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Mathematical Methods

2021

Trial Examination 1
(1 hour)

Instructions

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

Solve the following equations for *x*.

a. $(x-1)^{\frac{2}{3}}-(x-2)^{\frac{2}{3}}=0$ 2 marks

b. $e^{4x-4}=2e^{2x-2}+3$ 2 marks

Question 2

Consider the rule of function f, $f(x)=\dfrac{(x-1)(x^3+2x^2+2x+1)}{x^2-1}$.

a. Express the rule in simplest form. 2 marks

b. Find $f'(x)$. 1 mark

c. Sketch the graph of f in the interval $[-2,3]$. Label typical points with coordinates.

3 marks

Question 3

Random variable X has probability distribution $\text{Bi}(n, p)$ and $\Pr(X = 1) = (1 - p)^{n-2}$, $p < 1$. Find the possible values of n and the value(s) of p in terms of n.

3 marks

Question 4

Five samples of size n are taken from a large population. The 70% and 90% confidence intervals for sample proportion $\hat{P}$ are $(0.3896, 0.4104)$ and $(0.3836, 0.4164)$ respectively.

a. Find $\Pr\left(0.3836 < \hat{P} < 0.4104 \mid \hat{P} > 0.3836\right)$. 2 marks

b. Given $\Pr(-1.04 < Z < 1.04) \approx 0.70$, estimate the value of n to the nearest hundred.

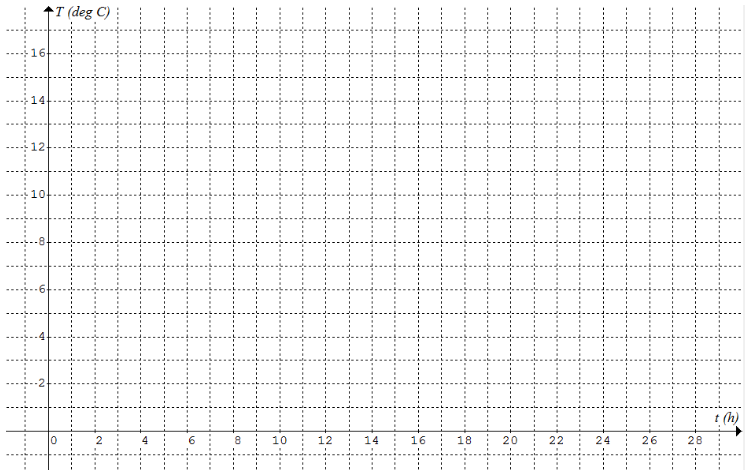
2 marks

Question 5

The outdoor air temperature (in $°C$) over a period of 24 hours is given by

$$T(t) = 8 - 5\sin\left(\pi\left(\frac{t}{12} - \frac{1}{6}\right)\right),\ t \in [0, 24].$$

a. Sketch a graph of the outdoor temperature $T(t)$. 2 marks



b. Determine the average temperature over the 24 hour period. 1 mark

c. Calculate the average rate of change in temperature from its minimum to its maximum.

1 mark

d. Over the 24 hour period, $T > \alpha$ for 20 hours. Determine the value of α. 2 marks

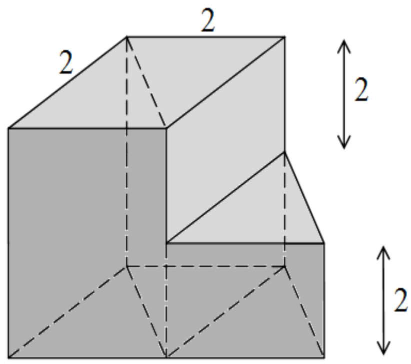
e. If you put your clock forward an hour for daylight saving, T in terms of daylight saving time t over the same 24 hour period can be expressed as $T(t) = 8 - 5\sin\left(\pi\left(\frac{t}{12} + b\right)\right)$.

Find the value of b. 2 marks

Question 6

The following diagram shows a large water tank with one section on top of a larger section. Both sections have the same height of 2 metres.

The top and the base planes are parallel, and they consist of two and three congruent equilateral triangles respectively. The area of each triangle is $\sqrt{3}$ m^2.



Initially ($t = 0$) the tank is full. At time $t \geq 0$ seconds the depth of water is h metres.

a. Write a piecewise function for $V(h)$ m^3 in simplest factorised form. 2 marks

b. Water is drained at a constant rate of $\frac{1}{500}$ m^3 per second.

Find $\frac{dh}{dt}$ in m s^{-1} when $h > 2$. 2 marks

Question 7

Given $f(x)=\log_e x$ and $g(x)=\sqrt{ax+b}$ such that $f(e)=g(e)$ and $f'(e)=g'(e)$,

a. find the values of a and b 2 marks

b. and show that $g'(x)>f'(x)$ for $x\in\left(\frac{e}{2},e\right)\cup(e,\infty)$. 3 marks

Question 8

Function $f(x)$ is odd, periodic and differentiable for $x\in R$.

The period of f is 7, $f(-2)=2$ and $f'(-2)=\frac{1}{3}$.

a. Find the value of $f(-5)$. 1 mark

b. Find the value of $f'(-5)$. 1 mark

c. Find the value of $\int_{-5}^{9} f(x)dx$. 1 mark

Question 9

Let $f(x)=e^{x}+\log_{e}x$, $b=f(a)$ and $g(x)=f^{-1}(x)$.

a. State the maximal domain of $f(x)$. 1 mark

b. Find $g'(b)$ in terms of a. 2 marks

End of Exam