The Mathematical Association of Victoria

Trial Examination 2022

MATHEMATICAL METHODS

Trial Written Examination 1 - SOLUTIONS

Question 1

a. $\frac{d}{dx}\left(-3e^{x^2+1}\right)$

$=-3e^{x^2+1}\times 2x$

$=-6xe^{x^2+1}$ **1A**

b. $f(x)=2\sin(2x)\cos\left(x+\frac{\pi}{4}\right)$

$f'(x)=2\sin(2x)\times -\sin\left(x+\frac{\pi}{4}\right)+\cos\left(x+\frac{\pi}{4}\right)\times 4\cos(2x)$

$f'(x)=-2\sin(2x)\sin\left(x+\frac{\pi}{4}\right)+4\cos\left(x+\frac{\pi}{4}\right)\cos(2x)$ **1M**

$f'(\pi)=-2\sin(2\pi)\sin\left(\frac{5\pi}{4}\right)+4\cos\left(\frac{5\pi}{4}\right)\cos(2\pi)$

$=0+4\left(-\frac{\sqrt{2}}{2}\right)\times 1$ **1M**

$=-2\sqrt{2}$ **1A**

Question 2

$\int_{-1}^{1}\left(\frac{1}{(3-2x)^2}+5\right)dx$

$=\int_{-1}^{1}\left((3-2x)^{-2}+5\right)dx$

$=\left[\frac{(3-2x)^{-1}}{-2\times -1}+5x\right]_{-1}^{1}$ **1M**

$=\left[\frac{1}{2(3-2x)}+5x\right]_{-1}^{1}$

$=\left(\frac{1}{2}+5\right)-\left(\frac{1}{10}-5\right)$ **1M**

$=\frac{11}{2}+\frac{49}{10}$

$=\frac{52}{5}$ **1A**

Question 3

a. $3\tan\left(2x-\frac{\pi}{2}\right)+\sqrt{3}=0$ for $x\in(0,2\pi)$

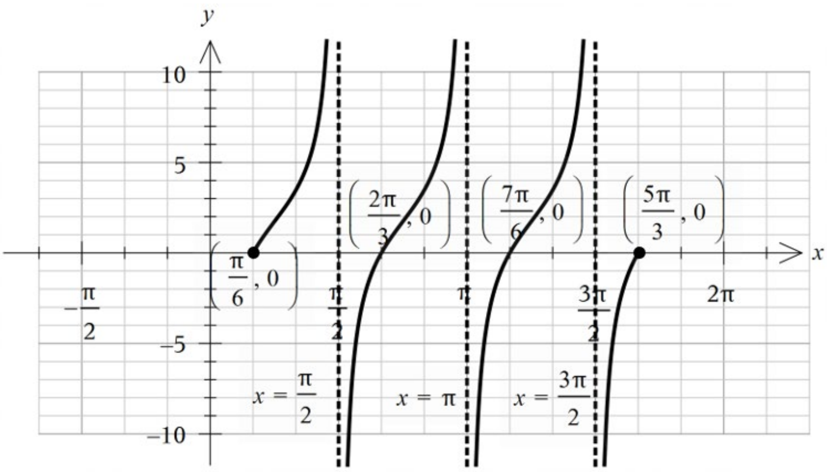
$\tan\left(2x-\frac{\pi}{2}\right)=-\frac{\sqrt{3}}{3}$

$2x-\frac{\pi}{2}=-\frac{\pi}{6},\frac{5\pi}{6},\frac{11\pi}{6},\frac{17\pi}{6}$ **1M**

$2x=\frac{2\pi}{6},\frac{8\pi}{6},\frac{14\pi}{6},\frac{20\pi}{6}$

$x=\frac{\pi}{6},\frac{2\pi}{3},\frac{7\pi}{6},\frac{5\pi}{3}$ **1M, 1A**

b. Shape **1A**
Coordinates of axial intercepts **1A**
Asymptotes with equations **1A**



Question 4

$2\log_3(x+2)-\log_3\left(2x^2+x-6\right)=2$

$\log_3\left((x+2)^2\right)-\log_3\left(2x^2+x-6\right)=2$

$\log_3\left(\frac{(x+2)^2}{2x^2+x-6}\right)=2$ **1M**

$\frac{(x+2)^2}{2x^2+x-6}=9$

$\frac{(x+2)^2}{(2x-3)(x+2)}=9$ **1M**

$\frac{x+2}{2x-3}=9,\ x\neq-2$

$x+2=9(2x-3)$

$x+2=18x-27$

$17x=29$

$x=\frac{29}{17}$ **1A**

Question 5

a. $p(x) = 2x^3 - 4x^2 + 6x - 12$

$p(x) = 2\left(x^3 - 2x^2 + 3x - 6\right)$

$p(x) = 2\left[x^2(x-2) + 3(x-2)\right]$

$\therefore p(x) = 2(x-2)\left(x^2+3\right)$ **1M**

$\left(x^2+3\right)$ cannot be factorised over R

$(x-2)$ is the only linear factor **1A**

b. $p(x) = 2x^3 - 4x^2 + 6x - 12$

$p'(x) = 6x^2 - 8x + 6$ **1M**

For stationary points $p'(x) = 0$

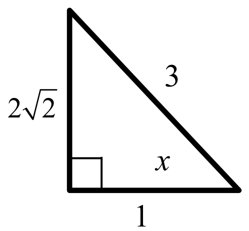
$\Delta = (-8)^2 - 4 \times 6 \times 6 = -80$

No solution as $\Delta < 0$ **1A**

Graph of p has no stationary points

Question 6

$\cos(x) = -\frac{1}{3}$ for $x \in \left[\pi, \frac{3\pi}{2}\right]$



a. $\sin\left(x + \frac{3\pi}{2}\right)$

$= -\cos(x)$

$= \frac{1}{3}$ **1A**

b. $\cos\left(x + \frac{\pi}{2}\right)$

$= -\sin(x)$ **1M**

$= \frac{2\sqrt{2}}{3}$ **1A**

Question 7

a. $X \sim \text{Bi}\left(3, \frac{3}{5}\right)$

$\Pr(X = 2)$

$= \binom{3}{2}\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)$

$= 3 \times \frac{9}{25} \times \frac{2}{5}$

$= \frac{54}{125}$ **1A**

$\Pr(X = 3)$

$= \binom{3}{3}\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^0$

$= \frac{27}{125}$ **1A**

OR

$1 - \left(\frac{8}{125} + \frac{36}{125} + \frac{54}{125}\right)$

$= \frac{27}{125}$ **1A**

x	0	1	2	3
$\Pr(X = x)$	$\frac{8}{125}$	$\frac{36}{125}$	$\frac{54}{125}$	$\frac{27}{125}$

b. $\Pr(X = 1 \mid X \geq 1)$

$= \frac{\Pr(X = 1 \cap X \geq 1)}{\Pr(X \geq 1)}$

$= \frac{\Pr(X = 1)}{\Pr(X \geq 1)}$

$= \frac{\frac{36}{125}}{\frac{117}{125}}$

$= \frac{36}{117} = \frac{4}{13}$ **1A**

c. $\text{sd}(X) = \sqrt{np(1-p)}$

$= \sqrt{3 \times \frac{3}{5} \times \frac{2}{5}}$ **1M**

$= \sqrt{\frac{18}{25}} = \frac{3\sqrt{2}}{5}$ **1A**

Question 7 (continued)

d. $X_1 \sim \text{Bi}\left(n, \frac{3}{5}\right)$

$\Pr\left(X_1 \geq 1\right) > 0.9$

$1 - \Pr\left(X_1 = 0\right) > 0.9$

$\Pr\left(X_1 = 0\right) \leq 0.1$

$\binom{n}{0}\left(\frac{3}{5}\right)^0\left(\frac{2}{5}\right)^n \leq 0.1$

$\left(\frac{2}{5}\right)^n \leq 0.1$ **1M**

$\left(\frac{2}{5}\right)^0 = 1,\ \left(\frac{2}{5}\right)^1 = 0.4,\ \left(\frac{2}{5}\right)^2 = 0.8,\ \left(\frac{2}{5}\right)^3 = \frac{8}{125} < 0.1$

$n = 3$ **1A**

Question 8

a. $\frac{d}{dx}\left(\frac{1}{2}(2x-1)\log_e(2x-1) - x\right) = \log_e\left(2x-1\right)$

$\frac{d}{dx}\left(\frac{1}{2}(2x-1)\log_e(2x-1) - x\right)$

$= \log_e\left(2x-1\right) + \frac{1}{2}\left(2x-1\right) \times \frac{2}{2x-1} - 1$

$= \log_e\left(2x-1\right) + 1 - 1$

$= \log_e\left(2x-1\right)$ **1M** *show that*

b. $f(x) = \log_e\left(2x-1\right) + 1$

Solve $\log_e\left(2x-1\right) + 1 = 0$

$2x - 1 = \frac{1}{e}$

$x = \frac{1}{2} + \frac{1}{2e}$ **1A**

$\int_{\frac{1}{2}+\frac{1}{2e}}^{2} \left(\log_e\left(2x-1\right) + 1\right) dx$

$= \left[\frac{1}{2}\left(2x-1\right)\log_e\left(2x-1\right) - x + x\right]_{\frac{1}{2}+\frac{1}{2e}}^{2}$ **1M**

$= \left[\frac{1}{2}\left(2x-1\right)\log_e\left(2x-1\right)\right]_{\frac{1}{2}+\frac{1}{2e}}^{2}$

$= \frac{3}{2}\log_e(3) - \frac{1}{2}\left(1 + \frac{1}{e} - 1\right)\log_e\left(1 + \frac{1}{e} - 1\right)$

$= \frac{3}{2}\log_e(3) - \frac{1}{2e}\log_e\left(\frac{1}{e}\right) = \frac{3}{2}\log_e(3) + \frac{1}{2e}$ **1A**

Question 9

a. $f(x)+g(x)=\sqrt{x+3}-x+1$

$\frac{d}{dx}\left(f(x)+g(x)\right)$

$=\frac{1}{2\sqrt{x+3}}-1$

Substitute $x=-\frac{11}{4}$

$=\frac{1}{2\sqrt{-\frac{11}{4}+3}}-1$

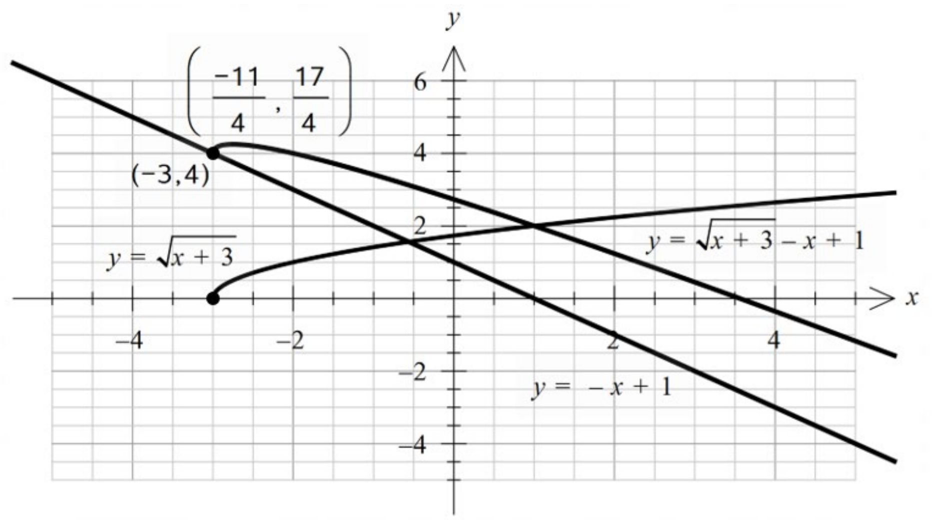
$=\frac{1}{2\sqrt{\frac{1}{4}}}-1$

$=1-1=0$ **1M** *Verify*

b. Shape **1A**

Axial intercepts and point of intersection with $y=-x+1$ in the correct positions **1A**

Coordinates **1A**



c. The graph of $h(x)$ and $h^{-1}(x)$ intersect along the line with equation $y=x$.

Solve $h(x)=x$ for *x*.

$\sqrt{x+3}-x+1=x$ **1M**

$\sqrt{x+3}=2x-1$

$x+3=(2x-1)^2$

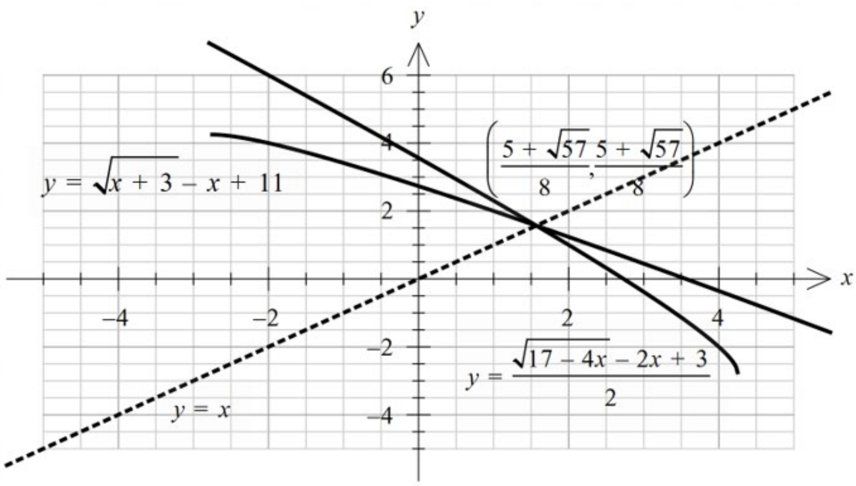
$x+3=4x^2-4x+1$

$4x^2-5x-2=0$

$x=\frac{5\pm\sqrt{57}}{8},\ x>0$

$\left(\frac{5+\sqrt{57}}{8},\frac{5+\sqrt{57}}{8}\right)$ **1A**

Question 9 (continued)



END OF SOLUTIONS