VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2022

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter | |

MATHEMATICAL METHODS

Written examination 1

Friday 27 May 2022

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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DO NOT WRITE IN THIS AREA

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. If $y = \sin(x^2 + 1)$, find $\frac{dy}{dx}$. 1 mark

b. If $f(x) = x^2 \log_e(x)$, find $f'(e)$. 2 marks

Question 2 (2 marks)

Find $f(x)$, given that $f(0) = 3$ and $f'(x) = \frac{2}{x+1} + 2\cos(x)$, where $x > -1$.

Question 3 (2 marks)

A marketing company wants to estimate the proportion of a population who regularly ride bicycles for exercise. The company randomly samples 100 people from this population and finds that 10 of these people regularly ride bicycles for exercise.

Using $z = 2$, find an approximate 95% confidence interval for the true proportion of the population who regularly ride bicycles for exercise.

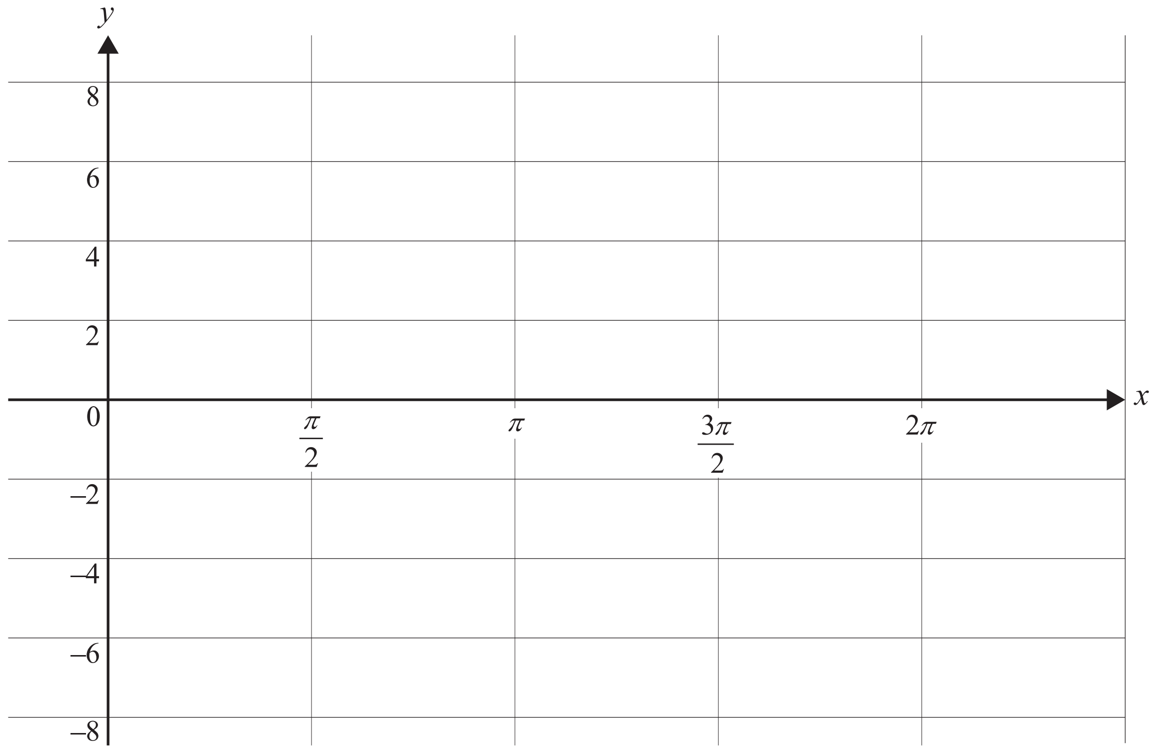
Question 4 (7 marks)

Consider the function $f : [0, 2\pi] \to R$, $f(x) = a\sin(x) + b$, given that $f\left(\frac{\pi}{2}\right) = 2$ and that $f\left(\frac{3\pi}{2}\right) = -8$.

a. Show that $a = 5$ and $b = -3$. 1 mark

Question 4 – continued

b. Sketch the graph of $y = f(x)$ on the set of axes below. Label the endpoints and the turning points with their coordinates. 2 marks



c. State the values of k for which the equation $f(x) + k = 0$, where $k \in R$, has no solution for x. 2 marks

d. Find the value of m for which $\int_0^{\pi} \left(f(x) + m\right) dx = 0$, where $m \in R$. 2 marks

Question 5 (4 marks)

Consider the function with rule $f(x) = e^{1-x}$.

a. Simplify $\dfrac{f(x)}{f(-x)}$. Express your answer in the form e^{kx}, where k is an integer. 1 mark

b. Show that $f(a) \times f(b) = f(a + b - 1)$. 1 mark

c. Solve $f(x) = 2$ for x. Give your answer in the form $x = \log_e\left(\dfrac{m}{n}\right)$, where $m, n \in R$. 2 marks

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Question 6 (6 marks)

In a particular city, the probability that it will snow on Monday is x^2.

If it does snow on Monday, the probability that it will snow on Tuesday is $\frac{1}{4}x$.

If it does not snow on Monday, the probability that it will snow on Tuesday is x.

a. **i.** Find an expression, in terms of x, that gives the probability that it will snow on both Monday and Tuesday. 1 mark

ii. Find the value of x if there is a 25% chance that it will snow on both Monday and Tuesday. 1 mark

b. **i.** Find an expression, in terms of x, that gives the probability that it will snow on Tuesday. 1 mark

ii. Find the value of x that will result in the highest probability that it will snow on Tuesday and find the probability for this value of x in the form $\frac{a}{b}$, where $a, b \in Z$. 3 marks

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Question 7 (7 marks)

a. Consider the function p, where $p : [1, \infty) \to R$, $p(x) = x^4 - x^3 - x^2 + x + 1$.

i. Find the value of a when $p^{-1}(a) = 2$, where $a \in R$. 2 marks

ii. Find the value of b when $p(b) = 1$, where $b > 0$. 2 marks

b. Find the rule and the domain of f^{-1}, the inverse of f, if $f : R\backslash\{2\} \to R,\ f(x) = \dfrac{x+3}{x-2}$. 3 marks

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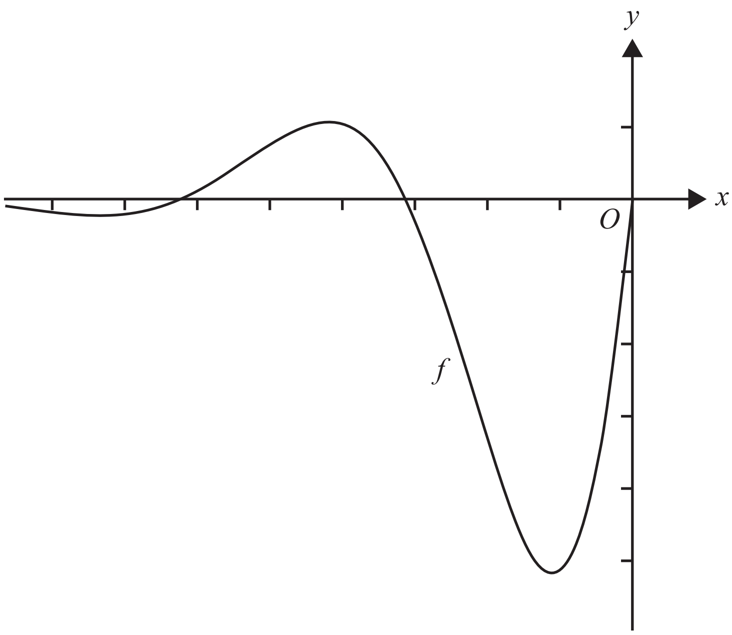
Question 8 (9 marks)

Part of the graph of the function f, where $f : (-\infty, 0] \rightarrow R, f(x) = e^x \sin(x)$, is shown below.



a. Find the general solution to $f(x) = 0$. 2 marks

b. Find the general solution to $f'(x) = 0$. 2 marks

Question 8 – continued

c. **i.** Show that $\frac{d}{dx}\left(e^x \sin(x) - e^x \cos(x)\right) = 2e^x \sin(x)$. 1 mark

ii. Hence, find the area bounded by the graph of the function f and the horizontal axis over the interval $x \in [-\pi, 0]$. 2 marks

d. Consider the function

$$g : R \to R,\ g(x) = \begin{cases} -\dfrac{2e^{\pi}}{1+e^{\pi}} e^x \sin(x) & -\pi \le x \le 0 \\ 0 & \text{elsewhere} \end{cases}$$

The continuous random variable X has the probability density function g.

If x_m is the value of x for which g has a maximum value, find $\Pr(X < x_m)$. 2 marks

END OF QUESTION AND ANSWER BOOK

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MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET

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