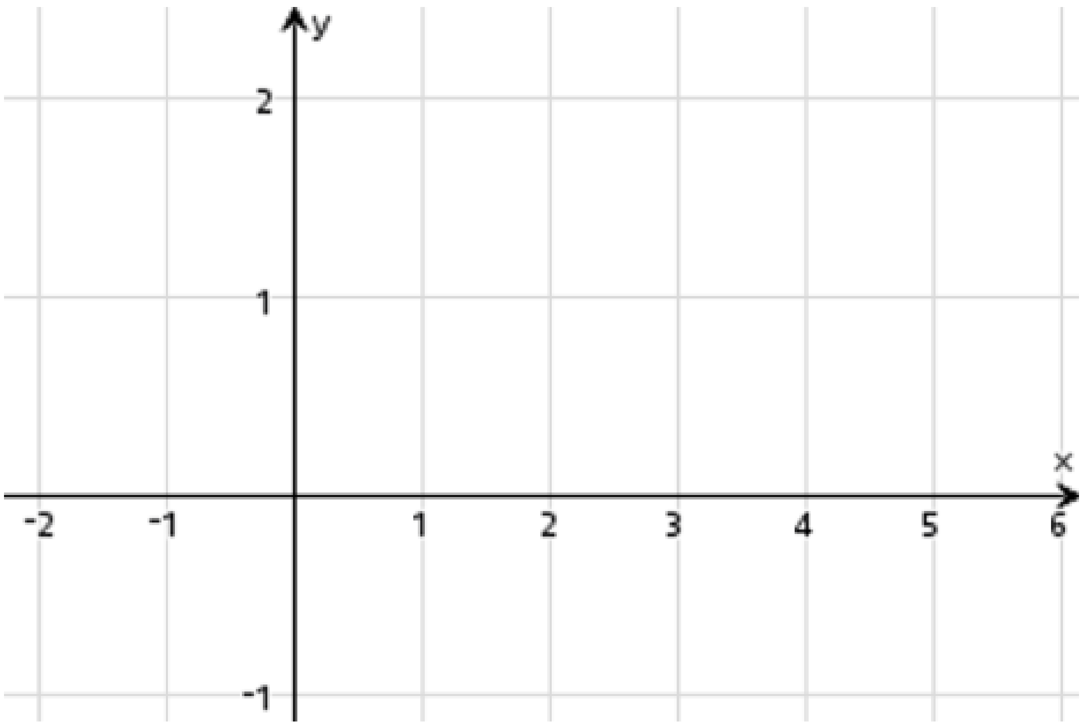
$$f(x) = \begin{cases} \dfrac{a}{(2x+1)^2} & 1 \le x \le 4 \\ 0 & \text{elsewhere} \end{cases}$$ where a is a positive real number.

i. Show that $a = 9$.

2 marks

ii. Sketch the graph of f on the axes below, stating the coordinates of the endpoints.

1 mark



b. Another random variable Y has a probability density function g given by

$$g(y)=\begin{cases}\dfrac{b}{2y+1} & 1\le y\le 4\\ 0 & \text{elsewhere}\end{cases}$$ where b is a positive real number.

Determine $E(Y)$, giving your answer in the form $\dfrac{p}{\log_e(p)}+q$ where $p\in Z^{+}$ and $q\in R$.

4 marks

End of question and answer book for the
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MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=na(ax+b)^{n-1}$		$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	$Area\approx\frac{x_n-x_0}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+...+2f(x_{n-2})+2f(x_{n-1})+f(x_n)\right]$		

Probability

$\Pr(A)=1-\Pr(A')$		$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$	
$\Pr(A\mid B)=\dfrac{\Pr(A\cap B)}{\Pr(B)}$			
mean	$\mu=E(X)$	variance	$\operatorname{var}(X)=\sigma^2=E\left((X-\mu)^2\right)=E(X^2)-\mu^2$
binomial coefficient	$\binom{n}{x}=\dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2\,p(x)$
binomial	$\Pr(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$	$\mu=np$	$\sigma^2=np(1-p)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)\,dx$	$\mu=\int_{-\infty}^{\infty}x\,f(x)\,dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2\,f(x)\,dx$

Sample proportions

$\hat{P}=\dfrac{X}{n}$		mean	$E(\hat{P})=p$
standard deviation	$\operatorname{sd}(\hat{P})=\sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p}-z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\ ,\ \hat{p}+z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET