Victorian Certificate of Education
2023

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 17 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 + 2 = 5 marks)

a. If $f(x)=\dfrac{\tan(2x)}{12x}$ and $f'\left(\dfrac{\pi}{6}\right)=\dfrac{p\pi+q}{\pi^2}$ determine the values of the real constants *p* and *q*.

3 marks

b. Given that $\int_{b-1}^{b+1}\dfrac{1}{x}\,dx=\log_e\left(\dfrac{5}{3}\right)$, determine the value of *b* where $b>-1$.

2 marks

Question 2 (3 marks)

Solve for *x*, $2^{x+3}=3^{x+2}$, giving your answer in the form $\frac{\log_e(c)}{\log_e(d)}$, where *c* and *d* are positive rational numbers.

Question 3 (4 marks)

a. Explain why the equation $\cos\left(\frac{2\pi x}{3}\right)=x^2$ has a root between $x=0$ and $x=1$.

1 mark

b. Using Newton's method, solve the equation $\cos\left(\frac{2\pi x}{3}\right)=x^2$ with a starting value of $x_0=\frac{3}{2}$, find the value of x_1 giving your answer in the form $\frac{a}{b}$ where $a,b\in Z^+$.

3 marks

Question 4 (3 marks)

An **approximate** 95% confidence interval for the proportion of adult males over 50 years of age who have high blood pressure is given by $(0.544, 0.736)$.

Given that $\Pr(-2 < Z < 2) = 0.95$, where Z is the standard normal random variable, determine the sample size used in the calculation of this confidence interval.

Question 5 (8 marks)

a. Find the general solution of $\sin(x)+\sqrt{3}\cos(x)=0$ for $x \in R$.

2 marks

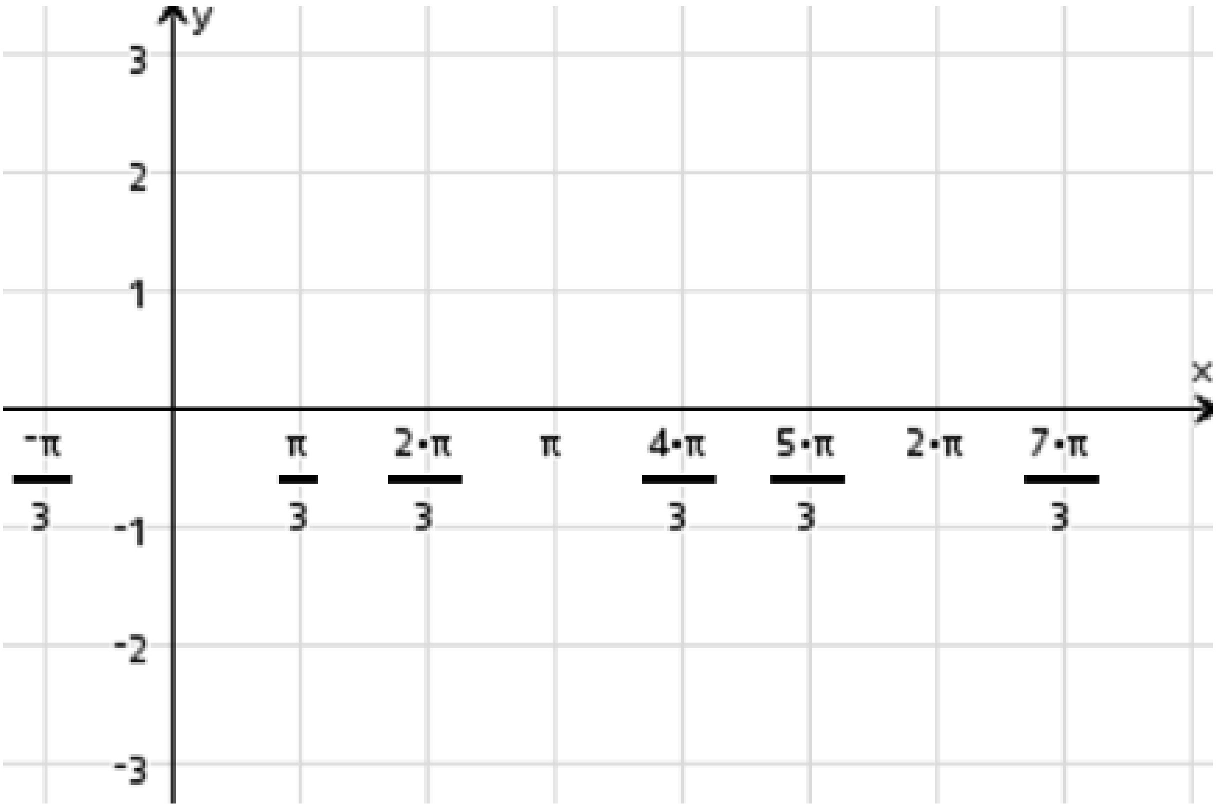
b. Consider the function $f:[0,2\pi]\to R,\ f(x)=\sin(x)+\sqrt{3}\cos(x)$.
Using calculus, determine the coordinates of the stationary points on the graph of $y=f(x)$.

3 marks

c. On the axes below, sketch the graphs of $y_1 = \sin(x)$ and $y_2 = \sqrt{3}\cos(x)$.

Hence or otherwise sketch the graph of $f:[0,2\pi] \to R,\ f(x) = \sin(x) + \sqrt{3}\cos(x)$, labelling all axial intercepts, endpoints and turning points with their coordinates.

3 marks



Question 6 (5 marks)

Given the two functions $f(x)=\dfrac{1}{x-3}$ and $g(x)=1+2\sin(x)$ defined on their maximal domains,

a. Determine whether or not the functions $f\circ g(x)$ or $g\circ f(x)$ exists, giving reasons for your answers, if the composite function exists, state its rule and domain.

2 marks

b. Find the values of x for which $f(x)<f^{-1}(x)$.

3 marks

Question 7 (6 marks)

The concentration of a drug in a body is given by $F(t)$ where t is the time in hours after the drug is takn. Initially the concentration o the drug is zero. The rate of change of concentration of the drug is given by $F'(t) = 50e^{-0.5t} - 0.4F(t)$.

a. Using the product rule, show that $\frac{d}{dt}\left(F(t)e^{0.4t}\right) = 50e^{-0.1t}$

2 marks

b. Hence show that $F(t) = 500\left(e^{-0.4t} - e^{-0.5t}\right)$

2 marks

c. The concentration of the drug increases to a maximum. Find the value of t when this maximum occurs. Give your answer in the form $a\log_e\left(\frac{b}{c}\right)$, where a,b and c are positive integers.

2 marks

Question 8 (3 marks)

a. Show that $\frac{d}{dx}\left[x^2\log_e(x)-\frac{x^2}{2}\right]=2x\log_e(x)$

1 mark

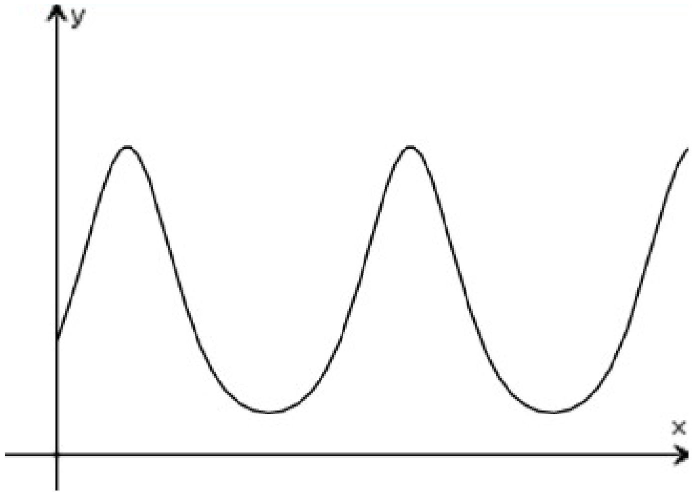
b. Given the continuous probability density function $f(x)=\begin{cases}\log_e(x) & \text{for } 1\le x\le e\\ 0 & \text{otherwise}\end{cases}$

Find $E(X)$.

2 marks

Question 9 (3 marks)

Part of the graph of the function $f:[0,\infty)\to R,\ f(x)=e^{\sin\left(\frac{\pi x}{2}\right)}$ is shown below.



a. Determine the coordinates of the first two maximum and the first two minimum turning points.

2 marks

b. State the period of the function.

1 mark

End of question and answer book for the
2023 Kilbaha VCE Mathematical Methods Trial Examination 1

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EXTRA WORKING PAGE

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right)=na(ax+b)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int \frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	$Area\approx\frac{x_n-x_0}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-2})+2f(x_{n-1})+f(x_n)\right]$		

Probability

$\Pr(A)=1-\Pr(A')$		$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$	
$\Pr(A\mid B)=\dfrac{\Pr(A\cap B)}{\Pr(B)}$			
mean	$\mu=E(X)$	variance	$\operatorname{var}(X)=\sigma^2=E\left((X-\mu)^2\right)=E(X^2)-\mu^2$
binomial coefficient	$\binom{n}{x}=\dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2\,p(x)$
binomial	$\Pr(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$	$\mu=np$	$\sigma^2=np(1-p)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)\,dx$	$\mu=\int_{-\infty}^{\infty}x\,f(x)\,dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P}=\dfrac{X}{n}$		mean	$E(\hat{P})=p$
standard deviation	$\operatorname{sd}(\hat{P})=\sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p}-z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET