TSSM
Creating VCE Success

2023 Trial Examination

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STUDENT NUMBER | | | | | | | | | Letter |

MATHEMATICAL METHODS

Units 3 & 4 - Written examination 2

Reading Time: 15 minutes
Writing Time: 2 Hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	20	20	20
2	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 18 pages.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

SECTION 1

Instructions for Section 1
A correct answer scores 1, an incorrect answer scores 0. Marks are not deducted for incorrect answers. If more than 1 answer is completed for any question, no mark will be given.

Question 1

The linear function $f: D \to R, f(x) = 1 - 2x$ has a range $(-1, 4]$.
The domain D is:

A. $\left[-\frac{3}{2}, 1\right)$
B. $[-7, 3)$
C. $\left(-\frac{3}{2}, 1\right]$
D. $(-7, 3)$
E. $(-1, 4]$

Question 2

Consider the graph of the function defined by $f: [0, 2\pi] \to R, f(x) = 2\cos(x)$.
The square of the length of the line segment joining the points on the graph for which $x = \frac{\pi}{2}$ and $x = \pi$ is:

A. $1 + \pi^2$
B. $2 + \frac{\pi^2}{2}$
C. $4 + \frac{\pi^2}{4}$
D. $1 + \frac{\pi^2}{4}$
E. $1 + \frac{\pi^2}{2}$

Question 3

The range and period of the function $g: R \to R, g(x) = 1 - 4\sin(2x)$ are:

A. $[-5, 3]$ and π
B. $[-5, 3]$ and $\frac{\pi}{2}$
C. $[-5, 3]$ and 2π
D. $[-3, 5]$ and $\frac{\pi}{2}$
E. $[-3, 5]$ and π

SECTION 1- continued

Question 4

Which of the following is the inverse function of $h: [3, \infty) \to R, h(x) = 2(x-3)^2 + 4$?

A. $h^{-1}: [-4, \infty) \to R, h^{-1}(x) = \sqrt{\frac{x-2}{4}} + 3$
B. $h^{-1}: [-4, \infty) \to R, h^{-1}(x) = \sqrt{\frac{x-4}{2}} + 3$
C. $h^{-1}: [-4, \infty) \to R, h^{-1}(x) = \sqrt{\frac{x-2}{4}} - 3$
D. $h^{-1}: [-4, \infty) \to R, h^{-1}(x) = \sqrt{\frac{x-4}{2}} - 3$
E. $h^{-1}: [-4, \infty) \to R, h^{-1}(x) = \sqrt{\frac{x-4}{2}}$

Question 5

The number of siblings, X, each student of a particular class has is a random variable with the following discrete probability distribution.

x	0	1	2	3	4
$\Pr(X = x)$	0.2	0.35	0.35	0	0.1

If two students are selected at random, the probability that they have the same number of siblings is:

A. 1.45
B. 0.35
C. 0.295
D. 0.04
E. 0

Question 6

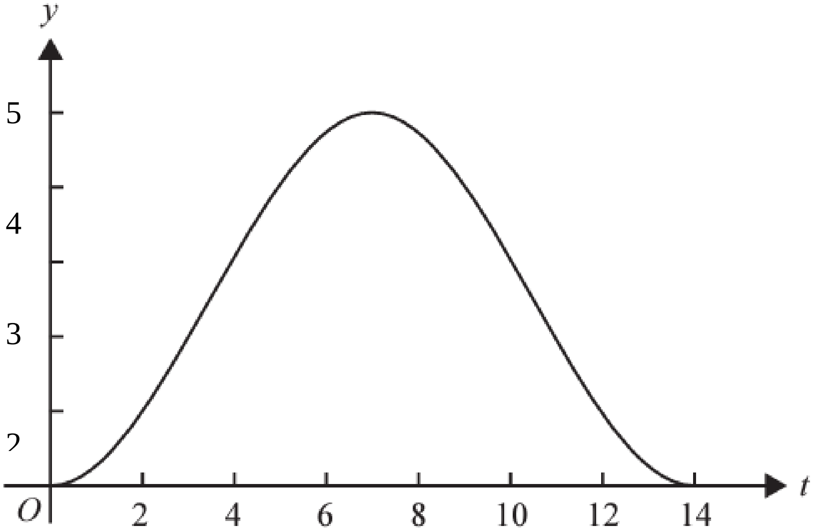
If $y = a\log_b(c - 2x)$ where $a, b, c > 0$ and $x < \frac{c}{2}$, then x is equal to:

A. $x = \frac{-2}{c-2x}$
B. $x = \frac{c-b^{\frac{y}{a}}}{2}$
C. $x = 2b^{ay} + c$
D. $x = a\log_{\mathrm{b}}(c - 2y)$
E. $x = c\log_{\mathrm{b}}(c - 2y)$

SECTION 1- continued
TURN OVER

Question 7

The height of the tide above mean sea level, y metres, can be modelled by the following graph where t is the hours after 3am.



The rule that best models this graph is:

A. $y = 5 - 5\cos\left(\frac{\pi t}{7}\right)$
B. $y = 5 + 5\sin\left(\frac{\pi t}{7}\right)$
C. $y = 2.5 - 2.5\cos\left(\frac{\pi t}{7}\right)$
D. $y = 2.5 + 2.5\sin\left(\frac{\pi t}{7}\right)$
E. $y = 2.5 - 2.5\cos\left(\frac{\pi t}{14}\right)$

Question 8

The 95% confidence interval for the proportion of goals scored from penalty kicks in a junior competition is calculated from a large sample to be $(0.428, 0.612)$.
The sample proportion from which this interval was constructed is:

A. 0.95
B. 0.428
C. 0.612
D. 0.5
E. 0.52

SECTION 1- continued

Question 9

Consider the graph of $y = 2x^3 - 2x^2 + 2$. The value for x at the point of inflection of y is:

A. 0.
B. 2
C. -2.
D. $\frac{1}{3}$
E. $-\frac{1}{3}$

Question 10

The function $f(x) = x^3 + mx^2 + n$ has one of its stationary points located at $(4, 100)$. The values of m and n respectively are:

A. $m = -6, n = 132$
B. $m = 6, n = 132$
C. $m = 132, n = -6$
D. $m = 132, n = 6$
E. $m = 4,\ n = 100$

Question 11

The random variable X has the following probability distribution, where $0 < a < 0.2$

x	-5	0	5
$\Pr(X = x)$	a	$4a$	$1 - 5a$

The mean of X is:

A. $5 - 30a$
B. $25 - 150a$
C. 0
D. 1
E. $30a + 5$

SECTION 1- continued
TURN OVER

Question 12

Consider the set of simultaneous equations below:

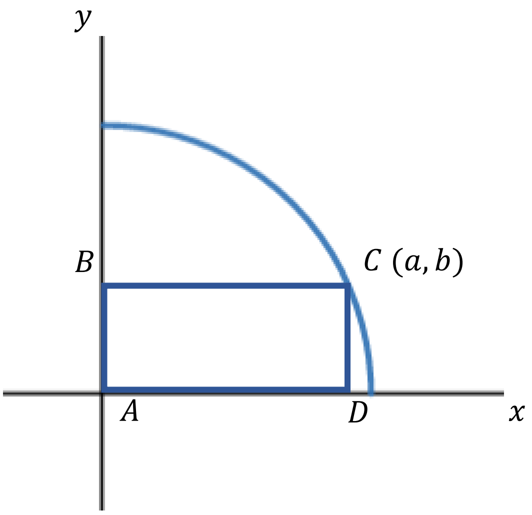
$$ax + 2y = b$$
$$5x + ay = 5$$

This set of equations will have no solutions provided:

A. $a \in R, b \in R\backslash\{\sqrt{10}\}$
B. $a = \sqrt{10}, b \in R$
C. $a = \sqrt{10}, b \in R\backslash\{\sqrt{10}\}$
D. $a = b = 10$
E. $a = b = \sqrt{10}$

Question 13

The rectangle $ABCD$ has vertices $A\ (0,0)$, $B(0,b)$, $C\ (a,b)$ and $D\ (a,0)$ where C lies on the graph of $y = \sqrt{16 - x^2}$, as shown below.



The maximum area of the rectangle and the corresponding value of x for which it occurs is:

A. $Area = 4, x = \sqrt{2}$
B. $Area = 8, x = 2$
C. $Area = 4, x = 2\sqrt{2}$
D. $Area = 4, x = 2$
E. $Area = 8, x = 2\sqrt{2}$

SECTION 1- continued

Question 14

For a random sample of four Australians, $\hat{P}$ is the random variable that represents the proportion who voted for a particular party at the last election.
Given that $\Pr\left(\hat{P}=0\right)=\frac{16}{625}$ then $\Pr(\hat{P}=1)$ must be equal to:

A. $\frac{81}{625}$
B. $\frac{16}{625}$
C. $\frac{96}{625}$
D. $\frac{216}{625}$
E. $\frac{216}{625}$

Question 15

The weights of packets of chocolates are normally distributed with a mean of 250 grams.
If 10.565% of packets are known to be greater than 265 grams then the standard deviation must be:

A. -12
B. 12
C. 144
D. -144
E. 15

Question 16

The iterative formula derived from Newton's method of solving the equation is $x^3-3x=0$ is:

A. $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$
B. $x_{n+1}=x_n+\frac{f\prime(x_n)}{f(x_n)}$
C. $x_{n+1}=\frac{3x_n-x_n^3}{3x_n^2-3}$
D. -3
E. $x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

Question 17

The area under the curve $y=(1-x)(x+1)(x-4)$ between $x=1$and $x=4$ is estimated to be closest to which of the following when using the trapezium method of width 0.5?

A. 15.75
B. 61.25
C. 15.3125
D. 20.125
E. 5.025

SECTION 1- continued
TURN OVER

Question 18

The continuous random variable, X, has a probability density function given by:

$$f(x) = \begin{cases} a\cos(2x), & 0 \le x \le \frac{\pi}{4} \\ 0 & \text{elsewhere} \end{cases}$$

The mean of X is:

A. $\frac{\pi+4}{2}$
B. $\frac{\pi-4}{2}$
C. $\frac{\pi}{4}$
D. $\frac{\pi+2}{4}$
E. $\frac{\pi-2}{4}$

Question 19

Consider the algorithm below, which uses Newton's method to solve an equation in the form $f(x) = 0$.

Inputs: $f(x)$, the function of x
$g(x)$, the derivative of $f(x)$
x_0, the initial guess
e, tolerable error
N, maximum number of iterations

Define Newton($f(x), g(x), x_0, e, N$)
While abs $f(x_1) > e$

$x_1 \leftarrow x_0 - f(x_0)/g(x_0)$
$x_0 \leftarrow x_1$
$Step \leftarrow Step + 1$

If $Step > N$
Return "Non Convergent"
Stop
End if
End while
Return x_1

The algorithm is implemented as follows:
Newton $(\sqrt{x} - 3x^2, -1/\sqrt{x} - 6x, 2, 0.001, 20)$

The number of iterations required for the algorithm to return a result is:

A. 8
B. 9
C. 10
D. 11
E. 12

Question 20

A box contains u blue marbles and v black marbles. Two marbles are drawn from the box. If the first marble is not replaced before the second marble is drawn, the probability that both marbles drawn are the same colour is:

A. $\frac{u^2+v^2-u-v}{(u+v)(u+v-1)}$

B. $\frac{1}{(u+v)}$

C. $\frac{u^2+v^2-u-v}{(u+v)^2}$

D. $\frac{u+v}{(u+v)(u+v-1)}$

E. $\frac{u+v}{(u+v)(u+v-2)}$

END OF SECTION 1
TURN OVER

SECTION 2

Instructions for Section 2
A decimal approximation will not be accepted if the question specifically asks for an **exact** answer. In questions worth more than one mark, appropriate working **must** be shown.
Marks are given as specified for each question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x) = 4\log_2(x-2) + 1$

a. State the maximal domain and range of $f(x)$.

2 marks

b. State the rule for the derivative function $f'(x)$.

1 mark

c. Find the equation of the tangent to $f(x)$ at $x = 6$

2 marks

SECTION 2- Question 1 - continued

d. Find the length of the line segment between the origin and the point of $f(x)$ that has a gradient of $\frac{1}{2\log_e 2}$.

3 marks

e. State a series of transformations that takes the graph of $f(x)$ to the graph of $y = \log_2(x)$

3 marks

Total 11 marks

Question 2

The temperature in a florist shop needs to be strictly controlled in order to best maintain the presentation of the flowers available.
On any particular day the temperature T °C, can be modelled by the $T(t) = 21 - 3.2\cos\left(\frac{\pi t}{6}\right)$, $0 \le t \le 24$ where t is the number of hours after 6am.

a. State the maximum temperature of the florist and the value(s) of t at when this occurs?

2 marks

b. State the period of the function T.

1 mark

There are two types of roses that require precise temperatures to be at full bloom.
Type A requires a temperature greater than $22°C$ whereas type B requires a temperature less than 23°C.

c. To the nearest percent, for what percentage of the day is the Type A rose in full bloom?

2 marks

d. To the nearest percent, for what percentage of the day is the Type B rose in full bloom?

2 marks

e. To the nearest minute, for how long are both Type A and Type B roses in full bloom?

2 marks

Total 9 marks

SECTION 2- continued

Question 3

In an attempt to show it can be done, Calum is flipping an unbiased coin to see if he can obtain 10 heads in a row.
Let X be the number of times the coin lands on heads when it is flipped 10 times.

a. Find $\Pr(X = 4)$, correct to four decimal places.

1 mark

b. Find $\Pr(X \leq 7)$, correct to four decimal places.

1 mark

c. Find $\Pr(X \geq 4 | X \leq 7)$ correct to three decimal places.

2 marks

d. Find the expected value and standard deviation of X.

3 marks

e. State the probability that Calum achieves his goal and obtains exactly 10 heads in his 10 coin flips.

1 mark
Total 8 marks
SECTION 2- continued

Question 4

Hannah's blood sugar level, A mmol/L, after she eats a high-sugar snack can be modelled by the function $A(t) = ate^{-\frac{t}{b}} + c$ where $t \geq 0$ is the number of minutes after eating the snack and $a, b, c \in R$.

The initial level was 8.1 mmol/L. 11.043 mmol/L was measured exactly two minutes later and a level of 8.370 was measured a further eight minutes after this.

a. Determine the values of a, b and c to the nearest integer.

3 marks

b. Using the rounded values found in part a, what is Hannah's maximum blood sugar level, correct to three decimal places?

1 mark

c. For how long is her blood sugar level above 10 mmol/L? Give your answer to the nearest second.

2 marks

d. State the rule for $A'(t)$.

1 mark

SECTION 2- Question 4 - continued

e. At what times, to two decimal places are the rate of increase and rate of decrease of the blood sugar level the greatest?

2 marks

At the same time, after eating the same snack, Freddie's blood sugar level can modelled by the function $B(t) = 3te^{-\frac{2t}{5}} + 7.9, t \geq 0.$

f. What was Freddie's initial blood sugar level?

1 mark

g. What is Freddie's maximum blood sugar level and at what time does this occur?

2 marks

h. For how long is Hannah's blood sugar level greater than Freddie's? Give your answer to the nearest second.

3 marks

Total 15 marks

SECTION 2- continued
TURN OVER

Question 5

The probability density function that describes the lifespan, X, in days, of a particular worm species is given by

$$f(x)=\begin{cases}a(4x^3-x^4), & 0\le x\le 4\\ 0, & \text{elsewhere}\end{cases}$$

a. Find the value of a.

2 marks

b. Find the mean life span of the worm species

2 marks

c. To the nearest percent, what percentage of the worm species have a life span greater than 3 days?

2 marks

d. What is the probability that a particular worm of this species lives less than 3 days, given that it lives greater than 2 days?

2 marks

SECTION 2- Question 5 - continued

The worm's length is normally distributed with a mean of 35mm and a standard deviation of 7mm.

e. Find the probability, correct to four decimal places, that a randomly selected worm of this species has a length between 30mm and 40mm.

1 mark

The shortest 10% of worms are considered to be small, whilst the longest 15% of worms are considered to be large. All other worms are considered standard.

f. In millimetres, find the largest length of a worm that is considered small and the shortest length of a worm that is considered large. Give both answers correct to two decimal places.

2 marks

A random sample 20 worms is collected. Let $\hat{P}$ be the proportion of worms in this sample with a standard length.

g. Correct to four decimal places, determine that probability that all 20 worms are standard lengths.

1 mark

h. Find the mean and standard deviation for $\hat{P}$, correct to four decimal places where appropriate.

3 marks

SECTION 2- Question 5 - continued
TURN OVER

i. Hence or otherwise, compute $\mu \pm 2\sigma$ and explain its meaning within this context.

2 marks

Total 17 marks

END OF QUESTION AND ANSWER BOOK