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## YEAR 12 *Trial Exam Paper*

### 2023

## MATHEMATICAL METHODS

### Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

**STUDENT NAME:**

## QUESTION AND ANSWER BOOK

### Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

#### Materials supplied

- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

#### Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

#### At the end of the examination

- You may keep the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** (3 marks)

a. Let  $y = \frac{2x}{3x+5}$ .

Find  $\frac{dy}{dx}$ .

1 mark

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b. Let  $f(x) = 3 - e^{1-2x}$ .

Evaluate  $f'\left(\frac{1}{2}\right)$ .

2 marks

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**Question 3** (6 marks)

Talula enjoys bushwalking and sometimes sees kangaroos on her walks. Let  $X$  be a random variable describing the number of kangaroos that Talula sees on any given walk. The probability distribution of  $X$  is given in the table below.

$x$	0	1	2	3
$\Pr(X=x)$	0.6	0.1	0.2	0.1

- a.** Find the mean, or expected value  $E(X)$ , of  $X$ .

1 mark

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- b.** In a particular week, Talula goes on three bushwalks. Find the probability that Talula sees one kangaroo on exactly two of those walks.

2 marks

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- c.** During another week, Talula went on two bush walks and she saw at least one kangaroo on each of those walks. Find the probability that Talula saw a total of three kangaroos on her walks during that week.

3 marks

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**Question 4** (6 marks)

Let  $f: R \rightarrow R, f(x) = 3 + 2\sin\left(\frac{x}{2}\right)$ .

- a.** The graph of  $f$  is transformed to produce the graph of the function  $g$ , where  $g(x) = f(x - c)$ .

- i.** If  $c > 0$ , find the smallest value of  $c$  such that the graph of  $g$  has a  $y$ -intercept at  $(0, 3)$ .

1 mark

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- ii.** If  $c \in R$ , state all values of  $c$  such that the graph of  $g$  has a  $y$ -intercept at  $(0, 3)$ .

1 mark

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- b.** Calculate the average value of  $f$  between  $x = 0$  and  $x = \pi$ .

2 marks

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- c.** The average value of  $f$  between  $x = \frac{5\pi}{4}$  and  $x = b$  is 3.

If  $b \in \left(\frac{5\pi}{4}, 8\pi\right]$ , state the possible values of  $b$ .

2 marks

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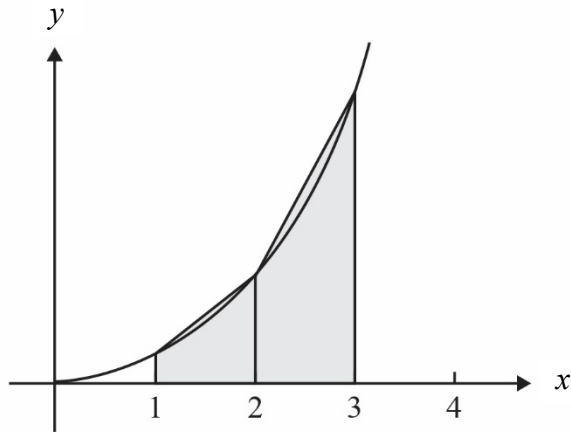


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**Question 5** (6 marks)

Let  $f: [0, \infty) \rightarrow \mathbb{R}, f(x) = e^x - 1$

- a.** Two trapeziums of equal width are used to approximate the area between the graph with equation  $y = f(x)$  and the  $x$ -axis between the lines with equations  $x = 1$  and  $x = 3$ . The heights of the left and right edges of each trapezium are the values of  $y = f(x)$ , as shown on the graph below.



Find the total area of the trapeziums.

2 marks

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- b.** Let  $g: [0, \infty) \rightarrow \mathbb{R}, g(x) = m - e^x$ , where  $m \geq 1$ .  
The graphs of  $y = f(x)$  and  $y = g(x)$  intersect when  $x = k$ .

Show that  $k = \log_e \left( \frac{m+1}{2} \right)$ .

1 mark

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**Question 6** (7 marks)

Consider the function  $f$  where  $f: [-1, \infty) \rightarrow \mathbb{R}, f(x) = \log_e(x + 2)$ .

a. Find  $f(-1)$ .

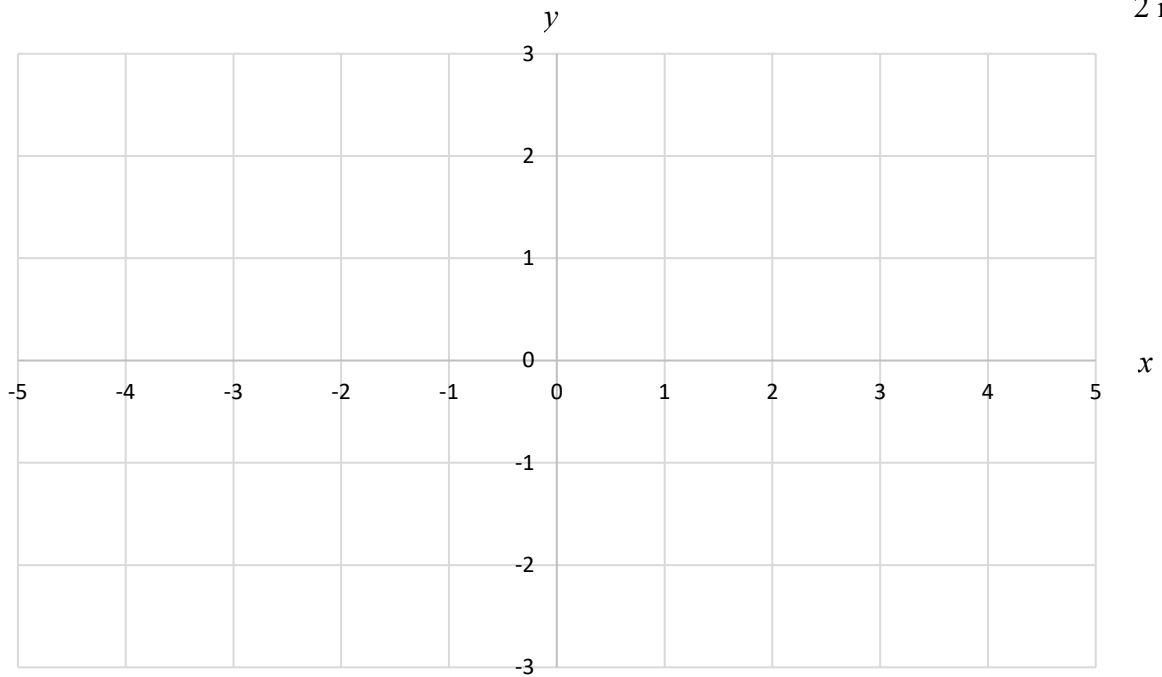
1 mark

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b. On the axes below, sketch the graph of  $f$ . Label any axial intercepts with their exact coordinates.

2 marks





**c.** Let  $g : [d, \infty) \rightarrow \mathbb{R}$ ,  $g(x) = x^2 - 17$ .

**i.** Find the smallest possible value of  $d$  such that  $f \circ g$  exists.

2 marks

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**ii.** For the value of  $d$  found in **part c.i.** state the range of  $f \circ g$ .

1 mark

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**d.** Let  $h : \mathbb{R} \rightarrow \mathbb{R}$ ,  $h(x) = x^2$ . State the range of  $f \circ h$ .

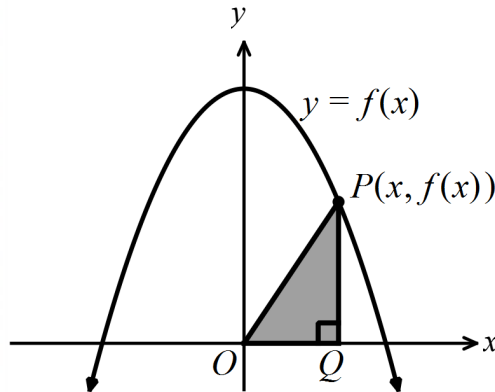
1 mark

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**Question 7** (4 marks)

The graph of  $f(x) = 9 - x^2$  is shown below. The point  $P$ , which has coordinates  $(x, f(x))$ , where  $1 \leq x \leq 2$ , lies on the graph of  $f$ .



Triangle  $OQP$  has vertex  $O$  at the origin and vertex  $Q$  lies on the  $x$ -axis. Angle  $PQO$  is a right angle.

- a. The area of triangle  $OQP$  is  $A$  square metres. Show that  $A = \frac{9}{2}x - \frac{1}{2}x^3$ .

1 mark

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- b. Find the minimum and maximum areas of triangle  $OQP$ .

3 marks

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**Question 8** (5 marks)

Let  $f: \left[-\frac{v}{6}, v\right] \rightarrow R, f(x) = \sqrt{6x + v} - 3$  where  $v$  is a positive real number.

- a. Find the value of  $f\left(-\frac{v}{6}\right)$ .

1 mark

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- b. The graphs of  $f$  and its inverse  $f^{-1}$  may have up to two points of intersection.  
Find the  $x$ -coordinates of any possible points of intersection of the graphs of  $f$  and  $f^{-1}$  in terms of  $v$ .

2 marks

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- c. Determine the set of values of  $v$  for which the graphs of  $f$  and  $f^{-1}$  have two points of intersection.

2 marks

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**END OF QUESTION AND ANSWER BOOK**