2023
VCE
Mathematical Methods
Year 12
Trial Examination 1

Detailed Answers

Kilbaha Education

Quality educational content

Kilbaha Education
PO Box 2227
Kew Vic 3101
Australia

Tel: (03) 9018 5376
kilbaha@gmail.com
https://kilbaha.com.au

Question 1

a. $f(x)=\dfrac{\tan(2x)}{12x}$ quotient rule

$$f\ (x)=\frac{\dfrac{2\times 12x}{\cos^2(2x)}-12\tan(2x)}{(12x)^2}=\frac{\dfrac{2x}{\cos^2(2x)}-\tan(2x)}{12x^2}$$ M1

$$f\left(\frac{\pi}{6}\right)=\frac{\dfrac{\frac{\pi}{3}}{\cos^2\left(\frac{\pi}{3}\right)}-\tan\left(\frac{\pi}{3}\right)}{12\left(\frac{\pi}{6}\right)^2}=\frac{\frac{\pi}{3}\times 4-\sqrt{3}}{\frac{\pi^2}{3}}$$ A1

$$f\left(\frac{\pi}{6}\right)=\frac{4\pi-3\sqrt{3}}{\pi^2},\quad p=4,\quad q=-3\sqrt{3}$$ A1

b. $$\int_{b-}^{b+}\frac{1}{x}\,dx=\log_e\left(\frac{5}{3}\right)$$

$$\left[\log_e(x)\right]_{b-}^{b+}=\log_e\left(\frac{5}{3}\right)$$

$$\log_e(b+1)-\log_e(b-1)=\log_e\left(\frac{5}{3}\right)$$ M1

$$\log_e\left(\frac{b+1}{b-1}\right)=\log_e\left(\frac{5}{3}\right)$$

$$\frac{b+1}{b-1}=\frac{5}{3}$$

$$3(b+1)=5(b-1)$$
$$3b+3=5b-5$$
$$2b=8$$
$$b=4$$ A1

Question 2

Method 1

$2^{x+3}=3^{x+2}$

$\log_e\left(2^{x+3}\right)=\log_e\left(3^{x+2}\right)$

$(x+3)\log_e(2)=(x+2)\log_e(3)$

$x\log_e(2)+3\log_e(2)=x\log_e(3)+2\log_e(3)$

$x\left(\log_e(3)-\log_e(2)\right)=3\log_e(2)-2\log_e(3)$

$x\log_e\left(\frac{3}{2}\right)=\log_e\left(2^3\right)-\log_e\left(3^2\right)$

$x\log_e\left(\frac{3}{2}\right)=\log_e(8)-\log_e(9)$

$x\log_e\left(\frac{3}{2}\right)=\log_e\left(\frac{8}{9}\right)$

$x=\dfrac{\log_e\left(\frac{8}{9}\right)}{\log_e\left(\frac{3}{2}\right)},\quad c=\frac{8}{9},\ d=\frac{3}{2}$

Method 2

$2^{x+3}=3^{x+2}$

$2^x\times 2^3=3^x\times 3^2$

$8\times 2^x=9\times 3^x$ M1

$\frac{8}{9}=\frac{3^x}{2^x}$

$\frac{8}{9}=\left(\frac{3}{2}\right)^x$

$\log_e\left(\frac{8}{9}\right)=\log_e\left(\left(\frac{3}{2}\right)^x\right)$ M1

$\log_e\left(\frac{8}{9}\right)=x\log_e\left(\frac{3}{2}\right)$

$x=\dfrac{\log_e\left(\frac{8}{9}\right)}{\log_e\left(\frac{3}{2}\right)},\quad c=\frac{8}{9},\ d=\frac{3}{2}$ A1

Question 3

a. Let $f(x)=\cos\left(\frac{2\pi x}{3}\right)-x^2$

$f(0)=\cos(0)-0=1,\quad f(1)=\cos\left(\frac{2\pi}{3}\right)-1=-\frac{1}{2}-1=-\frac{3}{2}$

Since $f(0)>0$ and $f(1)<0$ there is a sign change between $x=0$ and $x=1$ then there is a root between $x=0$ and $x=1$ A1

b. $x_0=\frac{3}{2}\quad f(x)=\cos\left(\frac{2\pi x}{3}\right)-x^2,\quad f\left(\frac{3}{2}\right)=\cos(\pi)-\frac{9}{4}=-1-\frac{9}{4}=-\frac{13}{4}$

$f'(x)=-\frac{2\pi}{3}\sin\left(\frac{2\pi x}{3}\right)-2x,\quad f'\left(\frac{3}{2}\right)=-\frac{2\pi}{3}\sin(\pi)-3=-3$ A1

$x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=\frac{3}{2}-\left(\dfrac{\frac{-13}{4}}{-3}\right)=\frac{3}{2}-\frac{13}{12}=\frac{18-13}{12}$ M1

$x=\frac{5}{12}$ A1

Question 4

$$\left(\hat{p}-z\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}\ ,\ \hat{p}+z\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}\right)=\left(0.544,0.736\right)$$

$2\hat{p}=0.736+0.544=1.28$ A1

$$\hat{p}=\frac{1.28}{2}$$

$$\hat{p}=0.64$$

$$2z\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}=0.736-0.544=0.192$$

$$z\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}=\frac{0192}{2}=0.096, \quad 95\%, \quad z\approx 2$$

$\sqrt{\frac{0.64\times 0.36}{n}}=\frac{0.096}{2}$ M1

$$\frac{048}{\sqrt{n}}=0048$$

$$\sqrt{n}=\frac{048}{0048}=10$$

$n=100$ A1

Question 5

a. $\sin\left(x\right)+\sqrt{3}\cos\left(x\right)=0$

$$\sin\left(x\right)=-\sqrt{3}\cos\left(x\right)$$

$\frac{\sin\left(x\right)}{\cos\left(x\right)}=-\sqrt{3}$ M1

$$\tan\left(x\right)=-\sqrt{3}$$

$$x=n\pi+\tan^{-}\left(-\sqrt{3}\right)$$

$x=n\pi-\frac{\pi}{3}=\frac{\pi}{3}\left(3n-1\right),\quad n\in Z$ A1

b. $f:[0,2\pi]\to R,\quad f(x)=\sin(x)+\sqrt{3}\cos(x)$

$f\;(x)=\cos(x)-\sqrt{3}\sin(x)=0$ for turning points

$\cos(x)=\sqrt{3}\sin(x)$

$\dfrac{\sin(x)}{\cos(x)}=\tan(x)=\dfrac{1}{\sqrt{3}}$ M1

$x=\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right),\pi+\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)=\dfrac{\pi}{6},\ \pi+\dfrac{\pi}{6}$

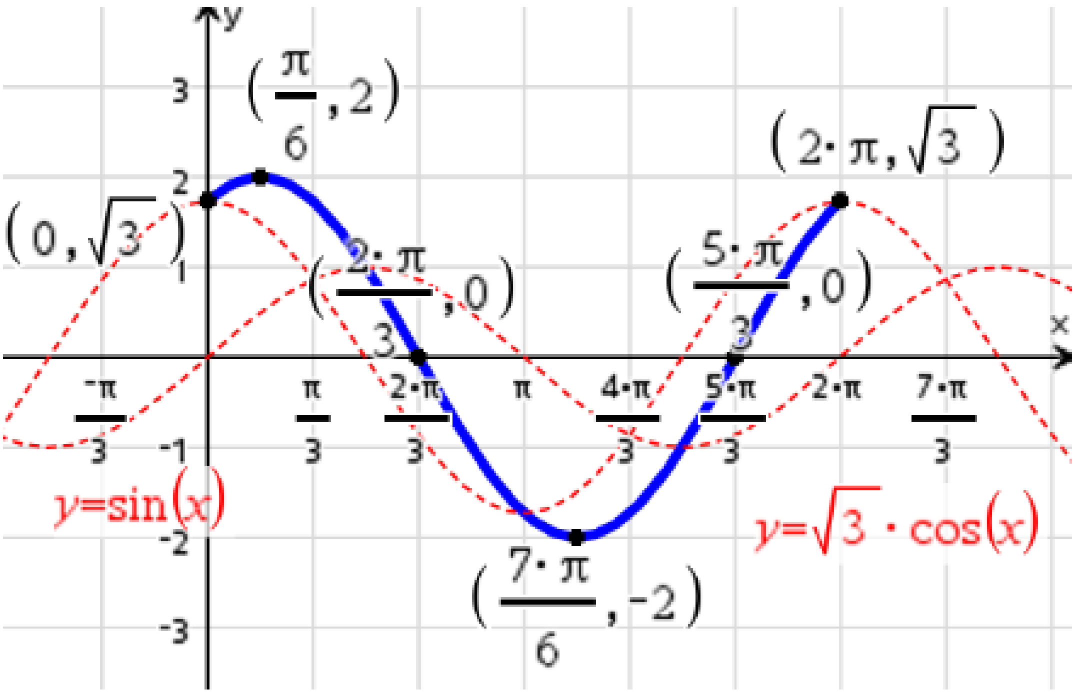
$x=\dfrac{\pi}{6}\ \dfrac{7\pi}{6}$ A1

$f\left(\dfrac{\pi}{6}\right)=\sin\left(\dfrac{\pi}{6}\right)+\sqrt{3}\cos\left(\dfrac{\pi}{6}\right)=\dfrac{1}{2}+\sqrt{3}\times\dfrac{\sqrt{3}}{2}=2,\quad\left(\dfrac{\pi}{6},2\right)$

$f\left(\dfrac{7\pi}{6}\right)=\sin\left(\dfrac{7\pi}{6}\right)+\sqrt{3}\cos\left(\dfrac{7\pi}{6}\right)=-\dfrac{1}{2}-\sqrt{3}\times\dfrac{\sqrt{3}}{2}=-2,\quad\left(\dfrac{7\pi}{6},-2\right)$ A1

c. Graph by addition of ordinates and from **a.** when $n=1\ \ x=\dfrac{2\pi}{3}$ and when $n=2\ \ x=\dfrac{5\pi}{3}$ these are where the graph crosses the x-axis $\left(\dfrac{2\pi}{3},0\right)$, $\left(\dfrac{5\pi}{3},0\right)$

endpoints $\left(0,\sqrt{3}\right)$, $\left(2\pi,\sqrt{3}\right)$ G3



Question 6 $f(x)=\frac{1}{x-3}$ and $g(x)=1+2\sin(x)$

a. Complete a function domain and range table

function	domain	range
$f(x)$	$R\setminus\{3\}$	$R\setminus\{0\}$
$g(x)$	R	$[-1,3]$

$f\,o\,g(x)$ exists if range $g \subseteq$ domain f, but $[-1,3] \not\subset R\setminus\{3\}$

so $f\,o\,g(x)$ does not exist A1

$g\,o\,f(x)$ exist, if range $f \subseteq$ domain g,

This is true, so $g\,o\,f(x)=g(f(x))=g\left(\frac{1}{x-3}\right)=1+2\sin\left(\frac{1}{x-3}\right)$ A1

and domain $g\,o\,f(x)$ is equal to the domain $f\ =R\setminus\{3\}$

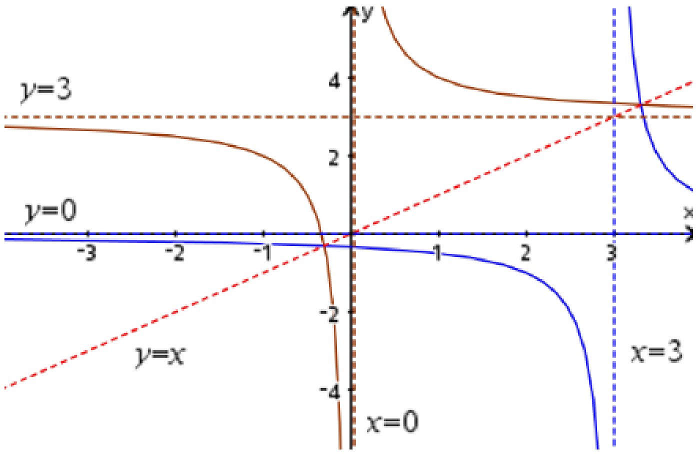
b.

$f \quad y=\frac{1}{x-3}$

swap x and y

$f^{-}:\quad x=\frac{1}{y-3},\quad y-3=\frac{1}{x},$

$f^{-}(x)=\frac{1}{x}+3$ A1



solving

$f(x)=f^{-}(x)$

$\frac{1}{x-3}=\frac{1}{x}+3$

$x=x-3+3x(x-3)$

$3x^2-9x-3=0$

$x^2-3x-1=0$

$x=\frac{3\pm\sqrt{13}}{2}$ M1

$f(x)<f^{-}(x)$

$\Rightarrow x\in\left(-\infty,\frac{3-\sqrt{13}}{2}\right)\cup(0,3)\cup\left(\frac{3+\sqrt{13}}{2},\infty\right)$ A1

Question 7

a. $F\ (t) = 50e^{-0.5} - 0.4F(t)$

$$\frac{d}{dt}\left(F(t)e^{0.4t}\right) = \frac{d}{dt}\left(F(t)\right)e^{0.4t} + F(t)\frac{d}{dt}\left(e^{0.4t}\right) \text{ using the product rule}$$

$$= F\ (t)e^{0.4t} + 0.4F(t)e^{0.4t}$$ A1

$$= \left(50e^{-0.5t} - 0.4F(t)\right)e^{0.4t} + 0.4F(t)e^{0.4t}$$

$$= 50e^{-0.1t} - 0.4F(t)e^{0.4t} + 0.4F(t)e^{0.4t} = 50e^{-0.1t}$$ M1

b. $\frac{d}{dt}\left(F(t)e^{0.4t}\right) = 50e^{-0.1t}$

$F(t)e^{0.4t} = \int 50e^{-0.1t}\, dt = \frac{50}{-0.1}e^{-0.1t} + c$ M1

$F(t)e^{0.4t} = -500e^{-0.1t} + c$

when $t = 0$ $F(0) = 0$, $0 = -500 + c$, $c = 500$ A1

$F(t)e^{0.4t} = 500 - 500e^{-0.1t} = 500\left(1 - e^{-0.1t}\right)$

$F(t) = \frac{500\left(1 - e^{-0.1}\right)}{e^{0.4}} = 500\left(e^{-0.4t} - e^{-0.5t}\right)$

c. $F(t) = 500\left(e^{-0.4t} - e^{-0.5t}\right)$

$F\ (t) = 500\left(-0.4e^{-0.4t} + 0.5e^{-0.5t}\right) = 0$ M1

$\frac{2}{5}e^{-\frac{2}{5}} = \frac{1}{2}e^{-\frac{}{2}}$

$\frac{5}{4} = \frac{e^{-\frac{2}{5}}}{e^{-\frac{}{2}}} - = e^{\frac{t}{2}-\frac{2t}{5}} = e^{\frac{t}{10}}$

$\frac{t}{10} = \log_e\left(\frac{5}{4}\right)$ A1

$t = 10\log_e\left(\frac{5}{4}\right)$

Question 8

a. $\frac{d}{dx}\left[x^2\log_e(x)-\frac{x^2}{2}\right]=2x\log_e(x)$ using the product rule

$x^2\frac{d}{dx}\left[\log_e(x)\right]+\log_e(x)\frac{d}{dx}\left(x^2\right)-\frac{d}{dx}\left(\frac{x^2}{2}\right)$

$=x^2\times\frac{1}{x}+2x\log_e(x)-x$ A1

$=x+2x\log_e(x)-x$

$=2x\log_e(x)$

b. $f(x)=\begin{cases}\log_e(x) & \text{for } 1\le x\le e\\ 0 & \text{otherwise}\end{cases}$

$E(X)=\int_1^e xf(x)\,dx=\int_1^e x\log_e(x)\,dx$

$E(X)=\frac{1}{2}\left[x^2\log_e(x)-\frac{x^2}{2}\right]_1^e$ from **a.** M1

$E(X)=\frac{1}{2}\left[\left(e^2\log_e(e)-\frac{e^2}{2}\right)-\left(\log_e(1)-\frac{1}{2}\right)\right]=\frac{1}{2}\left(e^2-\frac{e^2}{2}+\frac{1}{2}\right)$

$E(X)=\frac{1}{4}\left(e^2+1\right)$ A1

Question 9

a. $f:[0,\infty)\to R,\quad f(x)=e^{\mathrm{sn}\left(\frac{\pi x}{2}\right)}$ using the chain rule

$f\ (x)=\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)e^{\mathrm{sn}\left(\frac{\pi x}{2}\right)}=0$ for turning points

$\cos\left(\frac{\pi x}{2}\right)=0,\quad \frac{\pi x}{2}=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}$ A1

$x=1,3,5,7,\quad f(1)=f(5)=e,\quad f(3)=f(7)=\frac{1}{e}$

$(1,e),(5,e)$ maximum turning points

$\left(3,\frac{1}{e}\right),\left(7,\frac{1}{e}\right)$ minimum turning points A1

b. since $f(x)=f(x+4)$, $\sin\left(\frac{\pi x}{2}\right)=\sin\left(\frac{\pi(x+4)}{2}\right)=\sin\left(2\pi+\frac{\pi x}{2}\right)=\sin\left(\frac{\pi x}{2}\right)$

the period is 4 A1

End of detailed answers for the
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