2023 VCE Mathematical Methods Year 12 Trial Examination 2

Kilbaha Education

Quality educational content

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Victorian Certificate of Education
2023

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS
Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 33 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions, and sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Mark will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Which of the following graphs has a period of $\frac{1}{c}$, where $c \in R^+$

A. $y = \sin\left(\frac{\pi x}{c}\right)$

B. $y = \sin\left(\frac{2\pi x}{c}\right)$

C. $y = \sin(2\pi c x)$

D. $y = \tan\left(\frac{\pi x}{c}\right)$

E. $y = \tan(2\pi c x)$

Question 2

For the graph of $y = a + \frac{b}{\sqrt{x-c}}$ where $a, b, c \in R \setminus \{0\}$, which of the following is **false**?

A. The line $y = a$ is a horizontal asymptote.

B. The line $x = c$ is a vertical asymptote.

C. If $c > 0$ the graph does not cross the y-axis, if $c < 0$, the graph crosses the y-axis at the point $\left(0, a + \frac{b}{\sqrt{-c}}\right)$.

D. The maximal domain is $[c, \infty)$.

E. If $b > 0$ the range is (a, ∞) and if $b < 0$ the range is $(-\infty, a)$.

Question 3

A certain curve has its gradient given by $4\sin\left(\frac{x}{2}\right)$. If the curve crosses the *x*-axis at the origin, then the equation of the curve could be

A. $y=-8\cos\left(\frac{x}{2}\right)$

B. $y=8\left(1-\cos\left(\frac{x}{2}\right)\right)$

C. $y=-2\cos\left(\frac{x}{2}\right)$

D. $y=4\left(1-\cos\left(\frac{x}{2}\right)\right)$

E. $y=4\left(1+\cos\left(\frac{x}{2}\right)\right)$

Question 4

Several students were considering the functions $f(x)=2\log_e(x-a)$ and $g(x)=\log_e(x-a)^2$, defined on their maximal domains where $a \in R$.

Alan stated since by logarithms laws, $2\log_e(x-a)=\log_e(x-a)^2$ therefore the graphs of the two functions are the same.

Ben stated since by logarithms laws, $2\log_e(x-a)=\log_e(x-a)^2$ therefore the graphs of the two functions have the same maximal domain.

Colin stated that both graphs have the line $x=a$ as a vertical asymptote and the same range.

David stated the graphs are different and have different maximal domains.

Then

A. Only Alan is correct.

B. Only Ben is correct.

C. Only Colin is correct.

D. Both Alan and Ben are correct.

E. Both Colin and David are correct.

Question 5

The function $f(x)=4x^3-6x^2$ has its gradient decreasing for

A. $x\in\left(\frac{3}{2},\infty\right)$

B. $x\in(0,1)$

C. $x\in\left(0,\frac{1}{2}\right)$

D. $x\in(-\infty,0)\cup(1,\infty)$

E. $x\in\left(-\infty,\frac{1}{2}\right)$

Question 6

The approximate area bounded the curve $f(x)=\sqrt{2x+3}$, the coordinate axes, and the line $x=3$ using the trapezoidal rule with three equally spaced strips is

A. $\frac{1}{2}\left(\sqrt{3}+2\left(\sqrt{5}+\sqrt{7}\right)+3\right)$

B. $\frac{1}{2}\left(\sqrt{3}+\sqrt{5}+\sqrt{7}+3\right)$

C. $\sqrt{3}+\sqrt{5}+\sqrt{7}+\frac{3}{2}$

D. $\sqrt{3}+2\left(\sqrt{5}+\sqrt{7}\right)+3$

E. $9-\sqrt{3}$

Question 7

A box contains r red marbles and b blue marbles, where $r, b\in Z^+$ and $r>3$ and $b>3$.

Jack draws three marbles from the box, $\frac{3br^2}{(b+r)^3}$ represents the probability of drawing

A. one blue and two red marbles with replacement.

B. one blue and two red marbles without replacement.

C. two blue and one red marbles with replacement.

D. two blue and one red marbles without replacement.

E. three red marbles without replacement.

Question 8

The time taken to answer a question in minutes is assumed to be an independent random variable X with an exponential distribution that has the probability density function given by

$f(x)=\begin{cases}\lambda e^{-\lambda x} & x\geq 0\\ 0 & \text{otherwise}\end{cases}$ where λ is a positive constant.

Given that the mean time is two minutes, then the probability that the next question is answered in a time greater than the mean time is closest to

A. 0.018

B. 0.221

C. 0.368

D. 0.5

E. 0.779

Question 9

Approximately one in every 25 people in Australia are members of an AFL football team.

A C% confidence interval for the proportion of people who are not members of an AFL football team is $(0.889, 1.031)$, then C is closest to

A. 90

B. 91

C. 93

D. 95

E. 99

Question 10

If X is a binomial random variable where $n=15$ and $p=0.6$, then the probability that X exceeds the mean value given that it at least exceeds the value of the variance is closest to

A. 0.403

B. 0.404

C. 0.610

D. 0.611

E. 0.788

Question 11

Several students were considering the function $f: R \to R, \quad f(x) = 3\cos(2x) - 4$

Mia stated that the graph of the function crosses the *x*-axis an infinite number of times.

Jack stated that the graph has an infinite number of maximum turning points and also an infinite number of minimum turning points.

Zach stated that the graph has an infinite number of points of inflexion.

Then

A. Mia, Jack and Zach are all correct.

B. Both Jack and Zach are correct, Mia is incorrect.

C. Only Mia is correct.

D. Only Jack is correct.

E. Only Zach is correct

Question 12

The maximal domain of the function $f(x) = \log_e\left(\sqrt{b - \sqrt{x-a}}\right)$ where $a \in R$ and $b \in R^+$ is

A. $\left[a, a+b^2\right)$

B. $\left[a, a+b^2\right]$

C. $\left(a, a+b^2\right)$

D. $\left(a, a+b^2\right]$

E. $\left[a, b^2\right)$

Question 13

The widths of A4 pieces of paper are normally distributed with a mean of 148.5 mm and a standard deviation of 0.2 mm. In a ream of 500 sheets of paper, how many would be expected to not be within 0.1 mm of the mean width?

A. 154

B. 191

C. 192

D. 308

E. 309

Question 14

Which of the following does **not** correctly describe the general solution to the system of linear equations

$$x - y + z = 1$$
$$x + 2y - z = 3$$

A. $x = k,\ y = 4 - 2k,\ z = 5 - 3k,$ for all $k \in R$

B. $x = 2k,\ y = 4 - 4k,\ z = 5 - 6k,$ for all $k \in R$

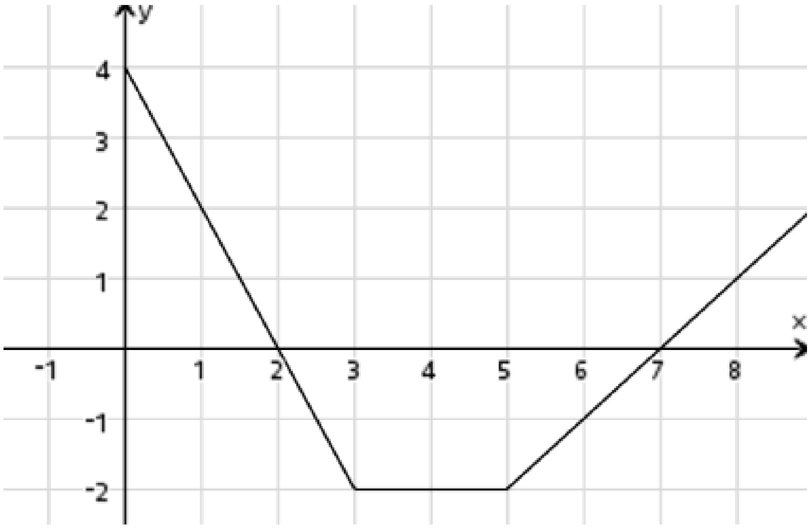
C. $x = \frac{1}{2}(4 - k),\ y = k,\ z = \frac{1}{2}(3k - 2),$ for all $k \in R$

D. $x = 2 - k,\ y = 2k,\ z = -1 + 3k,$ for all $k \in R$

E. $x = \frac{1}{3}(5 - k),\ y = \frac{1}{3}(3 + k),\ z = k,$ for all $k \in R$

Question 15

The graph below shows part of a hybrid function, the average value of the function over the interval $[0,8]$ is equal to



A. $-\frac{5}{16}$

B. $\frac{11}{16}$

C. $\frac{23}{16}$

D. $\frac{27}{16}$

E. $\frac{33}{16}$

Question 16

If $\log_3(y) = 4\log_2(x) + 1$ then

A. $y = 2x^4$

B. $y = 3x^4$

C. $y = \frac{3}{2}\log_3(2x^4)$

D. $y = 3x^{4\log_2(3)}$

E. $y = 3x^{\log_e(6)}$

Question 17

Given the system of linear simultaneous equations

$$\begin{aligned} ax - 4y &= 2a \\ -6x + (a+5)y &= b \end{aligned}$$

Which of the following is **false**?

A. When $a = -6$ and $b = -12$ there is a unique solution.

B. When $a \in R \setminus \{-8, 3\}$ and $b \in R$ there is an infinite number of solutions.

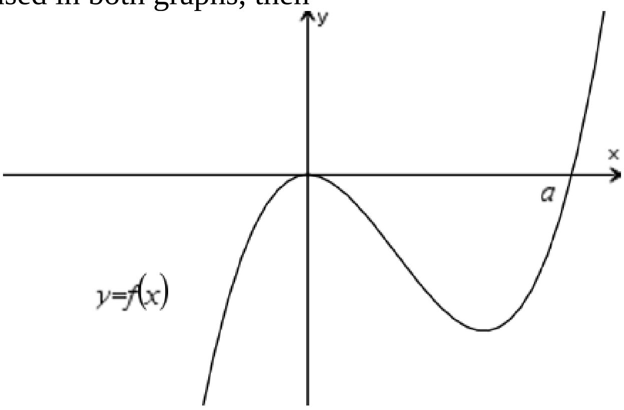
C. When $a = -8$ and $b = -12$ there is an infinite number of solutions.

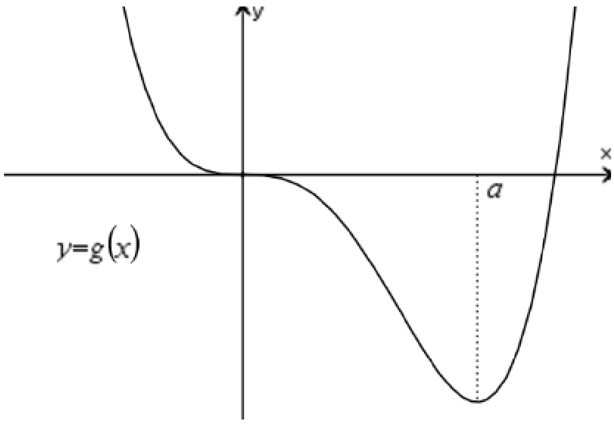
D. When $a = 3$ and $b = 4$ there is no solution.

E. When $a = 3$ and $b \neq -12$ there is no solution.

Question 18

The diagrams below shows the graphs of two functions $y = f(x)$ and $y = g(x)$, the same scale is used in both graphs, then





A. $f(x) = g^{-1}(x)$

B. $f^{-1}(x) = \frac{1}{g(x)}$

C. $f(x) = \int g(x)\,dx$

D. $g(x) = \int f(x)\,dx$

E. $f(x).g(x) = 1$

Question 19

The algorithm below, described in pseudocode, solves the equation $f(x)=0$ using the bisection method, with a tolerance and having a maximum number of iterations.

Inputs: $f(x)$, the function to solve equal to zero

xleft, the initial left estimate of the *x*-intercept of $f(x)$,

xright, the initial right estimate of the *x*-intercept of $f(x)$,

Constants: epsilon, the tolerance

maxiter, the maximum number of iterations

Define bisection$(f(x), \text{xleft}, \text{xright})$

If $f(\text{xleft}).f(\text{xright})>0$ **Then**

Print "Starting values incorrect, will not converge"

Print "Require $f(\text{xleft}).f(\text{xright})<0$"

Stop

EndIf

epsilon $\leftarrow 0.00001$

maxiter $\leftarrow 100$

$i \leftarrow 0$

While $-\text{epsilon} < \text{xleft} - \text{xright} < \text{epsilon}$ **Do**

$i \leftarrow i+1$

xmid $\leftarrow \dfrac{\text{xleft}+\text{xright}}{2}$

If $i \ge$ maxiter **Then**

Print "Did not converge after ",maxiter, " iterations"

Print "Maximum number of iterations exceeded"

Stop

EndIf

EndWhile

Return xmid

Print "Root converges to ", xmid

Print "After ",*i*, "iterations "

Which one of the following options would be most appropriate to fill the empty box?

A.

If $f(\text{xleft}).f(\text{xmid})<0$ **Then**
    $\text{xleft} \leftarrow \text{xmid}$
Else
    $\text{xmid} \leftarrow \text{xleft}$
EndIf

B.

If $f(\text{xleft}).f(\text{xmid})<0$ **Then**
    $\text{xright} \leftarrow \text{xmid}$
Else
    $\text{xmid} \leftarrow \text{xright}$
EndIf

C.

If $f(\text{xleft}).f(\text{xmid})<0$ **Then**
    $\text{xleft} \leftarrow \text{xmid}$
Else
    $\text{xright} \leftarrow \text{xmid}$
EndIf

D.

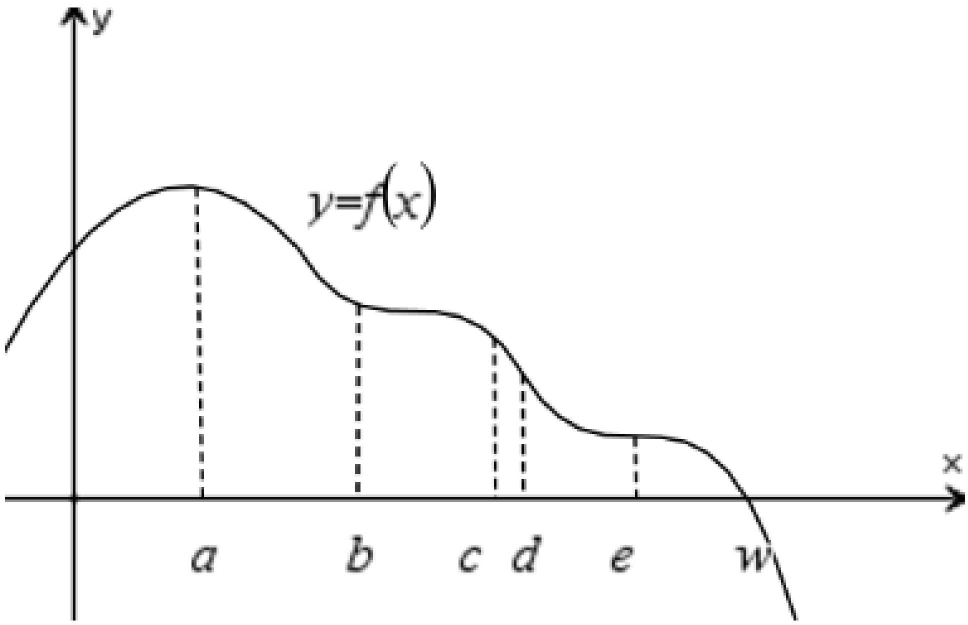
If $f(\text{xleft}).f(\text{xmid})<0$ **Then**
    $\text{xright} \leftarrow \text{xmid}$
Else
    $\text{xleft} \leftarrow \text{xmid}$
EndIf

E.

If $f(\text{xleft}).f(\text{xmid})<0$ **Then**
    $\text{xmid} \leftarrow \text{xleft}$
Else
    $\text{xmid} \leftarrow \text{xright}$
EndIf

Question 20

The diagram below, shows the graph of the function $y = f(x)$, the graph crosses the x-axis at $x = w$ as shown.



Newton's method is used to solve the equation $f(x) = 0$, with an initial starting value of x_0.

Which initial starting value will give the next approximation x_1 which is closest to solution $x = w$?

A. $x_0 = a$

B. $x_0 = b$

C. $x_0 = c$

D. $x_0 = d$

E. $x_0 = e$

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact answer must be given unless otherwise specified. In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (11 marks)

Consider the graph of the function $f : R \to R,\quad f(x) = -x^3 + bx^2 + cx$, where $b, c \in R$.

a. Determine the values of b and c when the graph of $y = f(x)$ has a turning point at $(3, 0)$.

2 marks

b. Determine the values of b and c when the graph of $y = f(x)$ has a turning point at $x = 4$ and $f(x) \geq 0$ for $x \in [0, 6]$ and the area bounded by the curve, the x-axis, the origin and the line $x = 6$ is 144 square units.

2 marks

c. Determine the values of b and c when the tangent to the graph of $y = f(x)$ at the point where $x = 2$ passes through the origin and is parallel to the line $y = 2x - 5$.

2 marks

d. Determine the values of b and c when the graph of $y = f(x)$ has a turning point at $x = 3$ and $f(x) \geq 0$ for $x \in [0, 4]$ and the area bounded by the curve, the x-axis, the origin and the line $x = 4$ when approximated by the trapezium rule with 4 equally spaced strips is 44 square units.

2 marks

e. Determine the values of b and c when the graph of $y = f(x)$ has a stationary point of inflexion at $x = 2$, and determine the y co-ordinate of this point of inflexion.

2 marks

f. Determine a range of values of b and c when the graph of $y = f(x)$ crosses the x-axis once and touches the x-axis at another point.

1 mark

Question 2 (12 marks)

A team of designers are working on the shape for a new slide. The distance *x* in metres represents the horizontal distance from the start of the ride and *y* is the vertical height in metres above ground level. When riders start and finish the ride they are horizontal and when they exit the ride, they are 9 metres horizontally from the start of the ride.

a. The first proposal for the design of the slide, design A is one cubic model, based on the function $g:[0,9]\to R,\ g(x)=px^3+qx^2+s$

For this design the riders start at a point $P(0,6)$ and finish at the point $D(9,0)$ and at both the start and finish of the ride the riders are horizontal.

i. Show that $p=\frac{4}{243},\ q=-\frac{2}{9}$ and $s=6$.

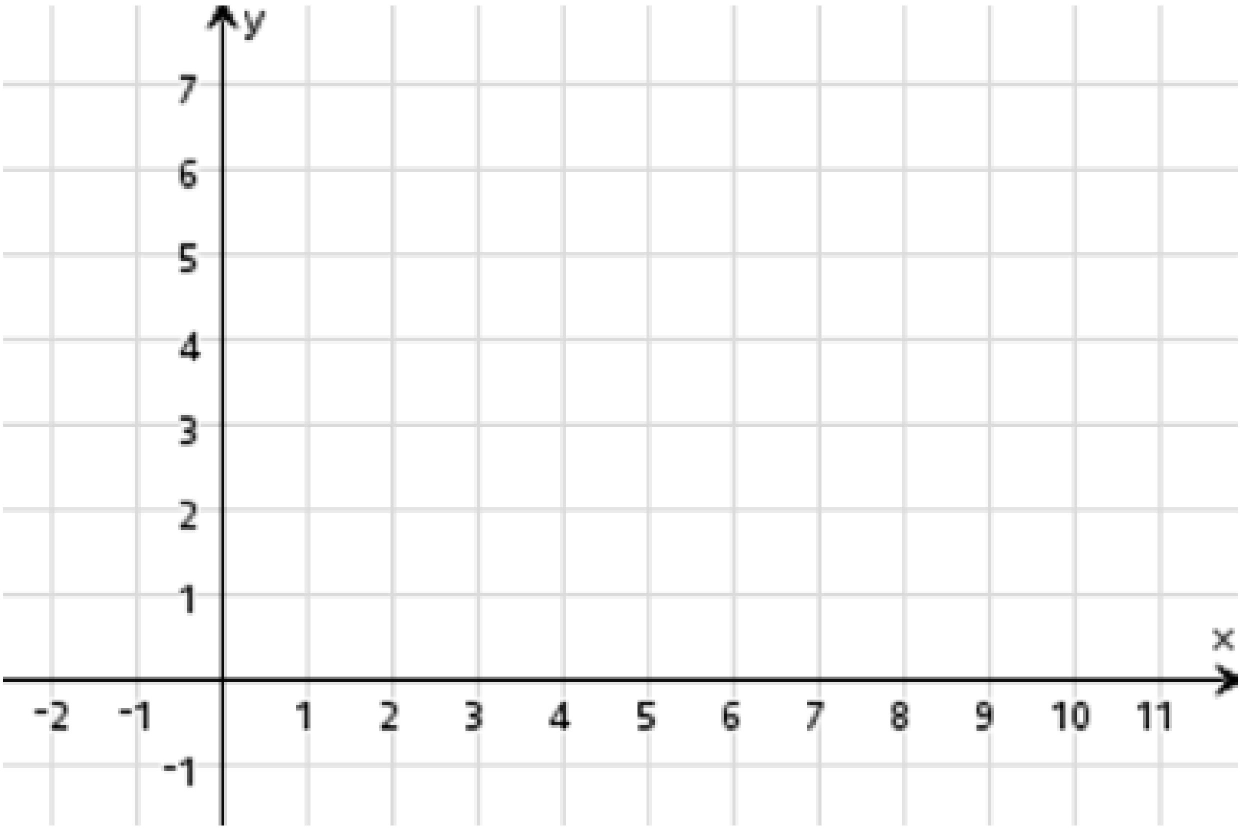
3 marks

ii. Determine the coordinates on the slide where the slope is the steepest.

2 marks

iii. Sketch the graph of the function $y = g\left(x\right)$ on the axes below.

1 mark



iv. Determine the average height of the ride.

1 mark

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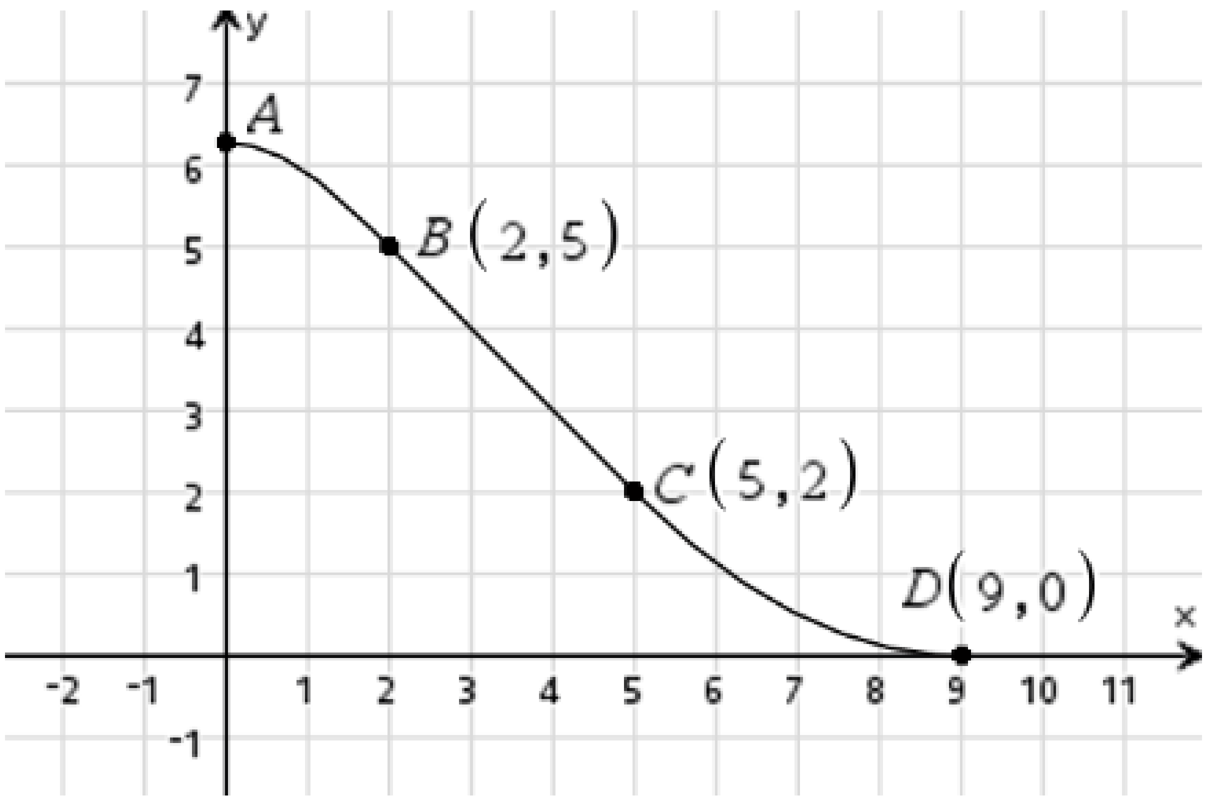
Another proposal for the design of the slide, design B is that the ride be comprised of a hybrid function consisting of three sections, as shown below.

The ride starts at the point *A*, which is slightly higher than the point *P* on the *y*-axis and passes through the points $B(2,5)$ and $C(5,2)$ and finishes at the point $D(9,0)$.

Each section is defined by its horizontal distance from the start of the ride. The section *AB* is defined for $x \in [0,2]$ and is part of a trigonometric curve, the section *BC* is defined for $x \in [2,5]$ and is part of a straight line, while the section *CD* is defined for $x \in [5,9]$ and is part of a quadratic curve. The curves are all continuous and the joins at the points *B* and *C* are both smooth.
The ride is defined by the graph of $y = f(x)$, where

$$y = f(x) = \begin{cases} R\cos(nx) + 5 & \text{for} \quad 0 \le x < 2 \\ mx + k & \text{for} \quad 2 \le x < 5 \\ ax^2 + bx + c & \text{for} \quad 5 \le x \le 9 \end{cases}$$



b. Determine the values of R, n, m, k, a, b and c.

4 marks

c. Which of the rides A or B has the steepest slope? Justify your answer.

1 mark

Question 3 (14 marks)

There are 4 different blood types A, B, AB and O. These names indicate whether the blood's red cells carry the A antigen, the B antigen, both A and B antigens, or neither antigen for type O. Each of the 4 blood groups can be classified as either Rhesus positive or Rhesus negative. In Australia the following table gives the percentages of people having these blood types.

Blood type	A	B	AB	O
positive	31	8	2	40
negative	7	2	1	9

a. An Australian person is selected at random, determine the probability that they have a blood type A, if it is known they have a positive Rhesus.

1 mark

b. In a random sample of 20 Australian people, determine the probability that more than 10% have a blood type B. Give your answer correct to four decimal places.

1 mark

c. At a Australian blood bank, people donate blood, the time required to donate one pint (or approximately 473 mls of blood) follows a normal distribution. It is found that 31% of donations exceed 12 minutes, while 16% of donations take less than 9 minutes. Determine the mean and standard deviation times to donate blood, giving your answers correct to one decimal place.

3 marks

It is found that in a life-time, only one third of Australians donate blood, yet 30% will need a blood transfusion at some time in their lives.

Let $\hat{P}$ represent the random variable that represents the proportion of Australians, who donate blood.

d.i. From a random sample of 50 Australians who donate blood, find the probability that $\hat{P}$ is greater than 30%. Give your answer correct to three decimal places.

2 marks

ii. From a random sample of 30 Australians who donate blood, find a 95% confidence interval for $\hat{P}$, giving your answers correct to three decimal places.

1 mark

e. Let $\hat{Q}_n$ represent the random variable that represents the proportion of *n* Australians, who will need a blood transfusion at some time in their lifetime. Find the least value of *n* for which $\Pr\left(\hat{Q}_n > \frac{1}{n}\right) > 0.95$.

2 marks

The amount of blood x in litres in the average adult human Australian male, can vary, and follows a probability density function given by $B(x) = \begin{cases} 1 - \dfrac{k}{x^2} & \text{for } 4 \le x \le 5 \\ 1 - \dfrac{k}{(x-10)^2} & \text{for } 5 < x \le 6 \end{cases}$

f.i. Show that $k = 10$.

2 marks

ii. Find the probability that the amount of blood in an adult human male lies within one standard deviation of the mean. Give your answer correct to four decimal places.

2 marks

Question 4 (11 marks)

Consider the function $f:[0,8]\to R,\quad f(x)=4+4\cos\left(\frac{\pi x}{8}\right)$

a. Determine the coordinates of the point on the graph of the function *f* that are at a minimum distance to the origin and determine this minimum distance.

Give all answers correct to three decimal places.

2 marks

b. Show that the function has a point of inflexion at the point $(4,4)$

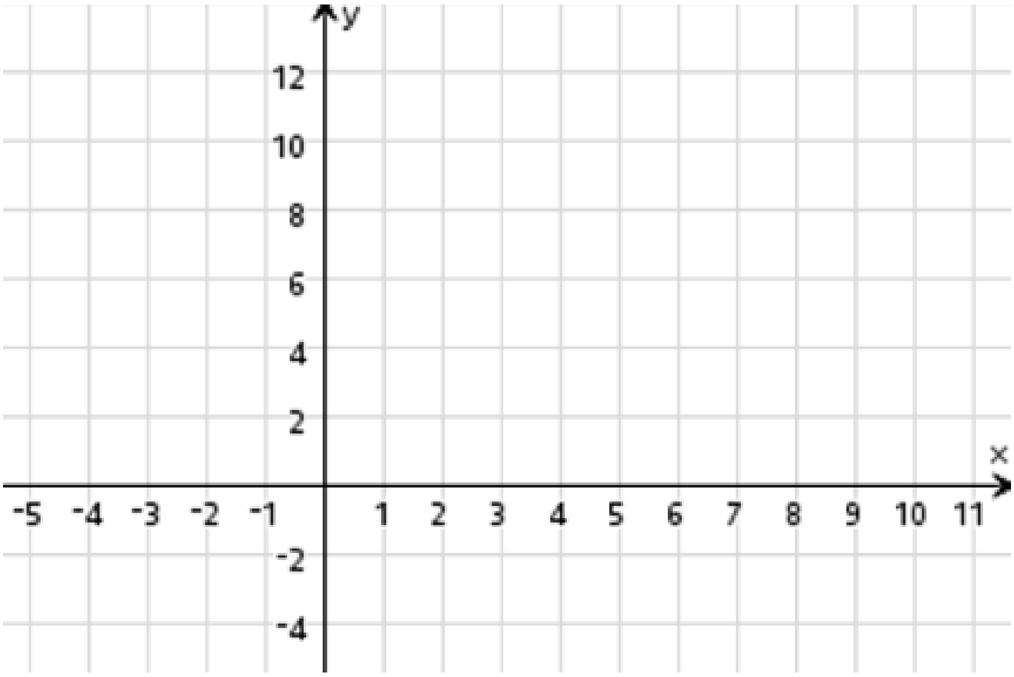
1 mark

c. Show that the equation of the tangent to the curve at the point of inflexion is given by $y=-\frac{\pi x}{2}+2(\pi+2)$.

1 mark

d. Sketch the graph of the function f and the draw the tangent to the graph at the point of inflexion on the axes below.

2 marks



e. Determine the area in the first quadrant between the graph of the function f the x and y axes and the tangent to the curve at the point of inflexion. Give your answer correct to three decimal places.

2 marks

Consider now the functions $g_1 : [0, 4b] \to R,\quad g_1(x) = 2b + 2b\cos\left(\frac{\pi x}{4b}\right)$ and

$g_2 : [0, 2b] \to R,\quad g_2(x) = b + b\cos\left(\frac{\pi x}{2b}\right)$ where $b > 0$.

f. Let $A(b)$ be the area between the graphs of g_1 and g_2 and the x and y axes in the first quadrant. Determine $A(b)$.

1 mark

g. Consider now the function $g : R \to R,\quad g(x) = \begin{cases} -x & \text{for } x < 0 \\ a + a\cos\left(\frac{\pi x}{2a}\right) & \text{for } 0 \le x \le 2a \\ x & \text{for } x > 2a \end{cases}$ where $a > 0$.

i. State the gradient function. 1 mark

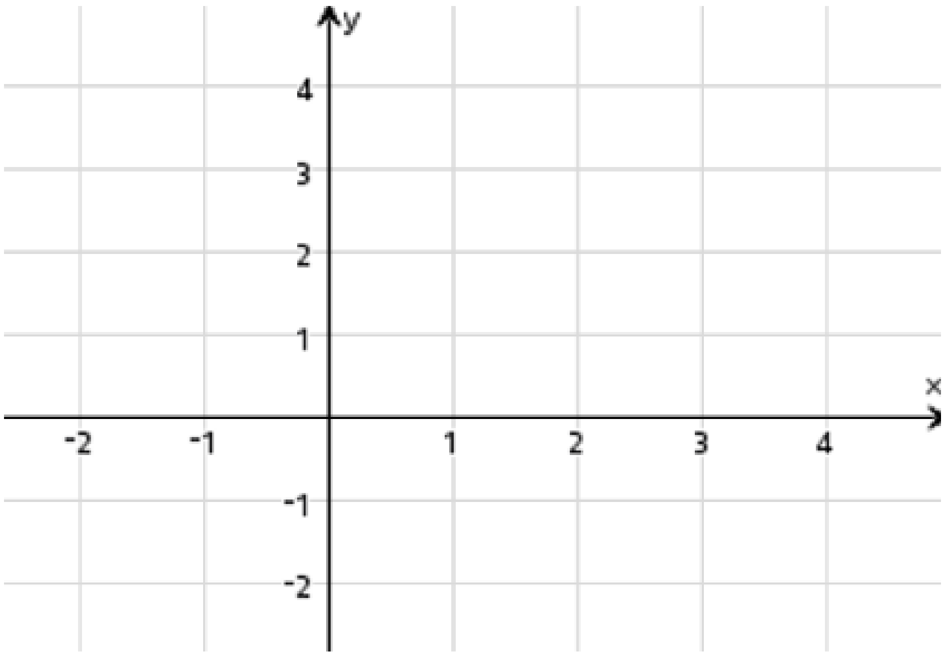
ii. For what values of x is the function g strictly increasing.

1 mark

Question 5 (12 marks)

a.i. Given the function $f: R \to R$, $f(x) = e^{2x}$, define the inverse function, $f^{-1}(x)$ and sketch the graphs of $y = f(x)$ and its inverse $y = f^{-1}(x)$ on the coordinate axes below.

2 marks



ii. The line $y = -x + 3$ intersects the graph of *f* at the point *U* and intersects the graph of the inverse function at the point *V*. Write down correct to three decimal places, the coordinates of the points *U* and *V*.

1 mark

iii. Determine the total area bounded by the function *f*, the inverse function, the line $y = -x + 3$ and the coordinates axes. Give your answer correct to four decimal places.

2 marks

b.i. Given the function $g:(0,\infty)\to R$, $g(x)=3\log_e(x)$, find the inverse function, $g^{-1}(x)$

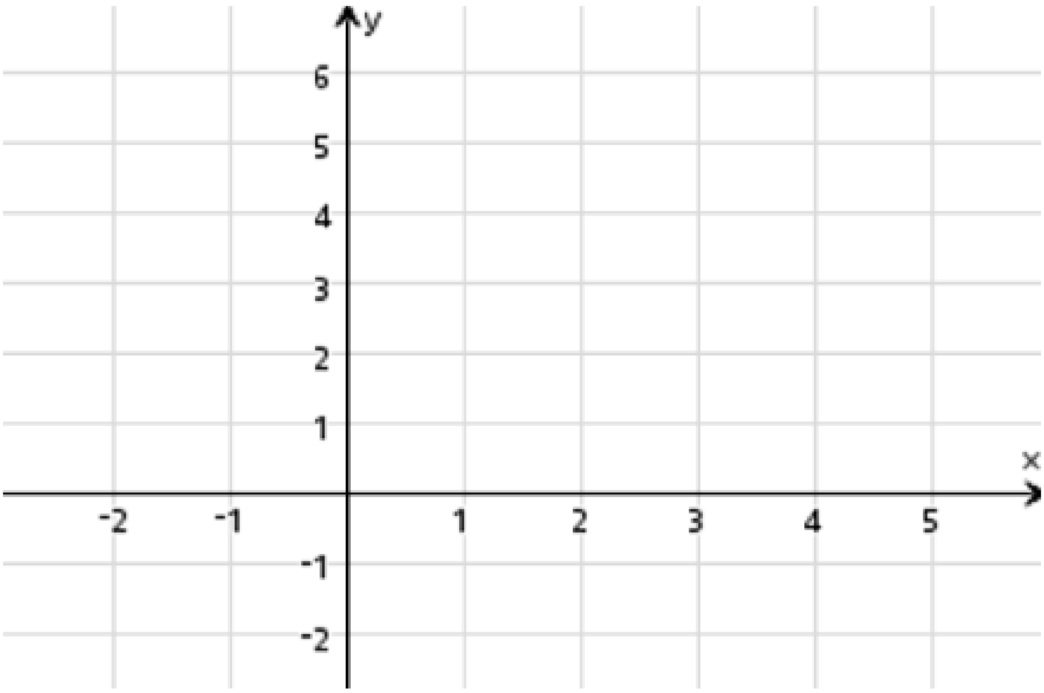
1 mark

ii. The function g and the inverse function $g^{-1}(x)$, intersect at the points $P(p,p)$ and $Q(q,q)$ where $p<q$. Write down the values of p and q correct to three decimal places.

1 mark

iii. Sketch the graphs of $y=g(x)$ and its inverse $y=g^{-1}(x)$ on the coordinate axes below.

1 mark



iv. Let T_1 be the tangent to the curve $y = g(x)$ at the point P, and let T_2 be the tangent to the curve $y = g^{-1}(x)$ at the point P. Find the angle between these tangents T_1 and T_2, giving your answer in degrees correct to two decimal places.

2 mark

c. Consider now the function $h: R \to R$, $h(x) = e^{kx}$ where $k \in R \setminus \{0\}$.
Find the inverse function h^{-1} and complete the following table below.

2 marks

function h and h^{-1}	values of k
do not intersect.	
have only one point of intersection.	
have two points of intersection.	
have three points of intersection	

END OF SECTION B

EXTRA WORKING SPACE

End of question and answer book for the
2023 Kilbaha VCE Mathematical Methods Trial Examination 2

Kilbaha Education **PO Box 2227** **Kew Vic 3101** **Australia**	**Tel: (03) 9018 5376** **kilbaha@gmail.com** **https://kilbaha.com.au**

MATHEMATICAL METHODS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left(\left(ax+b\right)^n\right)=na\left(ax+b\right)^{n-1}$		$\int\left(ax+b\right)^n dx=\frac{1}{a\left(n+1\right)}\left(ax+b\right)^{n+1}+c\ ,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$		$\int\frac{1}{x}dx=\log_e\left(x\right)+c,\ x>0$	
$\frac{d}{dx}\left(\sin\left(ax\right)\right)=a\cos\left(ax\right)$		$\int\sin\left(ax\right)dx=-\frac{1}{a}\cos\left(ax\right)+c$	
$\frac{d}{dx}\left(\cos\left(ax\right)\right)=-a\sin\left(ax\right)$		$\int\cos\left(ax\right)dx=\frac{1}{a}\sin\left(ax\right)+c$	
$\frac{d}{dx}\left(\tan\left(ax\right)\right)=\frac{a}{\cos^2\left(ax\right)}=a\sec^2\left(ax\right)$			
product rule	$\frac{d}{dx}\left(uv\right)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1}=x_n-\frac{f\left(x_n\right)}{f'\left(x_n\right)}$
trapezium rule approximation	$Area\approx\frac{x_n-x_0}{2n}\left[f\left(x_0\right)+2f\left(x_1\right)+2f\left(x_2\right)+...+2f\left(x_{n-2}\right)+2f\left(x_{n-1}\right)+f\left(x_n\right)\right]$		

Probability

$\Pr(A)=1-\Pr(A')$		$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$	
$\Pr(A\mid B)=\dfrac{\Pr(A\cap B)}{\Pr(B)}$			
mean	$\mu=E(X)$	variance	$\operatorname{var}(X)=\sigma^2=E\left((X-\mu)^2\right)=E(X^2)-\mu^2$
binomial coefficient	$\binom{n}{x}=\dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2\,p(x)$
binomial	$\Pr(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$	$\mu=np$	$\sigma^2=np(1-p)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)\,dx$	$\mu=\int_{-\infty}^{\infty}x\,f(x)\,dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2\,f(x)\,dx$

Sample proportions

$\hat{P}=\dfrac{X}{n}$		mean	$E(\hat{P})=p$
standard deviation	$\operatorname{sd}(\hat{P})=\sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p}-z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

										Letter
Figures										
Words										

SIGNATURE ____________________________________

SECTION A

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**

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