

Trial Examination 2023

MATHEMATICAL METHODS

Trial Written Examination 2 - SOLUTIONS

SECTION A: Multiple Choice

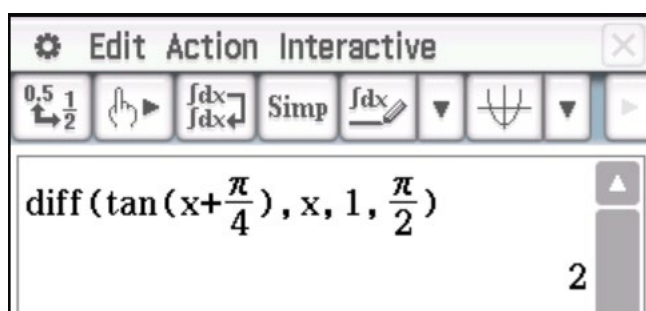
Question	Answer	Question	Answer
1	A	11	C
2	D	12	E
3	B	13	D
4	C	14	B
5	C	15	A
6	E	16	A
7	E	17	D
8	D	18	C
9	A	19	E
10	B	20	B

Question 1 **Answer A**

Gradient of $y = \tan\left(x + \frac{\pi}{4}\right)$ at $x = \frac{\pi}{2}$ is 2 using technology

OR

$$\frac{dy}{dx} = \sec^2\left(x + \frac{\pi}{4}\right) \text{ at } x = \frac{\pi}{2}, \frac{dy}{dx} = 2$$



Question 2 **Answer D**

Period of $g(x) = -3\sin\left(\frac{\pi}{8}x + 1\right)$

$$\text{Period} = \frac{2\pi}{\frac{\pi}{8}} = 16$$

Question 3 **Answer B**

$$f: (-\infty, a] \rightarrow \mathbb{R}, f(x) = \frac{1}{4}x^4 - 3x^3 - 6x^2 + 20x + 10$$

f^{-1} exists when original function f is one-to-one.

Stationary points are at $x = -2$, $x = 1$ and $x = 10$

Closest stationary point from $-\infty$ is at $x = -2$

So $a = -2$

define $f(x) = \frac{1}{4}x^4 - 3x^3 - 6x^2 + 20x + 10$ done

solve $\left(\frac{d}{dx}(f(x)) = 0, x\right)$

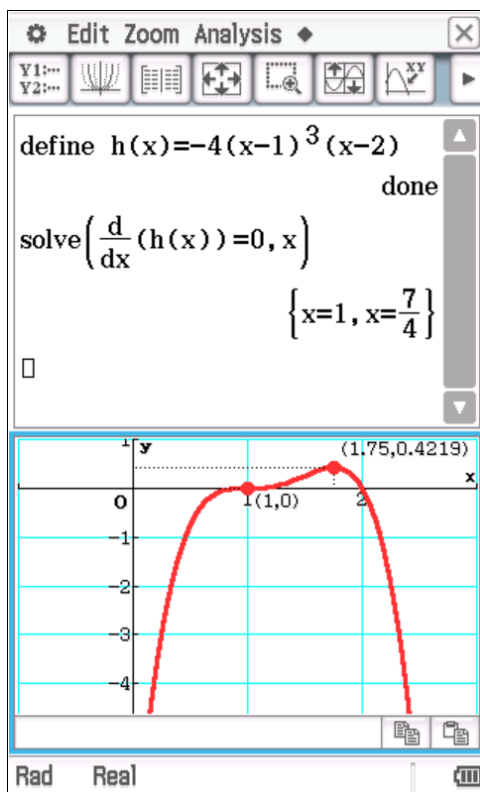
$\{x = -2, x = 1, x = 10\}$

Alg Standard Real Rad

Question 4 **Answer C**

$$h(x) = -4(x-1)^3(x-2) \text{ has stationary points at } x=1 \text{ and } x = \frac{7}{4}$$

Strictly increasing for $\left(-\infty, \frac{7}{4}\right]$



Question 5**Answer C**

$$2x + ky = 3$$

$$-3x - y = m$$

Using ratios, gradients will be the same when

$$-\frac{2}{3} = -k$$

$$k = \frac{2}{3}$$

y-intercepts will be the same when

$$-\frac{2}{3} = \frac{3}{m}$$

$$-2m = 9$$

$$m = -\frac{9}{2}$$

OR

Using $y = mx + c$

$$y = \frac{-2}{k}x + \frac{3}{k}$$

$$y = -3x - m$$

Gradients will be the same when

$$-\frac{2}{k} = -3$$

$$k = \frac{2}{3}$$

y-intercepts will be the same when

$$\frac{3}{k} = -m$$

$$\frac{3}{\frac{2}{3}} = -m$$

$$m = -\frac{9}{2}$$

Question 6**Answer E**

$$f : [0, \infty) \rightarrow R, f(x) = \sqrt{x}$$

Translate 2 units to the right and 3 units down

$$f_1 : [2, \infty) \rightarrow R, f_1(x) = \sqrt{x-2} - 3$$

Dilate by a factor of 4 from the x-axis

$$g : [2, \infty) \rightarrow R, g(x) = 4\sqrt{x-2} - 12$$

Question 7**Answer E**

Given $\int_a^5 f(x)dx = 3$ and $\int_5^b f(x)dx = -4$ where $a < 5 < b$

$$\begin{aligned} \int_a^b (2f(x)+1)dx &= \int_a^5 (2f(x)+1)dx + \int_5^b (2f(x)+1)dx \\ &= \int_a^5 (2f(x))dx + \int_a^5 (1)dx + \int_5^b (2f(x))dx + \int_5^b (1)dx \\ &= 2\int_a^5 (f(x))dx + 2\int_5^b (f(x))dx + \int_a^b (1)dx \\ &= 2 \times 3 + 2 \times -4 + b - a \\ &= -2 + b - a \\ &= -2 - a + b \end{aligned}$$

Question 8**Answer D**

The points are $(0,5)$ and $(x, (x-2)^2)$.

$$d = \sqrt{x^2 + ((x-2)^2 - 5)^2}$$

Solve $\frac{d}{dx} \sqrt{x^2 + ((x-2)^2 - 5)^2} = 0$ for x .

$$x = \frac{\pm\sqrt{6} + 2}{2}, x = 4$$

Minimum occurs when $x = \frac{-\sqrt{6} + 2}{2}$.

OR

$$f'(x) = 2(x-2)$$

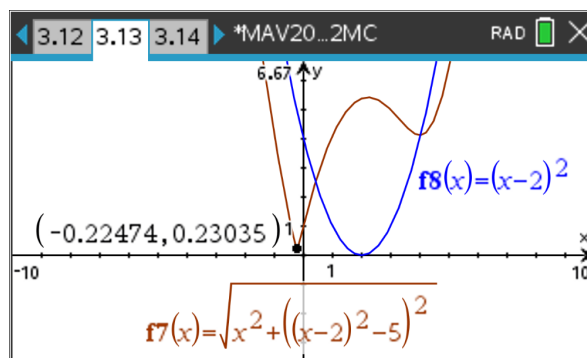
$$y = -\frac{x}{2(x-2)} + 5.$$

Solve $(x-2)^2 = -\frac{x}{2(x-2)} + 5$

$$x = \frac{\pm\sqrt{6} + 2}{2}, x = 4$$

Minimum occurs when $x = \frac{-\sqrt{6} + 2}{2}$.

$\text{solve}\left(\frac{d}{dx}\left(\sqrt{x^2 + ((x-2)^2 - 5)^2}\right) = 0, x\right)$
 $x = \frac{-(\sqrt{6}-2)}{2}$ or $x = \frac{\sqrt{6}+2}{2}$ or $x=4$
 $\sqrt{x^2 + ((x-2)^2 - 5)^2} \Big|_{x = \frac{-(\sqrt{6}-2)}{2}} = 0.23035091$



$\text{solve}\left((x-2)^2 = \frac{-x}{2(x-2)} + 5, x\right)$
 $x = \frac{-(\sqrt{6}-2)}{2}$ or $x = \frac{\sqrt{6}+2}{2}$ or $x=4$

Question 9**Answer A**

$$g: R \setminus \{1\} \rightarrow R, g(x) = \frac{1}{(x-1)^2}$$

$$\text{Average value} = \frac{1}{b-a} \int_a^b \left(\frac{1}{(x-1)^2}\right) dx = \frac{2}{5}$$

$$\text{Solve } \frac{d}{dx} \frac{1}{(x-1)^2} = -2 \text{ for } x$$

$$a = 2$$

$$\text{Solve } \frac{1}{b-2} \int_2^b \left(\frac{1}{(x-1)^2}\right) dx = \frac{2}{5} \text{ for } b$$

$$b = \frac{7}{2}$$

$\text{solve}\left(\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right) = -2, x\right)$ $x=2$
 $\text{solve}\left(\frac{1}{b-2} \cdot \int_2^b \frac{1}{(x-1)^2} dx = \frac{2}{5}, b\right)$ $b = \frac{7}{2}$

Question 10 **Answer B**

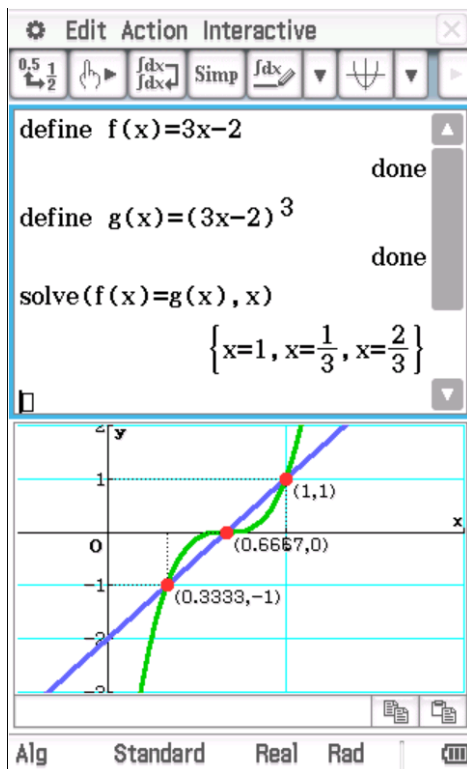
Intersection points between graphs of $f(x) = 3x - 2$ and $g(x) = (3x - 2)^3$ are at $x = \frac{1}{3}, x = \frac{2}{3}$ and $x = 1$

Area found by $\int_{\frac{1}{3}}^{\frac{2}{3}} (\text{cubic} - \text{linear}) dx + \int_{\frac{2}{3}}^1 (\text{linear} - \text{cubic}) dx$

$$\text{Area} = \int_{\frac{1}{3}}^{\frac{2}{3}} (g(x) - f(x)) dx + \int_{\frac{2}{3}}^1 (f(x) - g(x)) dx$$

$$\text{As } \int_{\frac{1}{3}}^{\frac{2}{3}} (g(x) - f(x)) dx = \int_{\frac{2}{3}}^1 (f(x) - g(x)) dx$$

$$\text{Area} = 2 \int_{\frac{2}{3}}^1 (f(x) - g(x)) dx = 2 \int_1^{\frac{2}{3}} (g(x) - f(x)) dx$$



Question 11**Answer C**

$$y + z = 2$$

$$-2x - 3y = 8$$

Let $y = \lambda$

$$\lambda + z = 2$$

$$z = 2 - \lambda$$

$$-2x - 3\lambda = 8$$

$$-2x = 8 + 3\lambda$$

$$x = -\frac{8 + 3\lambda}{2}$$

$$y = \lambda, x = -\frac{3\lambda + 8}{2}, z = -\lambda + 2 \text{ where } \lambda \in R$$

1.5 1.6 1.7 *MAV RAD

solve($y+z=2$ and $-2 \cdot x-3 \cdot y=8,x,z$)| $y=k$
 $x = \frac{-(3 \cdot k+8)}{2}$ and $z = -(k-2)$

solve($\left\{ \begin{array}{l} y+z=2 \\ -2 \cdot x-3 \cdot y=8 \end{array} \right\}, \{x,z\}$)| $y=k$
 $x = \frac{-(3 \cdot k+8)}{2}$ and $z = -(k-2)$

Edit Action Interactive

$\left\{ \begin{array}{l} y+z=2 \\ -2x-3y=8 \end{array} \right\}_{x,z}$
 $\left\{ x = \frac{-(3 \cdot y+8)}{2}, z = -y+2 \right\}$
 $\left\{ x = \frac{-(3 \cdot y+8)}{2}, z = -y+2 \right\} | y=k$
 $\left\{ x = \frac{-(3 \cdot k+8)}{2}, z = -k+2 \right\}$

Question 12**Answer E**

$$h(x) = \frac{\log_e(x-a)}{\log_e(x+a)}, \quad a > 0$$

Maximal domain for the intersection of:

Numerator: $x - a > 0 \therefore x > a$ Denominator: $x + a > 0 \therefore x > -a$ and $\log_e(x+a) \neq 0$ Giving (a, ∞)

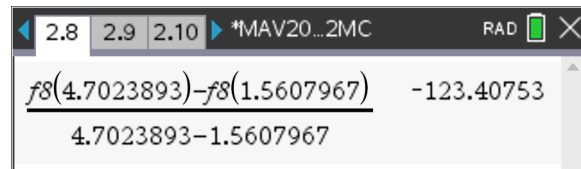
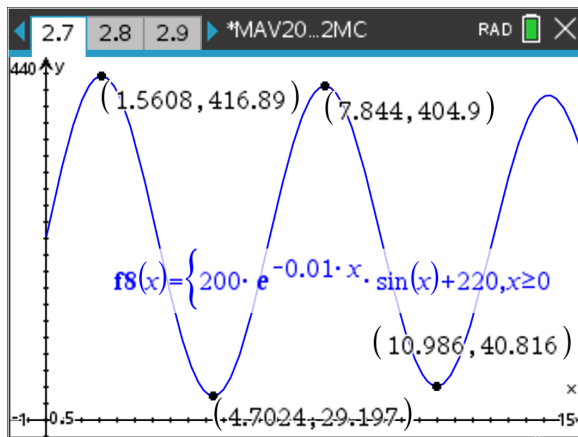
1.14 1.15 1.16 *MAV2023 RAD

domain($\frac{\ln(x-a)}{\ln(x+a)}, x$)| $a > 0$
 $\max(a, 1-a) < x < \infty$ and $a > 0$ or $1-a > x > a > 0$

Question 13**Answer D**

$$\text{Average rate of change} = \frac{s(4.70\dots) - s(1.56\dots)}{4.70\dots - 1.56\dots}$$

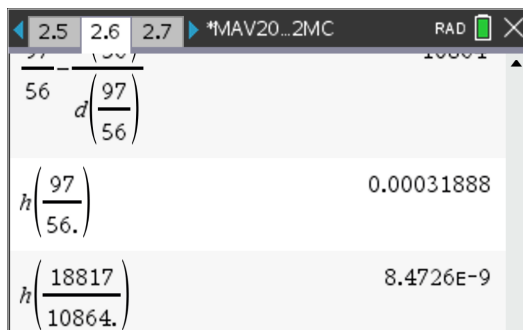
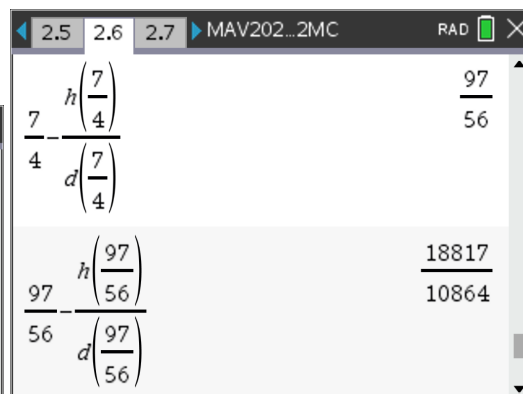
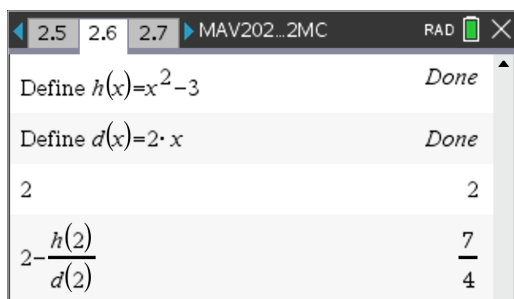
$$= -123.408 \text{ correct to three decimal places}$$



Question 14 **Answer B**

The program will stop when $h(x) < -0.0001$ or $h(x) > 0.0001$.

Iteration	x	$h(x)$
	2	1
1	1.75	0.0625
2	$\frac{97}{56} = 1.7321\dots$	0.0003...
3	$\frac{18817}{10864} = 1.7320\dots$	$8.4\dots \times 10^{-9}$



Question 15 **Answer A**

$$\begin{aligned} \text{Area} &= \frac{b-a}{2n}(f(1)+2f(2)+2f(3)+f(4)) \\ &= \frac{4-1}{6}(0+2f(2)+2f(3)+f(4)) \\ &= \frac{1}{2}(2f(2)+2f(3)+f(4)) \\ &= f(2)+f(3)+\frac{1}{2}f(4) \end{aligned}$$

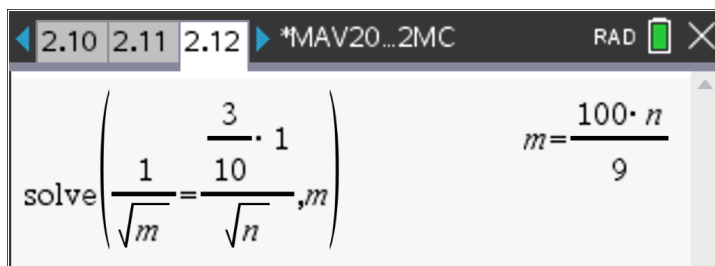
Question 16 **Answer A**

$$W_1 = 2z \frac{\sigma}{\sqrt{n_1}}, \quad W_2 = 2z \frac{\sigma}{\sqrt{n_2}}$$

$$W_2 = 2z \frac{\sigma}{\sqrt{n_2}} = 0.3 \times 2z \frac{\sigma}{\sqrt{n_1}}$$

$$\frac{1}{\sqrt{n_2}} = 0.3 \frac{1}{\sqrt{n_1}}$$

$$n_2 = \frac{100}{9} n_1$$



The screenshot shows a calculator window with the following content:

2.10 2.11 2.12 *MAV20...2MC RAD

solve $\left(\frac{1}{\sqrt{m}} = \frac{\frac{3}{10} \cdot 1}{\sqrt{n}}, m \right)$ $m = \frac{100 \cdot n}{9}$

Question 17 **Answer D**

Height (cm)	less than 40 cm	from 40 cm to 55 cm	greater than 55 cm
Proportion	0.105...	0.628...	0.265...
Cost (\$)	10	20	30

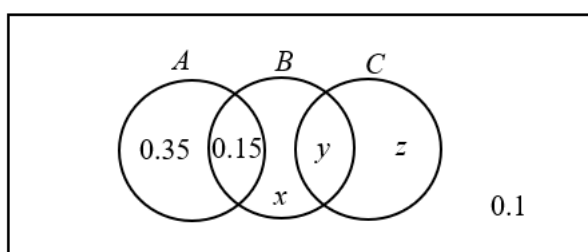
$$\begin{aligned} \text{Cost} &= (0.105... \times 10 + 0.628... \times 20 + 0.265... \times 30) \times 50 \\ &= \$1080.17 \end{aligned}$$

Calculator Window 1	Calculator Window 2
normCdf($-\infty, 40, 50, 8$)	0.10564984
normCdf($40, 55, 50, 8$)	0.62836469
normCdf($55, \infty, 50, 8$)	0.26598547
$0.10564983896266 \cdot 10 + 0.62836469324249$	
	21.603356
$21.603356288323 \cdot 50$	1080.1678

Calculator Window 3	
normCdf($-\infty, 40, 8, 50$)	0.1056497737
normCdf($40, 55, 8, 50$)	0.6283646973
normCdf($55, \infty, 8, 50$)	0.265985529

Question 18

Answer C



$$\Pr(A) = 0.5, \Pr(B) = 0.3 \text{ and } \Pr(C) = 0.35$$

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B) = 0.5 \times 0.3 = 0.15 \text{ independent events}$$

$$x + y = 0.15$$

$$0.5 + 0.15 + z + 0.1 = 1$$

$$z = 0.25$$

$$y = \Pr(B \cap C) = 0.1$$

Question 19

Answer E

$$\text{Solve } np(1-p) = 2 \text{ and } \binom{n}{2} p^2 (1-p)^{n-2} = \frac{512}{2187}$$

$$n = 9 \text{ and } p = \frac{1}{3}$$

$$\Pr(X < 2) = \Pr(X \leq 1) = 0.1431 \text{ correct to four decimal places}$$

Calculator Window 4	
$\text{solve}\left(\binom{n}{2} p^2 \cdot (1-p)^{n-2} = \frac{512}{2187} \text{ and } \right)$	
	$n=9. \text{ and } p=0.33333333$
$\text{binomCdf}\left(9, \frac{1}{3}, 0, 1\right)$	0.14306762

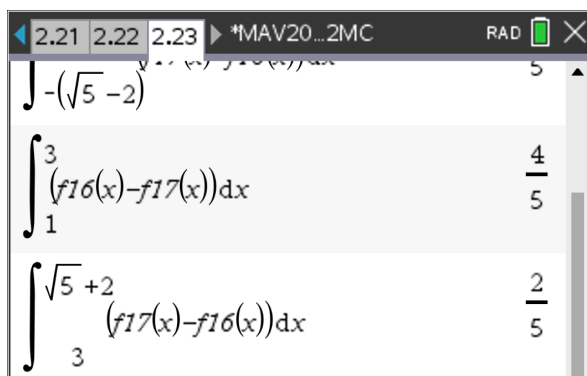
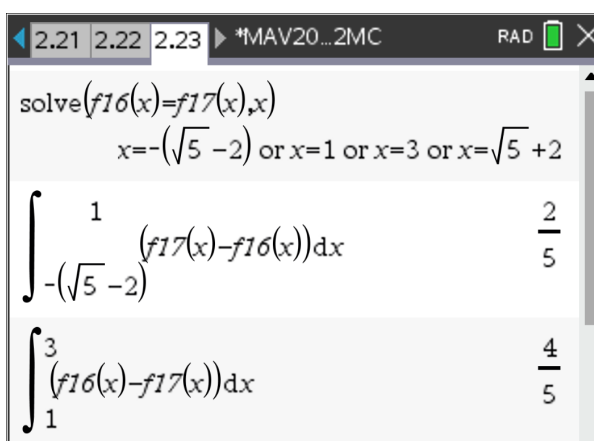
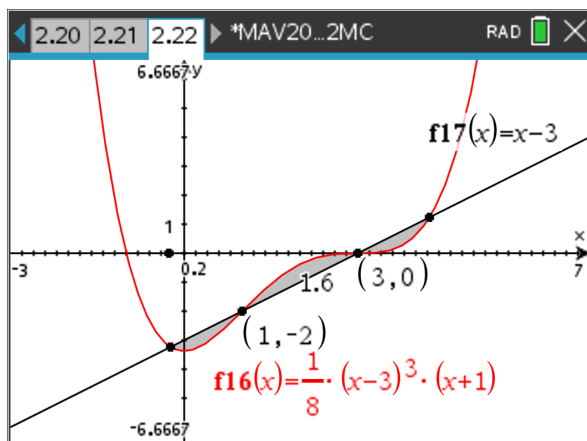
Question 20 **Answer B**

$$h(x) = \frac{1}{8}x^4 - x^3 + \frac{9}{4}x^2 - \frac{27}{8} = \frac{1}{8}(x-3)^3(x+1)$$

Points of inflection are at $(1, -2)$ and $(3, 0)$.

The equation of the line passing through these points is $y = x - 3$.

$$\begin{aligned} \text{Area} &= \int_{-\sqrt{5}+2}^1 (x-3-h(x))dx + \int_1^3 (h(x)-(x-3))dx + \int_3^{\sqrt{5}+2} (x-3-h(x))dx \\ &= 1.6 \end{aligned}$$



END OF SECTION A SOLUTIONS

SECTION B

Question 1

$$f(x) = \begin{cases} k & 0 \leq x \leq 100 \\ 20 \cos\left(\frac{\pi(x-100)}{200}\right) + 10 & 100 < x \leq 400 \end{cases}$$

a. $f(x)$ is **continuous** at $x = 100$.

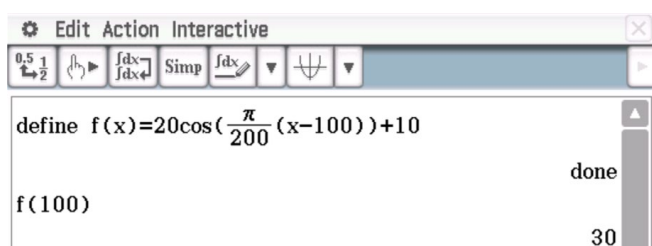
At the joining point

$$f(100) = 20 \cos\left(\frac{\pi(100-100)}{200}\right) + 10$$

$$= 20 \cos(0) + 10$$

$$k = 30$$

1A

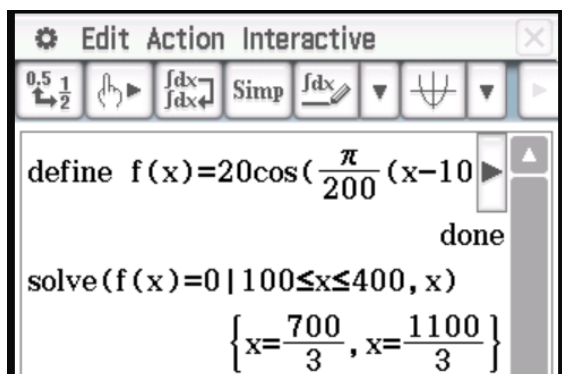


b. Solve $f(x) = 0$ to find the section of graph that runs under ground level.

$$\text{Gives } x = \frac{700}{3}, x = \frac{1100}{3}$$

$$\text{Answer } x \in \left(\frac{700}{3}, \frac{1100}{3}\right)$$

1A



c. Smooth at $x = 100$ requires same gradient as $x \rightarrow 100^-$ and $x \rightarrow 100^+$

As $x \rightarrow 100^-$, $y = 30$ giving gradient equal to zero.

As $x \rightarrow 100^+$,

$$f'(x) = -\frac{\pi}{10} \sin\left(\frac{\pi(x-100)}{200}\right)$$

$$f'(100) = -\frac{\pi}{10} \sin\left(\frac{\pi(100-100)}{200}\right) = 0 \text{ giving gradient equal to zero.}$$

Smooth at $x = 100$

1M

The screenshot shows a CAS calculator window titled "Edit Action Interactive". The function is defined as $f(x) = 20\cos\left(\frac{\pi}{200}(x-100)\right) + 10$. The derivative is calculated as $\frac{d}{dx}(f(x)) = \frac{-\sin\left(\frac{(x-100)\cdot\pi}{200}\right)\cdot\pi}{10}$. The interface includes a toolbar with various mathematical symbols and a mode selector at the bottom set to "Standard".

d.i. Points of inflection at $f''(x) = 0$

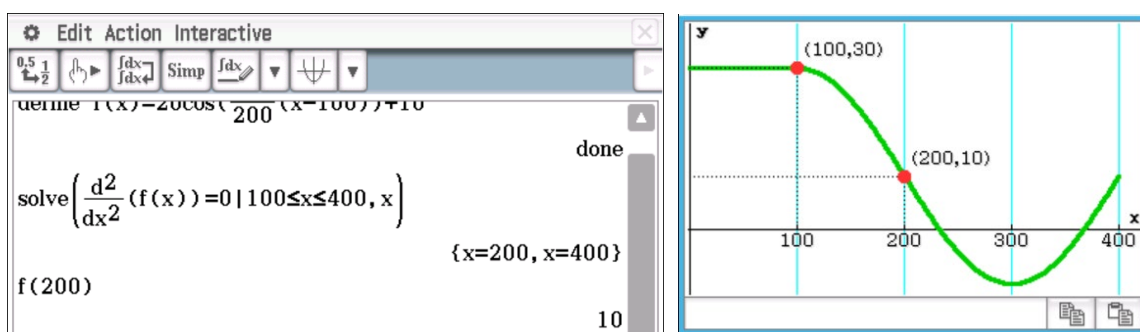
gives $x = 200$

(Note $f'(x)$ and $f''(x)$ do not exist at $x = 400$)

Point of inflection at $x = 200$ with concavity of graph changing either side of $x = 200$

Coordinates of point of inflection $(200, 10)$

1A



d.ii. Stationary points at $f'(x) = 0$

gives $x = 100, x = 300$

Strictly decreasing for $x \in [100, 300]$

1A

The screenshot shows a TI-84 Plus calculator interface. At the top, it says "Edit Action Interactive". Below that is a toolbar with various mathematical symbols. The main display area contains the following text:

```
define f(x)=20cos( $\frac{\pi}{200}(x-100)$ )+10
solve( $\frac{d}{dx}(f(x))=0 | 100 \leq x \leq 400, x$ )
{x=100, x=300}
```

The "done" button is visible on the right side of the screen.

e. Three supports of equal widths begin at $x = 100$ and end at $x = \frac{700}{3}$

$$\text{Width of each section} = \frac{\frac{700}{3} - 100}{3} = \frac{400}{9} = 44\frac{4}{9} \quad \mathbf{1A}$$

$$\text{Edges of sections: } x = 100, x = \frac{1300}{9}, x = \frac{1700}{9}, x = \frac{700}{3}$$

$$\text{Area of the cross-sections} = \frac{400}{2} \left(f(100) + f\left(\frac{700}{3}\right) + 2 \left(f\left(\frac{1300}{9}\right) + f\left(\frac{1700}{9}\right) \right) \right) \quad \mathbf{1M}$$

$$= \frac{400}{2} \left(f(100) + 2 \left(f\left(\frac{1300}{9}\right) + f\left(\frac{1700}{9}\right) \right) \right) \text{ as } f\left(\frac{700}{3}\right) = 0$$

$$= 2390.8 \text{ square metres correct to one decimal place}$$

1A

The screenshot shows a TI-84 Plus calculator interface. At the top, it says "Edit Action Interactive". Below that is a toolbar with various mathematical symbols. The main display area contains the following text:

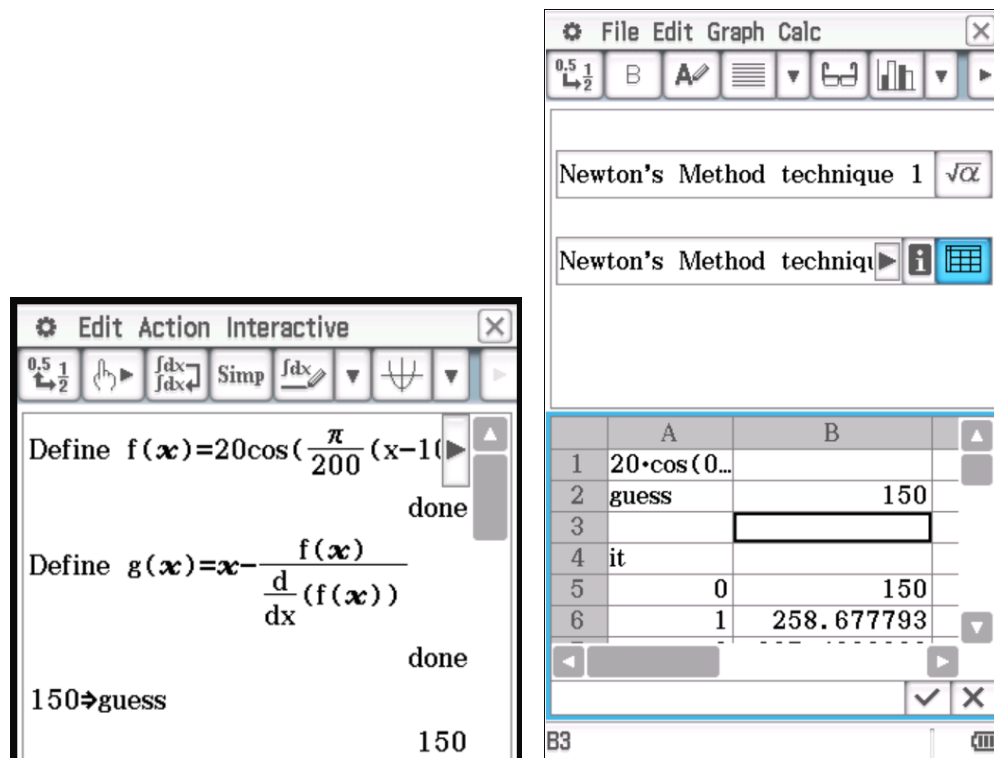
```
define f(x)=20*cos( $\frac{(x-100) \cdot \pi}{200}$ )+10
 $\frac{400}{18} (f(100) + f(\frac{700}{3}) + 2(f(\frac{1300}{9}) + f(\frac{1700}{9})))$ 
2390.837885
```

The "done" button is visible on the right side of the screen. At the bottom, there are tabs for "Alg", "Standard", "Real", and "Rad".

f.i. Newtons method, $x_0 = 150$

$$x_1 = 150 - \frac{f(150)}{f'(150)} \quad \mathbf{1M}$$

$$x_1 = 258.678 \quad \mathbf{1A}$$



f.ii. Newtons method, $x_0 = 150$

$x_3 = 233.1994015...$

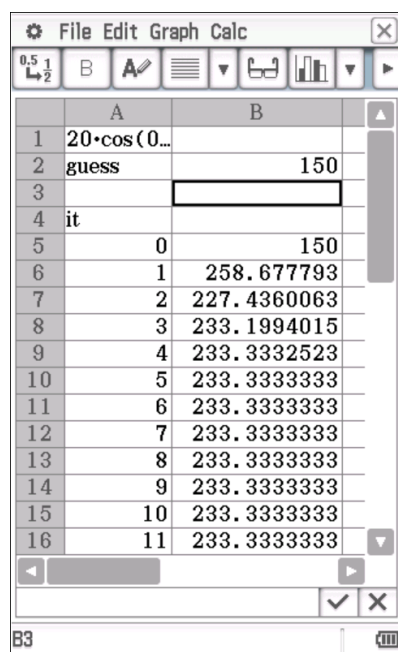
Solving $f(x) = 0$ gives x -intercepts of $x = \frac{700}{3}$ and $x = \frac{1100}{3}$

Near x -intercept $x = \frac{700}{3} = 233.333333.....$

Horizontal distance $233.333333... - 233.1994015... \quad \mathbf{1M}$

Distance = $0.133932...$

= 0.1339 metres correct to four decimal places $\quad \mathbf{1A}$



$$x - \frac{f(x)}{f'(x)} \Big|_{x=150} = 258.677793$$

$$x - \frac{f(x)}{f'(x)} \Big|_{x=ans} = 227.4360063$$

$$x - \frac{f(x)}{f'(x)} \Big|_{x=ans} = 233.1994015$$

$$\frac{700}{3} - 233.1994015 = 0.1339318333$$

Question 2

$$f(x) = \frac{p}{x-20} - 40 \text{ and } g(x) = \frac{q}{x+30} + 10$$

a. Graph of f has x -intercept at $x = 25$.

$$f(25) = 0$$

$$\text{Gives } 0 = \frac{p}{25-20} - 40 \Rightarrow \frac{p}{5} = 40$$

Shown $p = 200$. **1M**

b.i. Graph of g has a gradient of 1 at $x = -20$.

$$g'(x) = -\frac{q}{(x+30)^2}$$

$$g'(-20) = 1$$

$$\text{Gives } 1 = -\frac{q}{(-20+30)^2} \Rightarrow 1 = -\frac{q}{100}$$

Shown $q = -100$. **1M**

```

Edit Action Interactive
0.5 1/2  f dx  f dx  Simp  f dx  f dx
define f(x) = p/(x-20) - 40
done
define g(x) = q/(x+30) + 10
done
solve(f(25)=0, p)
{p=200}
solve(diff(g(x), x, 1, -20)=1, q)
{q=-100}
□

```

b.ii. Solve $g'(x) = 1$

$$x = -40, x = -20$$

Other value $x = -40$ **1A**

```

Edit Action Interactive
0.5 1/2  f dx  f dx  Simp  f dx  f dx
define f(x) = 200/(x-20) - 40
done
define g(x) = -100/(x+30) + 10
done
solve(d/dx(g(x))=1, x)
{x=-40, x=-20}

```

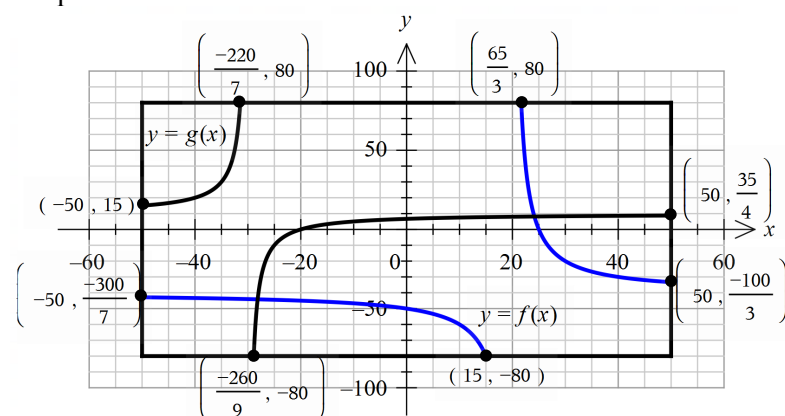

c.i. Points of intersection between f and g at $x = -2 \pm 6\sqrt{19}$
 $(-6\sqrt{19} - 2, -6\sqrt{19} - 18)$ and $(6\sqrt{19} - 2, 6\sqrt{19} - 18)$ **1A**

```

Edit Action Interactive
0.5 1/2 fdx fdx Simp fdx
define f(x) = 200 / (x-20) - 40
done
define g(x) = -100 / (x+30) + 10
done
solve(f(x)=g(x), x)
{x = -6*sqrt(19)-2, x = 6*sqrt(19)-2}
simplify(f(-6*sqrt(19)-2))
-6*sqrt(19)-18
simplify(f(6*sqrt(19)-2))
6*sqrt(19)-18

```

c.ii. Shape **1A**
 Endpoints **1A**



Solve $f(x) = -80$ gives $x = 15$

Solve $f(x) = 80$ gives $x = \frac{65}{3}$

$f(-50) = -\frac{300}{7}$, $f(50) = -\frac{100}{3}$

```

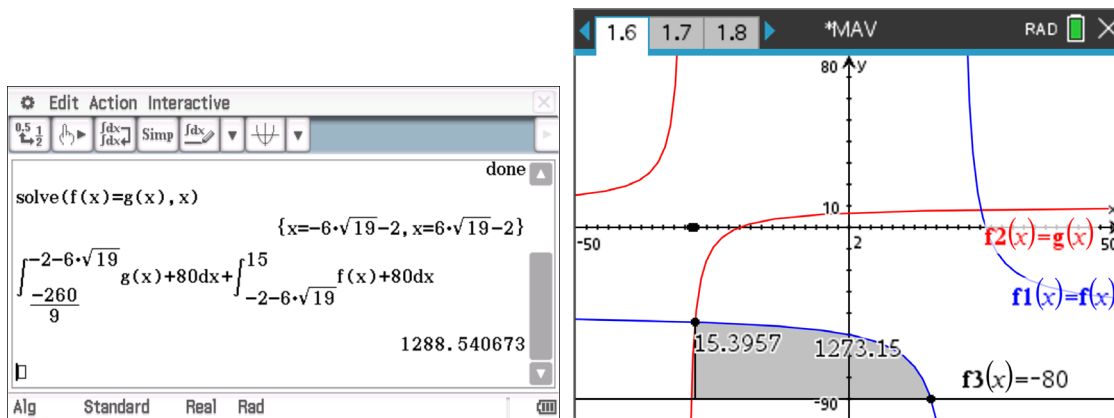
1.6 1.7 1.8 *MAV RAD
solve(f(x)=80,x) x = 65/3
solve(f(x)=-80,x) x = 15
f(50) -100/3
f(-50) -300/7

```

$$\mathbf{d.i.} \text{ Area} = \int_{\frac{-260}{9}}^{-2-6\sqrt{19}} (g(x) - (-80)) dx + \int_{-2-6\sqrt{19}}^{15} (f(x) - (-80)) dx \quad \mathbf{1A} \text{ correct terminals}$$

$$= \int_{\frac{-260}{9}}^{-2-6\sqrt{19}} (g(x) + 80) dx + \int_{-2-6\sqrt{19}}^{15} (f(x) + 80) dx \quad \mathbf{1A} \text{ correct functions}$$

d.ii. Area = 1289 sq km **1A**



d.iii. Area of the crop = 1288.54... sq km
 Area of farmland defined by the lines $x = \pm 50$ and $y = \pm 80$
 Area of the farm = $100 \times 160 = 16\,000$ sq km
 $\frac{1288.54...}{16\,000} \times 100\% = 8.05... \%$
 = 8% to the nearest percentage **1H**

e. Tangent to $f(x)$ at $x = 0$

$$y = -\frac{x}{2} - 50$$

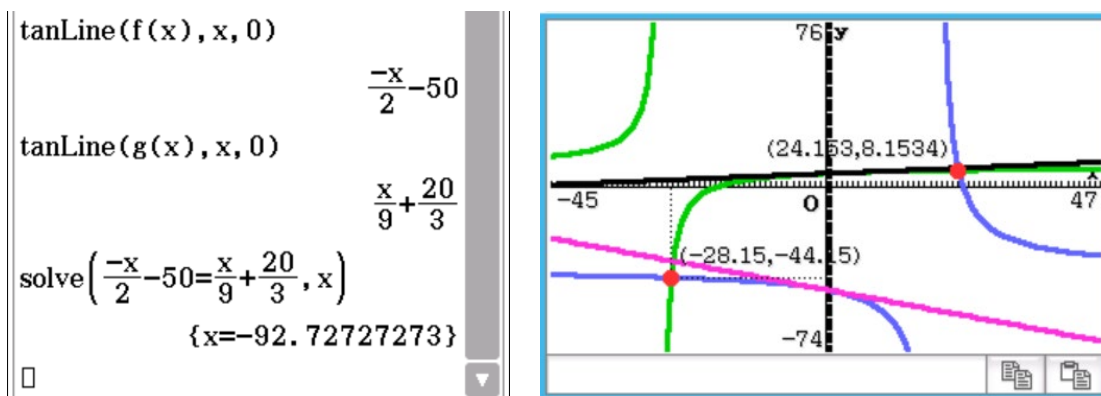
Tangent to $g(x)$ at $x = 0$

$$y = \frac{x}{9} + \frac{20}{3}$$

Equate tangents give point of intersection at $x = -92.7272... \quad \mathbf{1M}$

Outside the of domain of $x \in [-50, 50]$

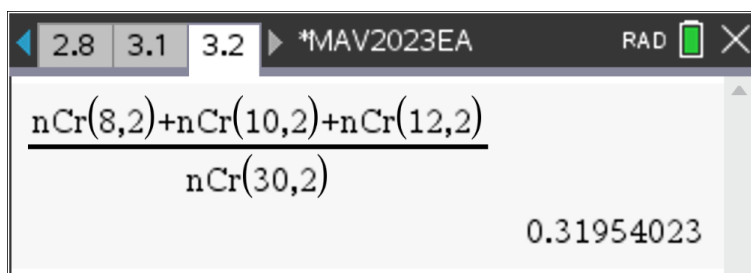
Water pipes do not meet on his property **1A**

**Question 3**

a. $\text{Pr}(2 \text{ black socks}) + \text{Pr}(2 \text{ green socks}) + \text{Pr}(2 \text{ blue socks})$

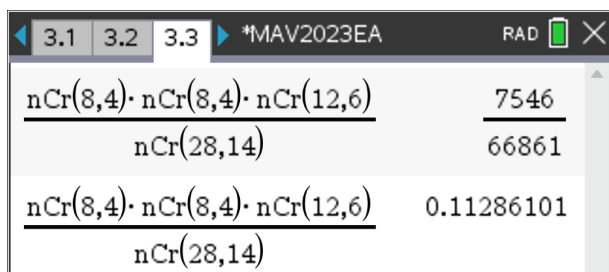
$$= \frac{\binom{8}{2} + \binom{10}{2} + \binom{12}{2}}{\binom{30}{2}} \quad \mathbf{1M}$$

$$= 0.3195 \quad \mathbf{1A}$$



$$\mathbf{b.} \frac{\binom{8}{4} \binom{8}{4} \binom{12}{6}}{\binom{28}{14}}$$

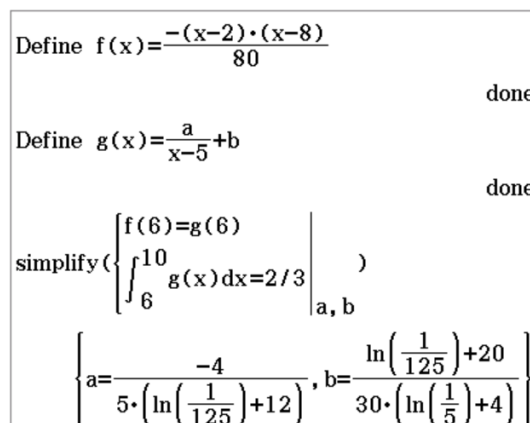
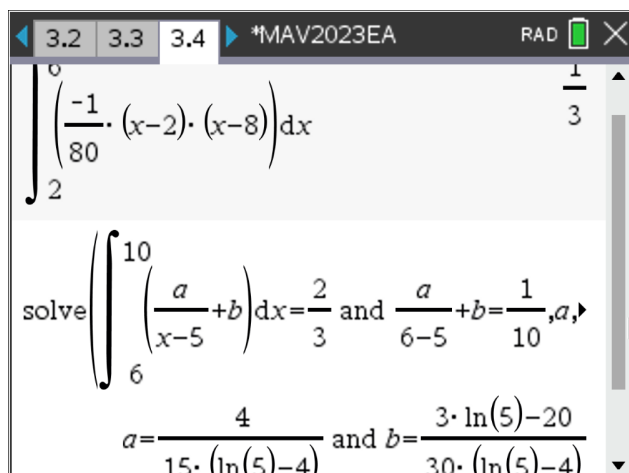
$$= 0.1129 \quad \mathbf{1A}$$



c. $\int_2^6 \left(-\frac{1}{80}(x-2)(x-8) \right) dx = \frac{1}{3}, -\frac{1}{80}(6-2)(6-8) = \frac{1}{10}$

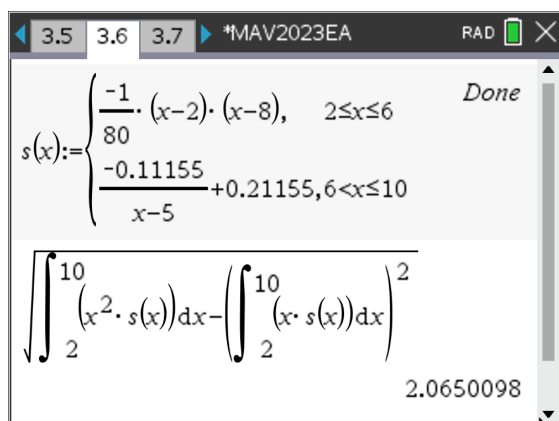
Solve $\int_6^{10} \left(\frac{a}{x-5} + b \right) dx = \frac{2}{3}$ and $\frac{a}{6-5} + b = \frac{1}{10}$ for a and b **1M**

$$a = \frac{4}{15(\log_e(5)-4)} = \frac{-4}{5\left(\log_e\left(\frac{1}{125}\right)+12\right)} \text{ and } b = \frac{3\log_e(5)-20}{30(\log_e(5)-4)} = \frac{\log_e\left(\frac{1}{125}\right)+20}{30\left(\log_e\left(\frac{1}{5}\right)+4\right)} \quad \mathbf{1A}$$



d.i. $\text{sd}(X) = \sqrt{\int_2^{10} (x^2 \times s(x)) dx - \left(\int_2^{10} (x \times s(x)) dx \right)^2}$ **1M**

= 2.065 **1A**

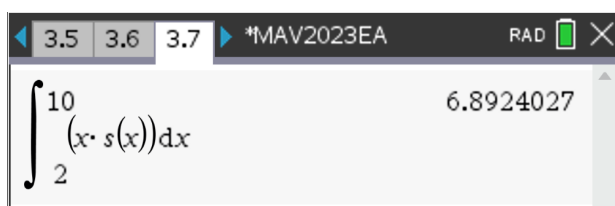
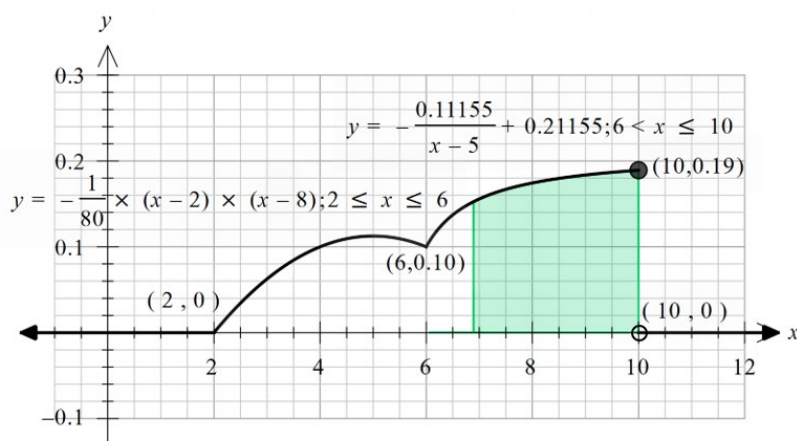


d.ii. Coordinates with open and closed circles **1A**

Shape (must draw along axis) **1A**

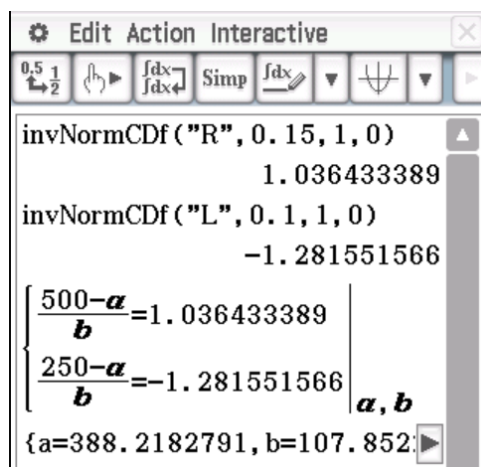
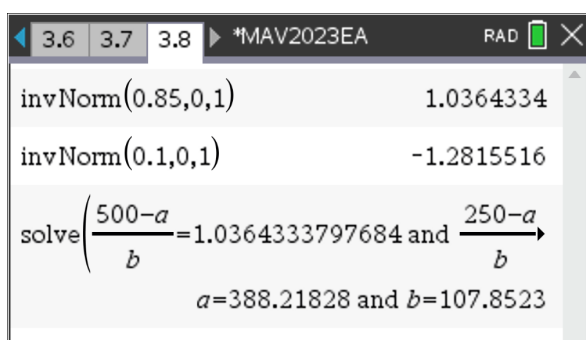
Shading $E(X) \approx 6.892$ **1H**

There is no need to show the equations.



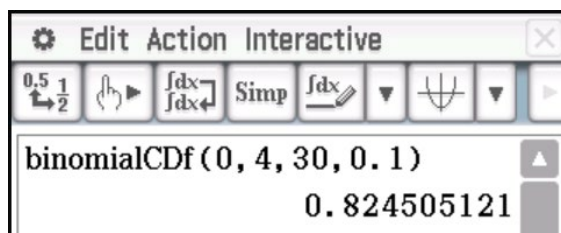
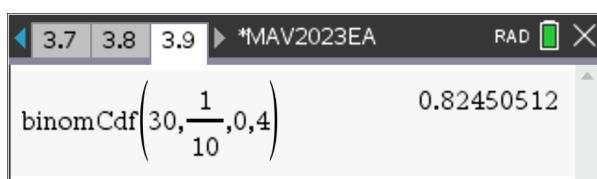
e. Solve $\frac{500-\mu}{\sigma} = 1.036\dots$ and $\frac{250-\mu}{\sigma} = -1.281\dots$ **1M**

$\mu = 388$ mm and $\sigma = 108$ mm **1A**



f. $L \sim \text{Bi}\left(30, \frac{1}{10}\right)$ **1M**

$\Pr(L < 5) = 0.8245$ **1A**



g. (0.1385, 0.3615) **1A**

0.04 is outside the confidence interval. If Maya did 100 such samples, she would expect 99 of the confidence intervals to contain p . It is highly likely the farmer is incorrect but further statistical testing needs to be carried out. **1M**

zInterval_1Prop 25,100,0.99: stat.results	
"Title"	"1-Prop z Interval"
"CLower"	0.13846332
"CUpper"	0.36153668
"p̂"	0.25
"ME"	0.11153668
"n"	100.

Question 4

a. $g: (-\infty, \infty) \rightarrow R, g(x) = \log_e(x^2 + px + 2)$

$$\Delta = p^2 - 8 < 0 \quad \mathbf{1M}$$

$$-2\sqrt{2} < p < 2\sqrt{2} \quad \mathbf{1A}$$

solve(p ² -8<0,p)	
	$-2 \cdot \sqrt{2} < p < 2 \cdot \sqrt{2}$

b. g^{-1} will not exist, as g will be a 'many to one function' or g 'fails the horizontal line test' or 'there exist two x -values for some y -values'. **1A**

c. Solve $\log_e(x^2 + 2) = 4$

$$a = -\sqrt{e^4 - 2} \text{ and } b = \sqrt{e^4 - 2} \quad \mathbf{1A}$$

solve(ln(x ² +2)=4,x)	
	$x = -\sqrt{e^4 - 2} \text{ or } x = \sqrt{e^4 - 2}$

d. h is symmetrical about the y -axis.

$$\text{Average value} = \frac{1}{\sqrt{e^4 - 2}} \int_0^{\sqrt{e^4 - 2}} (\log_e(x^2 + 2)) dx = 2.537... \quad \mathbf{1H}$$

Sketch the graphs of $y = 2.537...$ and h and find the bounded area.

$$\text{Area} = 6.480... \quad \mathbf{1M}$$

$$\text{Volume} = 2 \times 6.480...$$

$$= 12.96 \text{ dm}^3 \quad \mathbf{1A}$$

OR

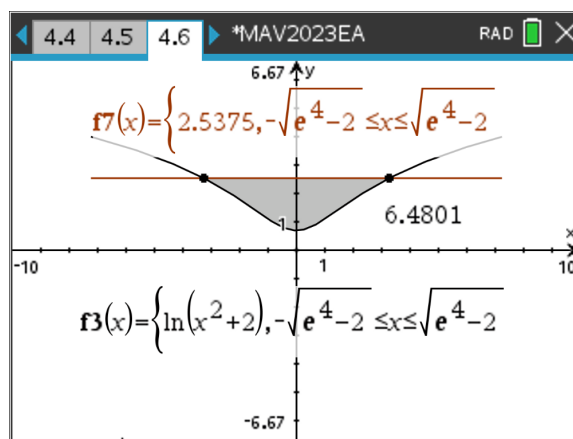
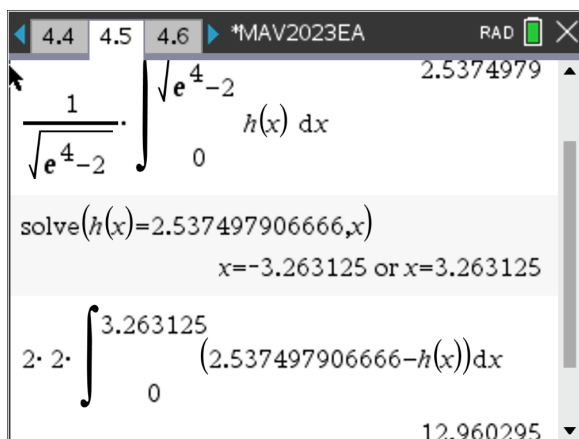
Solve $h(x) = 2.537\dots$ for x .

$x = -3.263\dots$ or $x = 3.263\dots$

Volume = length \times area bounded by $y = 2.537\dots$ and $h(x)$

$$\text{Volume} = 2 \times 2 \times \int_0^{3.263\dots} (2.537\dots - h(x)) dx \quad \mathbf{1M}$$

$$= 12.96 \text{ dm}^3 \quad \mathbf{1A}$$

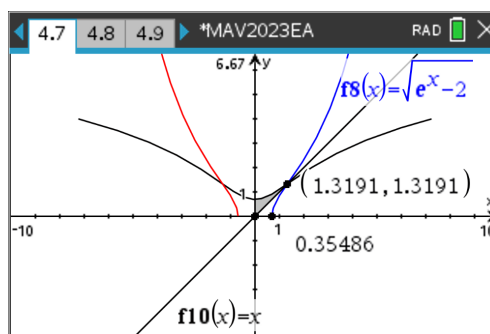
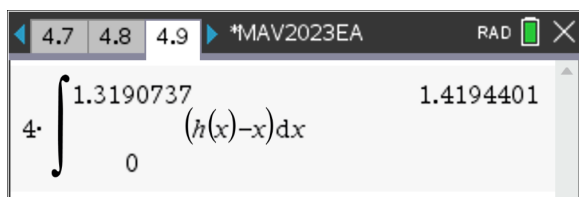


e. Sketch h and $y = x$ and find the bounded area between the y -axis, h and $y = x$. **1M**

$$\int_0^{1.319\dots} (h(x) - x) dx = 0.3548\dots \quad \mathbf{1A}$$

$$4 \int_0^{1.319\dots} (h(x) - x) dx$$

$$= 1.42 \text{ dm}^2 \quad \mathbf{1H}$$



OR

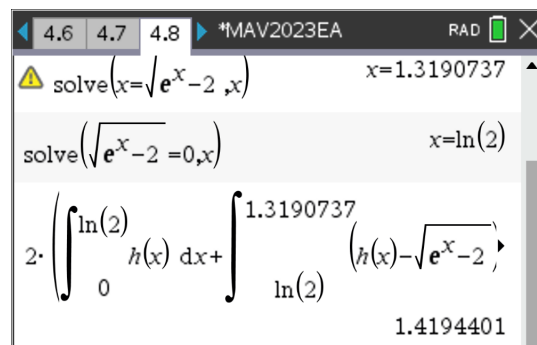
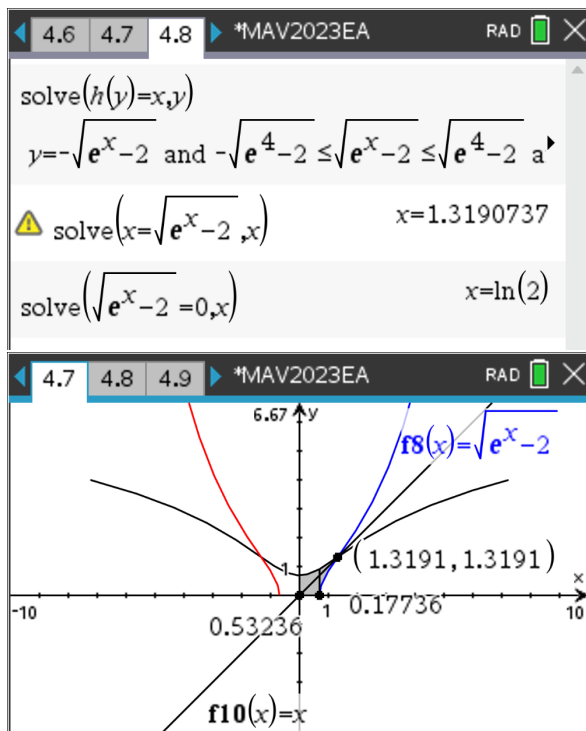
The equation of RHS branch is $y = \sqrt{e^x - 2}$ (part of the inverse relation of h) **1A**

Solve $\sqrt{e^x - 2} = 0$, $x = \log_e(2)$

Solve $\sqrt{e^x - 2} = x = h(x)$, $x = 1.319\dots$

$$2 \left(\int_0^{\log_e(2)} (h(x)) dx + \int_{\log_e(2)}^{1.319\dots} (h(x) - \sqrt{e^x - 2}) dx \right) \quad \mathbf{1M}$$

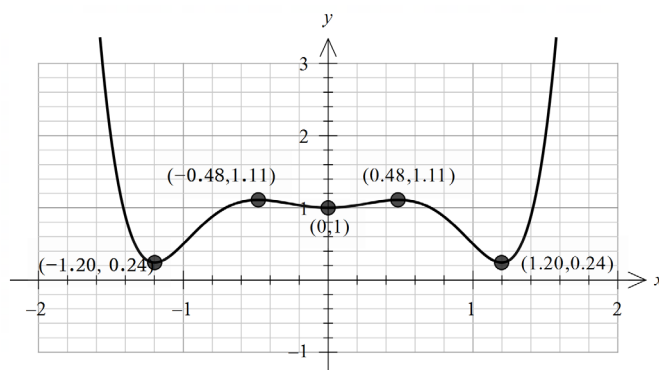
= 1.42 dm² **1A**



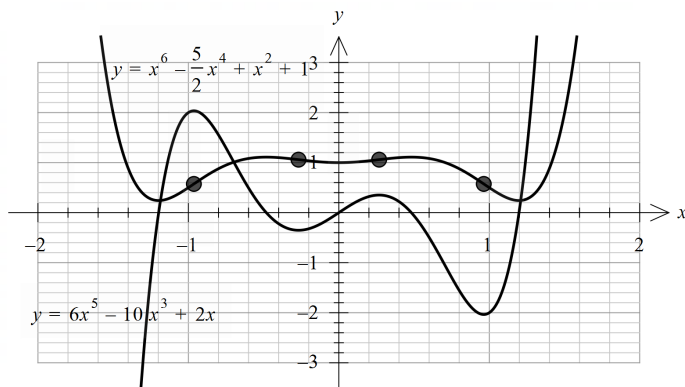
Question 5

$f : R \rightarrow R, f(x) = x^6 - \frac{5}{2}x^4 + x^2 + 1$

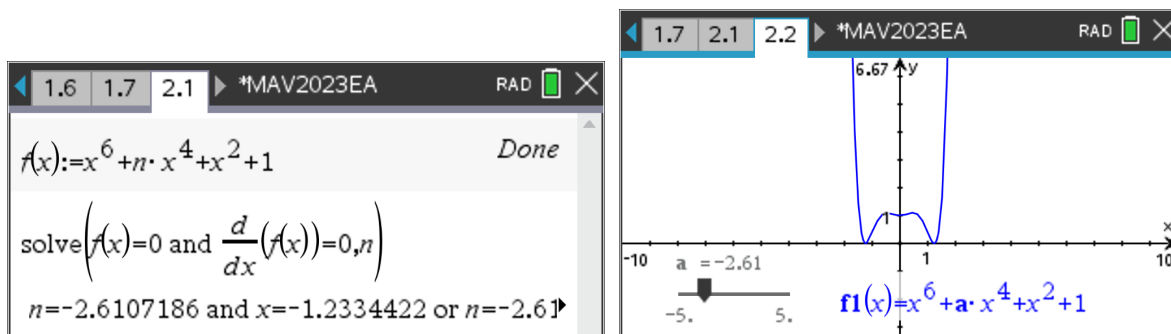
a. Points labelled correctly. **1A**



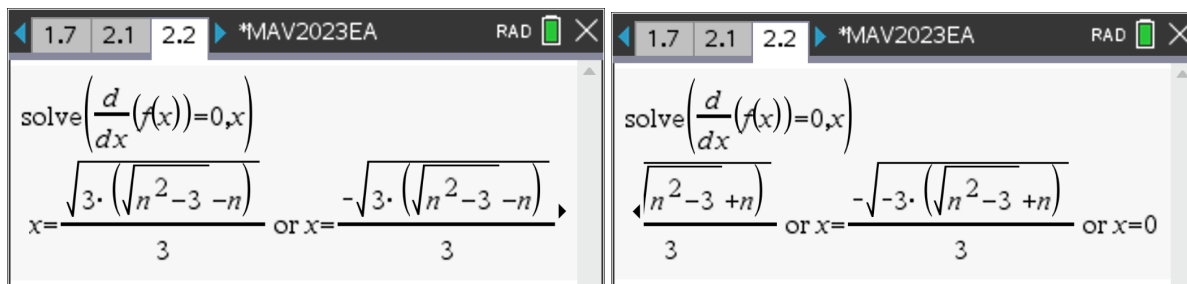
b. 4 **1A**



c. Solve $f(x) = 0$ and $f'(x) = 0$ **1M
 $a \geq -2.611$ **1A****



d. $x = \frac{\pm\sqrt{3(\sqrt{a^2-3}-a)}}{3}$, $x = \frac{\pm\sqrt{-3(\sqrt{a^2-3}+a)}}{3}$, $x = 0$ **1A**



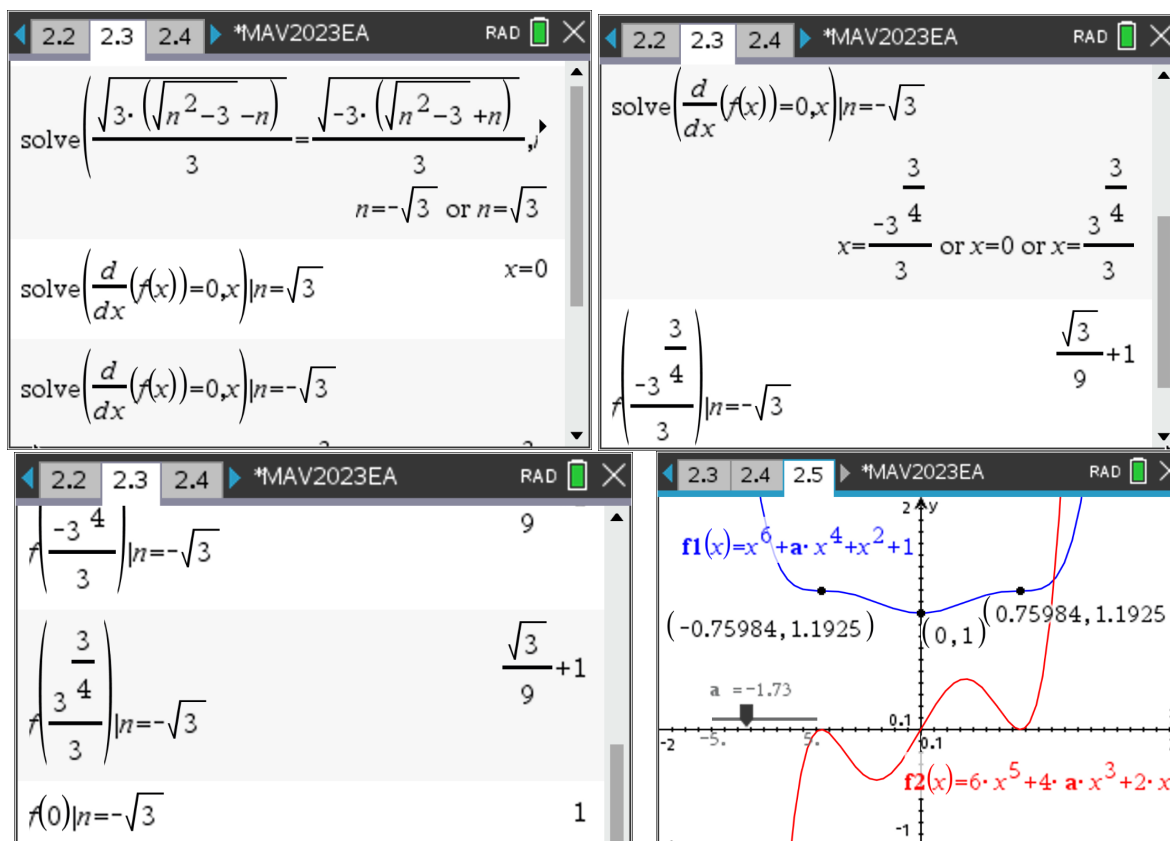
e. Solve $\frac{\sqrt{3(\sqrt{a^2-3}-a)}}{3} = \frac{\sqrt{-3(\sqrt{a^2-3}+a)}}{3}$ for a

$a = -\sqrt{3}$ **1A**

$\left(-3^{\frac{1}{4}}, \frac{\sqrt{3}}{9} + 1\right)$ stationary point of inflection

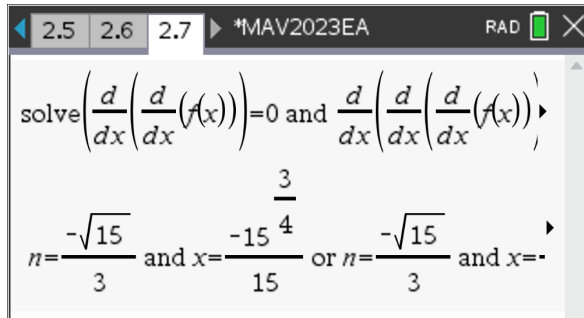
$\left(3^{\frac{1}{4}}, \frac{\sqrt{3}}{9} + 1\right)$ stationary point of inflection **1A both**

$(0,1)$ local minimum **1A**



f. Solve $f''(x) = 0$ and $f'''(x) = 0$ for a . **1M**

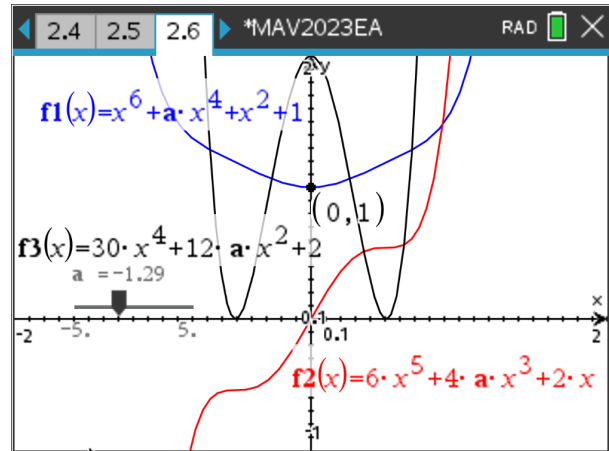
$$a \geq \frac{-\sqrt{15}}{3} \quad \mathbf{1A}$$



2.5 2.6 2.7 *MAV2023EA RAD

$$\text{solve}\left(\frac{d}{dx}\left(\frac{d}{dx}(f(x))\right)\right)=0 \text{ and } \frac{d}{dx}\left(\frac{d}{dx}\left(\frac{d}{dx}(f(x))\right)\right)$$

$$n = \frac{-\sqrt{15}}{3} \text{ and } x = \frac{-15}{15} \text{ or } n = \frac{-\sqrt{15}}{3} \text{ and } x = \frac{3}{15}$$



END OF SOLUTIONS