Neap

Trial Examination 2022

VCE Mathematical Methods Units 1&2

Written Examination 1

Suggested Solutions

Question 1 (3 marks)

$\log_{14}(x-2)+\log_{14}(x+3)=1$

$\log_{14}\left[(x-2)(x+3)\right]=\log_{14}14$

$\log_{14}(x^2+x-6)=\log_{14}14$

$x^2+x-6=14$ M1

OR

$\log_{14}\left[(x-2)(x+3)\right]=1$

$\log_{14}(x^2+x-6)=1$

$14^1=x^2+x-6$ M1

Then:

$x^2+x-20=0$

$(x+5)(x-4)=0$ M1

$x+5=0$ or $x-4=0$

$x=-5$ or $x=4$

However, for $\log_a(b)$ to exist, $b>0$. Hence, $x\neq-5$. Therefore, $x=4$. A1

Question 2 (3 marks)

Method 1:

$P(x) = 2x^3 - 5x^2 + 3x - 4$

Dividing by $x - 2$ means that finding $P(2)$ gives the remainder.

$P(2)=2\times2^3-5\times2^2+3\times2-4$

$=16-20+6-4$

$=-2$ M1

Hence, the remainder is –2.

Factorising $P(x)$ gives $P(x) = 2x^2(x - 2) - x(x - 2) + 1(x - 2)$. M1

Therefore, the quotient is $2x^2 - x + 1$.

$P(x) = (2x^2 - x + 1)(x - 2) - \frac{2}{(x-2)}$ A1

Method 2:

Performing long division of $P(x)$ by $(x-2)$ gives:

$$
\begin{array}{r}
2x^2 - x + 1 \\
x-2\,\overline{)\,2x^3 - 5x^2 + 3x - 4} \\
\underline{-(2x^3 - 4x^2)} \\
-x^2 + 3x \\
\underline{-(-x^2 + 2x)} \\
x - 4 \\
\underline{-(x-2)} \\
-2
\end{array}
$$

M2

Note: Award 2 marks if the long division used for Method 2 is correct. Award only 1 mark if the long division used for Method 2 is partially correct (that is, if no final A1 mark can be awarded).

Therefore, $P(x) = (2x^2 - x + 1)(x-2) - \dfrac{2}{(x-2)}$. A1

Question 3 (5 marks)

a. For the table of data to be a probability distribution, all the probabilities must sum to 1.

Therefore:

$$
\begin{aligned}
2a + 3a + a + a + 3a + 2a &= 1 \\
12a &= 1 \\
a &= \frac{1}{12}
\end{aligned}
$$

A1

Hence:

x	1	2	3	4	5	6
Pr($X = x$)	$2a$	$3a$	a	a	$3a$	$2a$
	$\frac{2}{12} = \frac{1}{6}$	$\frac{3}{12} = \frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{3}{12} = \frac{1}{4}$	$\frac{2}{12} = \frac{1}{6}$

Note: The third row of the table is not necessary to receive the mark.

b.

$$
\begin{aligned}
\Pr(X < 3) &= \frac{1}{6} + \frac{1}{4} \\
&= \frac{2}{12} + \frac{3}{12} \\
&= \frac{5}{12}
\end{aligned}
$$

A1

*Note: Consequential on answer to **Question 3a.***

c.

$$
\begin{aligned}
\Pr(1 < X \le 4) &= \frac{3}{12} + \frac{1}{12} + \frac{1}{12} \\
&= \frac{5}{12}
\end{aligned}
$$

A1

*Note: Consequential on answer to **Question 3a.***

d. $\Pr(X < 3 \mid X \le 5) = \dfrac{\frac{5}{12}}{\frac{10}{12}}$ M1

$= \dfrac{1}{2}$ A1

Note: Consequential on answer to ***Question 3a.***

Question 4 (7 marks)

a. For $f : \mathbb{R} \to \mathbb{R}, f(x) = 2x(x^3 - 2x^2 + 2x) - 3$.

$$\begin{aligned} f(1) &= 2 \times 1 \times (1^3 - 2 \times 1^2 + 2 \times 1) - 3 \\ &= 2(1 - 2 + 2) - 3 \\ &= -1 \end{aligned}$$ A1

b. **i.** $$\begin{aligned} f(x) &= 2x(x^3 - 2x^2 + 2x) - 3 \\ &= 2x^4 - 4x^3 + 4x^2 - 3 \end{aligned}$$

$f'(x) = 8x^3 - 12x^2 + 8x$ A1

ii. $f'(x) = 8x^3 - 12x^2 + 8x$

$$\begin{aligned} f'(-1) &= 8 \times (-1)^3 - 12 \times (-1)^2 + 8 \times (-1) \\ &= -8 - 12 - 8 \\ &= -28 \end{aligned}$$ A1

Note: Consequential on answer to ***Question 4b.i.***

c. To find any stationary points, solve $f'(x) = 0$.

$f'(x) = 0$

$8x^3 - 12x^2 + 8x = 0$

$4x(2x^2 - 3x + 2) = 0$

$4x = 0$ or $2x^2 - 3x + 2 = 0$

$x = 0$ or $2x^2 - 3x + 2 = 0$ M1

For $2x^2 - 3x + 2 = 0$:

Finding the discriminant for $2x^2 - 3x + 2$ gives:

$\Delta = b^2 - 4ac$, where $a = 2$, $b = -3$ and $c = 2$

$= (-3)^2 - 4 \times 2 \times 2$

$= 9 - 16$

$= -7$

Since the discriminant is less than zero, no solutions exist.

Therefore, there is only one stationary point, which occurs when $x = 0$. M1

The y-coordinate of the stationary point is found by substituting $x = 0$ into the original equation.

$f(0) = 2 \times 0 \times (0^3 - 2 \times 0^2 + 2 \times 0) - 3$

$= -3$

Testing either side of $f'(0) = 0$ to see if the gradient is positive or negative gives, for example:

$f'(-1) = 8(-1)^3 - 12(-1)^2 + 8(-1) = -28$

Therefore, the gradient is negative.

$f'(1) = 8(1)^3 - 12(1)^2 + 8(1) = 4$

Therefore, the gradient is positive.

Note: Allow other correct calculations of the gradient.

Hence, the gradient profile is:

Negative gradient: $f'(-1) = -28$

Stationary point: $f'(0) = 0$

Positive gradient: $f'(1) = 4$ M1

Therefore, the coordinates of the stationary point are (0, –3).

As this is an upright, quartic graph, the stationary point is a minimum. A1

Note: Consequential on answer to ***Question 4b.i.***

Question 5 (4 marks)

a. For tangent graphs, the general form is $f(x) = a\tan(xn)$, where the period is $P = \frac{\pi}{n}$.

Given that $h(x) = 2\tan\left(\frac{x}{4}\right)$, $n = \frac{1}{4}$.
Hence:

$$P = \frac{\pi}{\frac{1}{4}}$$
$$= 4\pi$$

The graph shown is symmetrical about the y-axis and the point (0, 0). Therefore, the asymptotes must be at -2π and 2π.

Hence, $c = 2\pi$. A1

b. To find the average rate of change of h between $x = 0$ and $x = \pi$, the values of $h(0)$ and $h(\pi)$ must be found.

$$h(0) = 2\tan(0)$$
$$= 0$$

Therefore, the coordinates at $h(0)$ are (0, 0). M1

$$h(\pi) = 2\tan\left(\frac{\pi}{4}\right)$$
$$= 2 \times 1$$
$$= 2$$

Therefore, the coordinates at $h(\pi)$ are $(\pi, 2)$. M1

The average rate of change can be found by calculating the gradient of the line between the points (0, 0) and $(\pi, 2)$.

$$\text{average rate } (a \text{ to } b) = \frac{f(b) - f(a)}{b - a}$$
$$= \frac{2 - 0}{\pi - 0}$$
$$= \frac{2}{\pi}$$
A1

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Question 6 (5 marks)

a. $2\cos(2x)+1=0$ for $x \in [0, \pi]$

$$2\cos(2x)=-1$$

$$\cos(2x)=-\frac{1}{2}$$

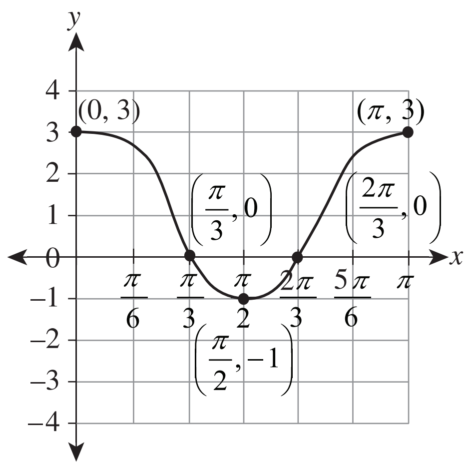
$$2x=\frac{2\pi}{3},\frac{4\pi}{3}$$ M1

$$x=\frac{2\pi}{6},\frac{4\pi}{6}$$

$$=\frac{\pi}{3},\frac{2\pi}{3} \text{ as } x \in [0, \pi]$$ A1

b. $P=\frac{2\pi}{n}=\frac{2\pi}{2}=\pi$

The graph has an amplitude of 2, and the mid-line is at $y = 1$.



endpoints at (0, 3) and (π, 3) A1

x-intercepts at $\left(\frac{\pi}{3}, 0\right)$ *and* $\left(\frac{2\pi}{3}, 0\right)$ *and minimum point at* $\left(\frac{\pi}{2}, -1\right)$ A1

correct shape A1

Note: Consequential on answer to ***Question 6a.***

Question 7 (3 marks)

The general equation of a quadratic is $y = ax^2 + bx + c$.

Given that the coordinates of the y-intercept are (0, –7), $c = -7$ as this is the y-intercept.

Substituting the point (–3, –70) into the general equation gives:

$$-70 = a(-3)^2 + b(-3) - 7$$
$$-70 = 9a - 3b - 7$$
$$9a - 3b + 63 = 0$$
$$b = 3a + 21 \qquad \text{(equation 1)}$$

Substituting the point (3, 2) into the general equation gives:

$$2 = a(3)^2 + b(3) - 7$$
$$2 = 9a + 3b - 7$$
$$3a + b - 3 = 0 \qquad \text{(equation 2)}$$

Substituting equation 1 into equation 2 gives:

$$3a + 3a + 21 - 3 = 0$$ M1

$$6a + 18 = 0$$
$$a = -3$$

Substituting $a = -3$ into equation 1 gives:

$$b = 3 \times (-3) + 21$$
$$= 12$$

Hence, $y = -3x^2 + 12x - 7$ A1

Converting the equation into turning point form gives:

$$y = -3\left(x^2 - 4x + \frac{7}{3}\right)$$
$$= -3\left(x^2 - 4x + 4 - 4 + \frac{7}{3}\right)$$
$$= -3\left[(x-2)^2 - 4 + \frac{7}{3}\right]$$
$$= -3\left[(x-2)^2 - \frac{5}{3}\right]$$
$$y = -3(x-2)^2 + 5$$ A1

Question 8 (4 marks)

a. The total number of students is 5. Hence, the number of arrangements is 5! = 120. A1

b. The total number of arrangements are 3! × 2! × 3 = 36. If the three girls are sitting together, they could be sitting at the start of the group OR with one boy on each side of them OR at the end of the group. A1

c. **i.** $^3C_3 \times {}^2C_1 = 2$ A1

ii. At least two girls means two girls or three girls.

$$^3C_3 \times {}^2C_1 + {}^3C_2 \times {}^2C_2 = 2 + 3$$
$$= 5$$ A1

*Note: Consequential on answer to **Question 8c.i.***

Question 9 (2 marks)

$$\lim_{x \to \frac{1}{3}} \frac{6x^2 + 7x - 3}{3x - 1} = \lim_{x \to \frac{1}{3}} \frac{(3x - 1)(2x + 3)}{3x - 1} \quad \text{(factorising the quadratic)}$$ M1

$$= \lim_{x \to \frac{1}{3}} 2x + 3,\ x \neq \frac{1}{3}$$

$$= 2\left(\frac{1}{3}\right) + 3 \quad \text{(taking the limit)}$$

$$= \frac{11}{3}$$ A1

Question 10 (4 marks)

a. $f'(x) = -2x(3x - 2) - 4$

$= -6x^2 + 4x - 4$

Integrating the equation gives:

$$f(x) = \frac{-6x^3}{3} + \frac{4x^2}{2} - 4x + c$$

$$= -2x^3 + 2x^2 - 4x + c$$ M1

Given that $f(-2) = 8$, solving for c gives:

$8 = -2 \times (-2)^3 + 2 \times (-2)^2 - 4(-2) + c$

$8 = 32 + c$

$c = -24$ M1

Hence, $f(x) = -2x^3 + 2x^2 - 4x - 24$ as required.

b. $$\int_0^2 \left(3x^2 - 4x + 6\right) \cdot dx = \left[\frac{3x^3}{3} - \frac{4x^2}{2} + 6x\right]_0^2$$ M1

$$= \left[x^3 - 2x^2 + 6x\right]_0^2$$

$$= \left(2^3 - 2(2)^2 + 6(2)\right) - 0$$

$$= 12$$ A1