Question 1

a. $f(x)=\log_e\left(\sqrt{\left(x^3+1\right)}\right)=\log_e\left(\left(x^3+1\right)^{\frac{1}{2}}\right)=\frac{1}{2}\log_e\left(x^3+1\right)$

$f'(x)=\frac{1}{2}\frac{\frac{d}{dx}\left(x^3+1\right)}{\left(x^3+1\right)}=\frac{3x^2}{2\left(x^3+1\right)}$ M1

$f'(2)=\frac{3\times 4}{2(8+1)}$

$f'(2)=\frac{2}{3}$ A1

b. $\frac{d}{dx}\left(\frac{x}{\sqrt{4x+9}}\right)=\frac{px+r}{(4x+9)^n}$ using the quotient rule

$u=x,\quad v=\sqrt{4x+9}=(4x+9)^{\frac{1}{2}}$

$\frac{du}{dx}=1\quad \frac{dv}{dx}=4\times\frac{1}{2}\times(4x+9)^{-\frac{1}{2}}=\frac{2}{\sqrt{4x+9}}$ M1

$\frac{d}{dx}\left(\frac{x}{\sqrt{4x+9}}\right)=\frac{\sqrt{4x+9}-\frac{2x}{\sqrt{4x+9}}}{4x+9}$

$=\frac{4x+9-2x}{\sqrt{4x+9}\,(4x+9)}=\frac{2x+9}{(4x+9)^{\frac{3}{2}}},\quad p=2,\quad q=9,\quad n=\frac{3}{2}$ A1

Question 2

a. $3^{x^2+6x}=\frac{1}{243}=3^{-5}$

$x^2+6x=-5$

$x^2+6x+5=0$

$(x+5)(x+1)=0$

$x=-5,\ -1$ A1

b. $\log_2\left(x^2+4\sqrt{2}\right)+\log_2\left(x^2-4\sqrt{2}\right)=5$

$\log_2\left(\left(x^2+4\sqrt{2}\right)\left(x^2-4\sqrt{2}\right)\right)=5$

$\left(x^2+4\sqrt{2}\right)\left(x^2-4\sqrt{2}\right)=2^5$ M1

$x^4-32=32$

$x^4=64,\quad x^2=8$

$x=\pm 2\sqrt{2}$ A1

Question 3

$$(1)\quad 2ax - 2by = 5 \quad \Rightarrow \quad 2by = 2ax - 5$$
$$(2)\quad (1-3b)x + 12y = 2 - 4b \quad \Rightarrow \quad 12y = (3b-1)x + 2 - 4b$$
$$(1)\quad y = \frac{ax}{b} - \frac{5}{2b} \qquad (2)\quad y = \frac{(3b-1)x}{12} + \frac{1-2b}{6}$$

for an infinite number of solutions the gradients and the y-intercepts are both equal, so

$$(3)\ \frac{a}{b} = \frac{3b-1}{12} \qquad (4)\ -\frac{5}{2b} = \frac{1-2b}{6}$$ M1

$$(3)\ a = \frac{b(3b-1)}{12} \qquad (4)\ -30 = 2b(1-2b) = 2b - 4b^2$$
$$4b^2 - 2b - 30 = 0$$
$$2b^2 - b - 15 - 0$$ M1
$$(2b+5)(b-3) = 0$$

$$b = 3 \qquad a = \frac{3(9-1)}{12} = 2, \quad \text{or} \quad b = -\frac{5}{2} \quad a = \frac{-\frac{5}{2}\left(-\frac{15}{2}-1\right)}{12} = -\frac{5}{2} \times \frac{-17}{2} \times \frac{1}{12} = \frac{85}{48}$$

an infinite number of solutions when $a = 2,\ b = 3$ or $a = \frac{85}{48},\ b = -\frac{5}{2}$ A1

Question 4

$$n = 6, \quad \hat{P} = \frac{X}{n} = \frac{X}{6} \qquad X \stackrel{d}{=} Bi(n = 6, p = ?)$$
$$\Pr\left(\hat{P} = \frac{1}{3}\right) = \Pr(X = 2) = \Pr\left(\hat{P} = \frac{1}{2}\right) = \Pr(X = 3)$$

$$\Pr(X = 2) = \binom{6}{2} p^2 (1-p)^4 = \Pr(X = 3) = \binom{6}{3} p^3 (1-p)^3$$ A1

$$\frac{6\times5}{2} p^2 (1-p)^4 - \frac{6\times5\times4}{3\times2} p^3 (1-p)^3 = 0$$ M1
$$5p^2(1-p)^3\left[3(1-p) - 4p\right] = 5p^2(1-p)^3(3-7p) = 0$$

since $0 < p < 1, \quad p = \frac{3}{7}, \quad X \stackrel{d}{=} Bi\left(n = 6, p = \frac{3}{7}\right)$

$$\Pr\left(\hat{P} = 1\right) = \Pr(X = 6) = \binom{6}{6}\left(\frac{3}{7}\right)^6\left(\frac{4}{7}\right)^0 = \left(\frac{3}{7}\right)^6, \quad a = 3, \quad b = 7, \quad n = 6$$ A1

Question 5

$$f(x)=\begin{cases}\sqrt{5-x^2}, & x\le 2\\ ax^2+bx, & x>2\end{cases}\quad,\quad \text{let } g(x)=ax^2+bx$$

Since the function is continuous at $x=2$,

$$f(2)=1=g(2)\quad\Rightarrow(1)\quad 4a+2b=1$$ A1

Since the join is smooth, the gradients are also equal.

$$f'(x)=\frac{d}{dx}\left(\sqrt{5-x^2}\right)=\frac{-x}{\sqrt{5-x^2}}$$

$$g'(x)=2ax+b$$ M1

$$f'(2)=-2=g'(2)\quad\Rightarrow(2)\quad 4a+b=-2$$

$$(1)-(2)\quad b=3,\quad 4a=-5,\quad a=-\frac{5}{4}$$ A1

Question 6

$$y=f'(x)=5\sin\left(\frac{x}{2}\right)+me^{-2x}+4$$

at the origin the gradient is zero, $f'(0)=0$

$$0=5\sin(0)+me^{0}+4=m+4=0,$$
$$m=-4$$ A1

$$y=f(x)=\int\left(5\sin\left(\frac{x}{2}\right)-4e^{-2x}+4\right)dx$$
$$y=f(x)=-10\cos\left(\frac{x}{2}\right)+2e^{-2x}+4x+c$$ A1

$$f(0)=0,\quad 0=-10\cos(0)+2e^{0}+c=-8+c,$$
$$c=8$$ A1

$$y=f(x)=-10\cos\left(\frac{x}{2}\right)+2e^{-2x}+4x+8$$

Question 7

a. $2\sin^2(2x)+\cos(2x)-1=0$

$2(1-\cos^2(2x))+\cos(2x)-1=0$

$2-2\cos^2(2x)+\cos(2x)-1=0$ M1

$-2\cos^2(2x)+\cos(2x)+1=0$

$2\cos^2(2x)-\cos(2x)-1=0$

$(2\cos(2x)+1)(\cos(2x)-1)=0$

$\cos(2x)=-\frac{1}{2} \quad , \quad \cos(2x)=1$ A1

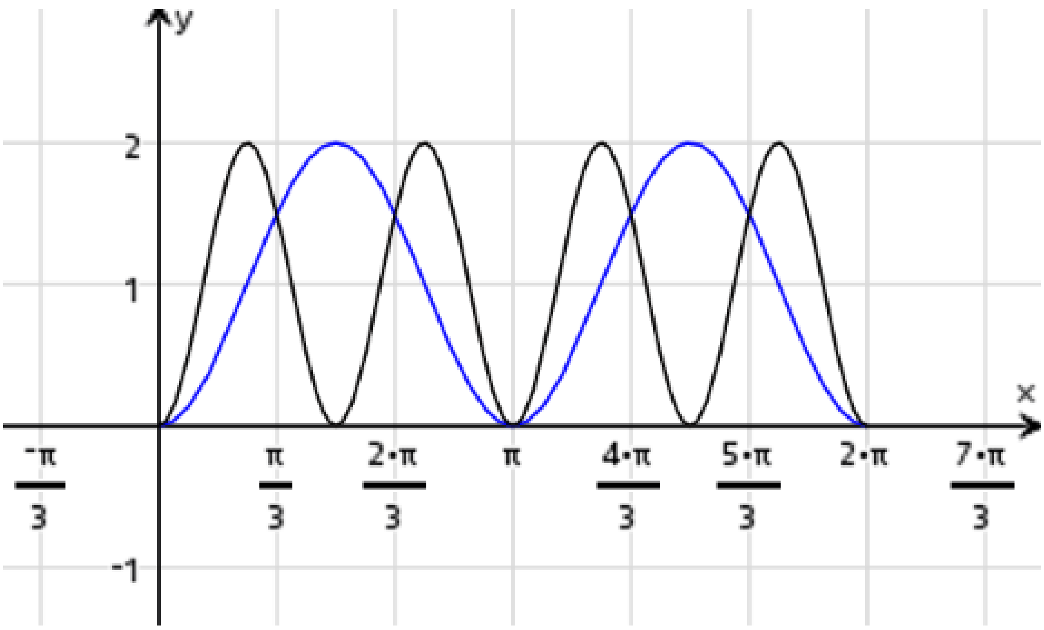
$2x=2n\pi\pm\cos^{-1}\left(-\frac{1}{2}\right),\ 2x=2n\pi\pm\cos^{-1}(1)$

$2x=2n\pi\pm\frac{2\pi}{3} \quad , \quad 2x=2n\pi\pm 0$

$x=\frac{\pi}{3}(3n\pm1)\ ,\ x=n\pi \quad n\in Z.$ A1

b. The two graphs intersect at $x=0,\frac{\pi}{3},\frac{2\pi}{3},\pi,\frac{4\pi}{3},\frac{5\pi}{3},2\pi$ from **a.**

$y_1=2\sin^2(2x)$ $\quad$ $y_2=1-\cos(2x)$ has a period of π G2



$2\sin^2(2x)<1-\cos(2x), \quad \frac{\pi}{3}<x<\frac{2\pi}{3}\ ,\ \frac{4\pi}{3}<x<\frac{5\pi}{3}$

$\left(\frac{\pi}{3},\frac{2\pi}{3}\right)\cup\left(\frac{4\pi}{3},\frac{5\pi}{3}\right)$ A1

Question 8

a. completing the square

$$h(x)=6+3x-x^2=-\left(x^2-3x+\frac{9}{4}\right)+6+\frac{9}{4}=\frac{33}{4}-\left(x-\frac{3}{2}\right)^2$$

range $h=\left(-\infty,\frac{33}{4}\right]$

$f(x)=\log_e(x-2)$ domain $x-2>0 \Rightarrow x>2$, range R

	$f(x)$	$h(x)$
domain	$(2,\infty)$	R
range	R	$\left(-\infty,\frac{33}{4}\right]$

Since range $h \not\subset$ domain f, so $f\circ h(x)$ does not exist. A1

b. solving $g(x)=6+3x-x^2=2$

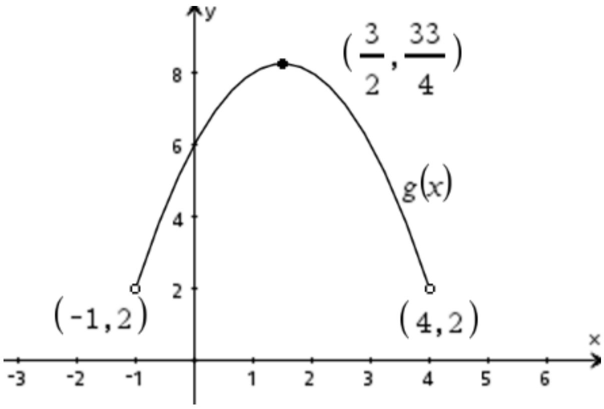
$\Rightarrow \quad x^2-3x-4=0$

$(x-4)(x+1)=0$ M1

$\Rightarrow \quad x=-1, 4$

now $g(-1)=g(4)=2$, $g\left(\frac{3}{2}\right)=\frac{33}{4}$, so if we now restrict the domain of g,

as $D=(-1,4)=$ domain $f\circ g(x)$ and the range of $g=\left(2,\frac{33}{4}\right]$ A1



so range $g\subseteq$ domain f, so now $f\circ g(x)$ exist.

$$f\circ g(x)=f(g(x))=f\left(6+3x-x^2\right)=\log_e\left(6+3x-x^2-2\right)$$

$f\circ g(x):(-1,4)\to R,\quad f\circ g(x)=\log_e\left(4+3x-x^2\right)=\log_e\left((4-x)(x+1)\right)$ A1

Question 9

The two y values are equal at $x=b$ so that $(1)\quad \sin(b)=\sin(b-\alpha)+c$

$y=\sin(x),\ \dfrac{dy}{dx}=\cos(x)$ and $y=\sin(x-\alpha)+c,\ \dfrac{dy}{dx}=\cos(x-\alpha)$

The gradients are also equal at $x=b$ so that $(2)\quad \cos(b)=\cos(b-\alpha)$ A1

square (1) $\sin^2(b)=\sin^2(b-\alpha)+2c\sin(b-\alpha)+c^2$

square (2) $\cos^2(b)=\cos^2(b-\alpha)$ now add M1

$$\sin^2(b)+\cos^2(b)=\sin^2(b-\alpha)+\cos^2(b-\alpha)+2c\sin(b-\alpha)+c^2$$
$$1=1+2c\sin(b-\alpha)+c^2$$

$$2c\sin(b-\alpha)+c^2=c\left(2\sin(b-\alpha)+c\right)=0$$

$c=-2\sin(b-\alpha)=2\sin(\alpha-b)$ since $c>0$ and $0<b<\alpha<\dfrac{\pi}{2}$ M1

from (1) $\sin(b)=\sin(b-\alpha)+c=\sin(b-\alpha)-2\sin(b-\alpha)$

$$\sin(b)=-\sin(b-\alpha)=\sin(\alpha-b)$$

$$b=\alpha-b$$
$$2b=\alpha$$
$$b=\frac{\alpha}{2}$$

$$c=2\sin\left(\alpha-\frac{\alpha}{2}\right)$$

$$c=2\sin\left(\frac{\alpha}{2}\right)$$ A1

Question 10

a.i. $f(x)=\begin{cases}\dfrac{a}{(2x+1)^2} & 1\le x\le 4\\ 0 & \text{elsewhere}\end{cases}$

since it is a probability density function the total area under the curve is 1.

$$\int_1^4 \frac{a}{(2x+1)^2}\,dx=1$$

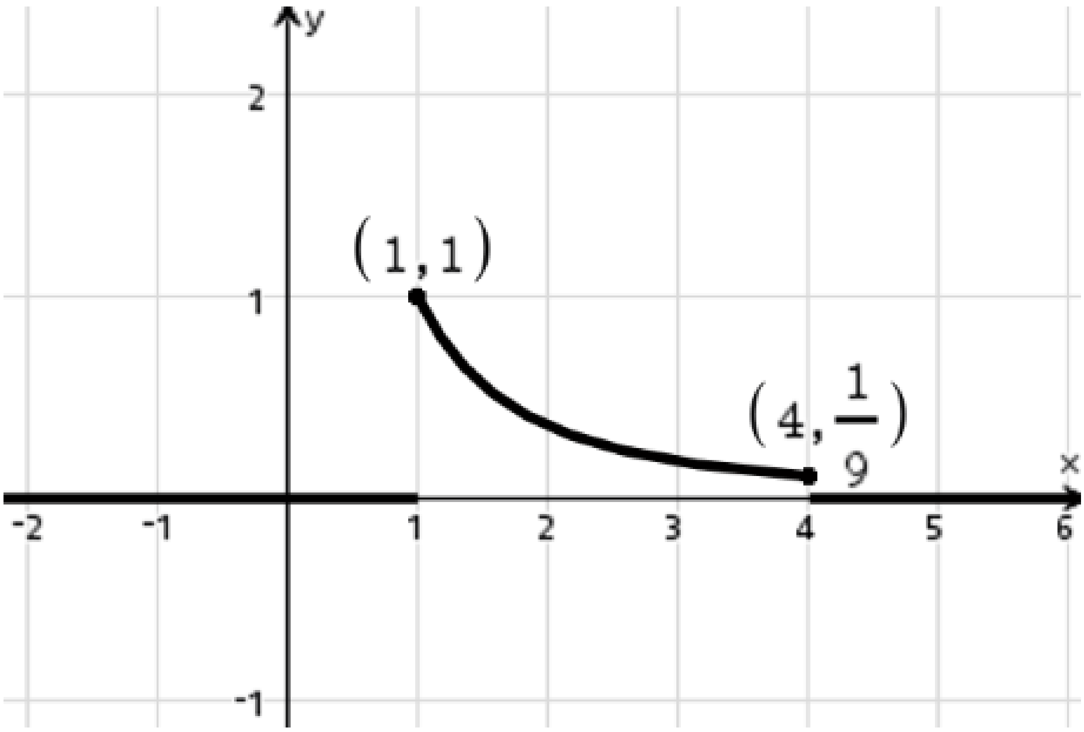
$$a\left[-\frac{1}{2(2x+1)}\right]_1^4=1$$ A1

$$a\left(-\frac{1}{18}+\frac{1}{6}\right)=\frac{a(3-1)}{18}=1$$ M1

$a=9$

$$f(x)=\begin{cases}\dfrac{9}{(2x+1)^2} & 1\le x\le 4\\ 0 & \text{elsewhere}\end{cases}$$

ii. $f(1)=1,\quad f(4)=\dfrac{1}{9}$ the graph is part of a truncus graph G1



b. $g(y)=\begin{cases}\dfrac{b}{2y+1} & 1\le y\le 4\\ 0 & \text{elsewhere}\end{cases}$

since it is a probability density function the total area under the curve is 1.

$$\int_1^4 \frac{b}{2y+1}\,dy=1$$

$$b\left[\frac{1}{2}\log_e(2y+1)\right]_1^4=1$$

$$\frac{b}{2}\left(\log_e(9)-\log_e(3)\right)=\frac{b}{2}\log_e(3)=1$$

$$b=\frac{2}{\log_e(3)}$$ A1

$$E(Y)=\frac{2}{\log_e(3)}\int_1^4 \frac{y}{2y+1}\,dy$$

$$E(Y)=\frac{1}{\log_e(3)}\int_1^4\left(\frac{2y}{2y+1}\right)dy=\frac{1}{\log_e(3)}\int_1^4\left(\frac{2y+1-1}{2y+1}\right)dy$$ M1

$$E(Y)=\frac{1}{\log_e(3)}\int_1^4\left(1-\frac{1}{2y+1}\right)dy$$

$$=\frac{1}{\log_e(3)}\left[y-\frac{1}{2}\log_e(2y+1)\right]_1^4$$

$$=\frac{1}{\log_e(3)}\left(4-\frac{1}{2}\log_e(9)-1+\frac{1}{2}\log_e(3)\right)=\frac{1}{\log_e(3)}\left(3-\frac{1}{2}\log_e(3)\right)$$ A1

$$=\frac{3}{\log_e(3)}-\frac{1}{2},\quad p=3,\ q=-\frac{1}{2}$$ A1

End of detailed answers for the
2024 Kilbaha VCE Mathematical Methods Trial Examination 1

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