The Mathematical Association of Victoria

Trial Exam 2022

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room : any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages,
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Find $\frac{d}{dx}\left(-3e^{x^2+1}\right)$. 1 mark

b. Let $f(x)=2\sin(2x)\cos\left(x+\frac{\pi}{4}\right)$.

Find $f'(\pi)$. 3 marks

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Question 2 (3 marks)

Evaluate $\int_{-1}^{1}\left(\frac{1}{(3-2x)^2}+5\right)dx$.

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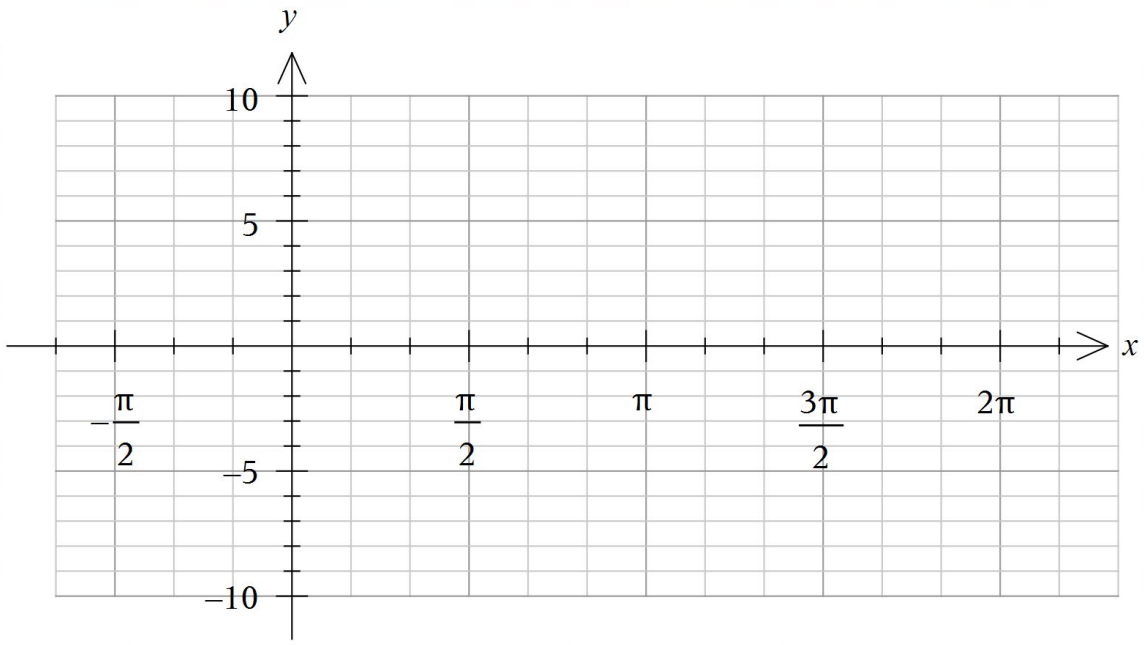
Question 3 (6 marks)

a. Find the solutions to the equation $3\tan\left(2x-\frac{\pi}{2}\right)+\sqrt{3}=0$ for $x\in(0,2\pi)$. 3 marks

b. Sketch the graph of $y=3\tan\left(2x-\frac{\pi}{2}\right)+\sqrt{3}$ for $x\in\left[\frac{\pi}{6},\frac{5\pi}{3}\right]$ on the set of axes below.

Label the axial intercepts with their coordinates and any asymptotes with their equations. 3 marks



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Question 4 (3 marks)
Solve $2\log_3(x+2) - \log_3\left(2x^2 + x - 6\right) = 2$ for x.

Question 5 (4 marks)
Consider the polynomial $p(x) = 2x^3 - 4x^2 + 6x - 12$.

a. Factorise $p(x)$ over R, and explain why there is only one linear factor. 2 marks

b. Show that the graph of p has no stationary points. 2 marks

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Question 6 (3 marks)

Given that $\cos(x) = -\frac{1}{3}$ for $x \in \left[\pi, \frac{3\pi}{2}\right]$, evaluate

a. $\sin\left(x + \frac{3\pi}{2}\right)$ 1 mark

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b. $\cos\left(x + \frac{\pi}{2}\right)$ 2 marks

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Question 7 (7 marks)
There are three red balls and two blue balls in a bag. Three balls are drawn from the bag with replacement. Let X represent the number of red balls drawn.

a. Complete the following probability distribution table for X. 2 marks

x	0	1	2	3
$\Pr(X = x)$	$\frac{8}{125}$	$\frac{36}{125}$		

b. Given that at least one red ball is drawn, find the probability that only one of the balls is red. 1 mark

c. Find $\text{sd}(X)$. 2 marks

n balls are now drawn from the bag with replacement.

d. What is the least number of balls that need to be drawn to ensure that the probability of getting at least one red ball is greater than 0.9? 2 marks

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Question 8 (4 marks)

a. Show that $\frac{d}{dx}\left(\frac{1}{2}(2x-1)\log_e(2x-1)-x\right)=\log_e\left(2x-1\right)$. 1 mark

b. Hence find the area bounded by the graph of $f(x)=\log_e\left(2x-1\right)+1$, the x-axis and the line $x=2$. 3 marks

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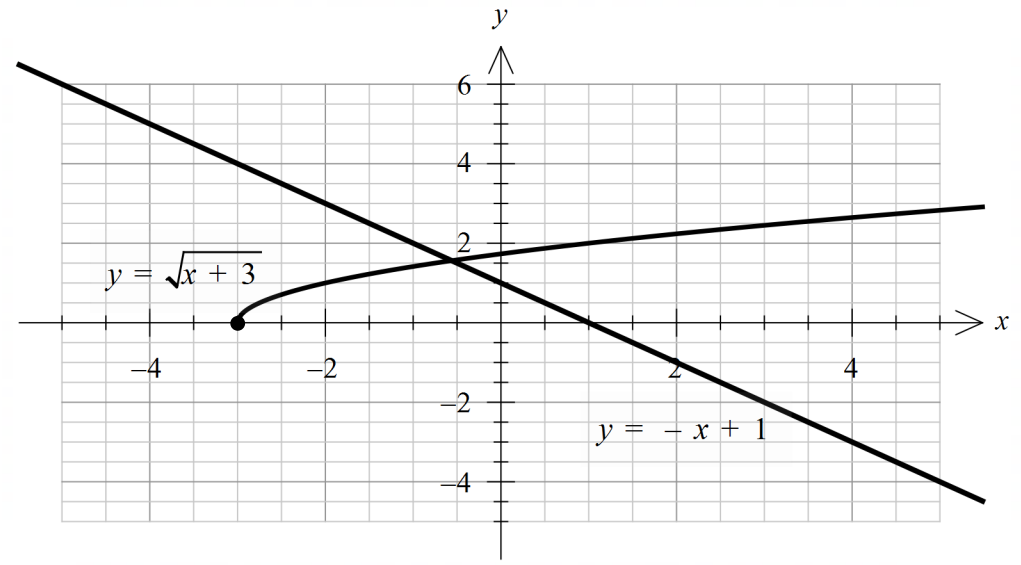
Question 9 (6 marks)

The graphs of $f:[-3,\infty) \to R, f(x) = \sqrt{x+3}$ and $g: R \to R, g(x) = -x+1$ are shown below.



The graph of $f(x)+g(x)$ has one turning point and no other stationary points.

a. Verify that the x-coordinate of the turning point of the graph of $f(x)+g(x)$ is $-\frac{11}{4}$. 1 mark

b. Sketch the graph of $f(x)+g(x)$ on the set of axes above. Label the endpoint and the turning point with their coordinates. 3 marks

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Let $h:\left[-\frac{11}{4},\infty\right) \to R$, where $h(x)=\sqrt{x+3}-x+1$.

c. Find the coordinates of the point where $h(x)=h^{-1}(x)$. 2 marks

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END OF QUESTION AND ANSWER BOOK

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