



MATHEMATICAL METHODS 2020

Unit 3

Key Topic Test 5 – Exponential & Logarithmic Functions Technology Free

Recommended writing time*: 45 minutes

Total number of marks available: 30 marks

SOLUTIONS

Question 1

a. $e^{2x} - 2 = 0$
 $e^{2x} = 2$
 $2x = \log_e 2$
 $x = \frac{1}{2} \log_e 2$ or $\log_e \sqrt{2}$ 1 mark

b. $4^x = 2^{x-5}$
 $2^{2x} = 2^{x-5}$
 $2x = x - 5$
 $x = -5$ 1 mark

c. $4^x - 8 \times 2^x = -12$
 $2^{2x} - 8 \times 2^x + 12 = 0$ 1 mark

Let $a = 2^x$
 $a^2 - 8a + 12 = 0$
 $(a - 6)(a - 2) = 0$ 1 mark

$a = 2, 6$
 $2^x = 2, x = 1$

 $2^x = 6, x = \log_2 6$ 1 mark

d. $e^t + 5 = 6e^{-t}$
 $e^t(e^t + 5) = e^t(6e^{-t})$ 1 mark

$e^{2t} + 5e^t = 6$
 $e^{2t} + 5e^t - 6 = 0$
 Let $e^t = a$

$a^2 + 5a - 6 = 0$
 $(a + 6)(a - 1) = 0$
 $a = -6, 1$

$e^t = -6, \text{no solution}$ 1 mark
 $e^t = 1, t = 0$ 1 mark

Question 2

a. $2 - x > 0$
 $x < 2$
 $a = 2$ 1 mark

b. Let $x = \log_e(2 - y)$ 1 mark

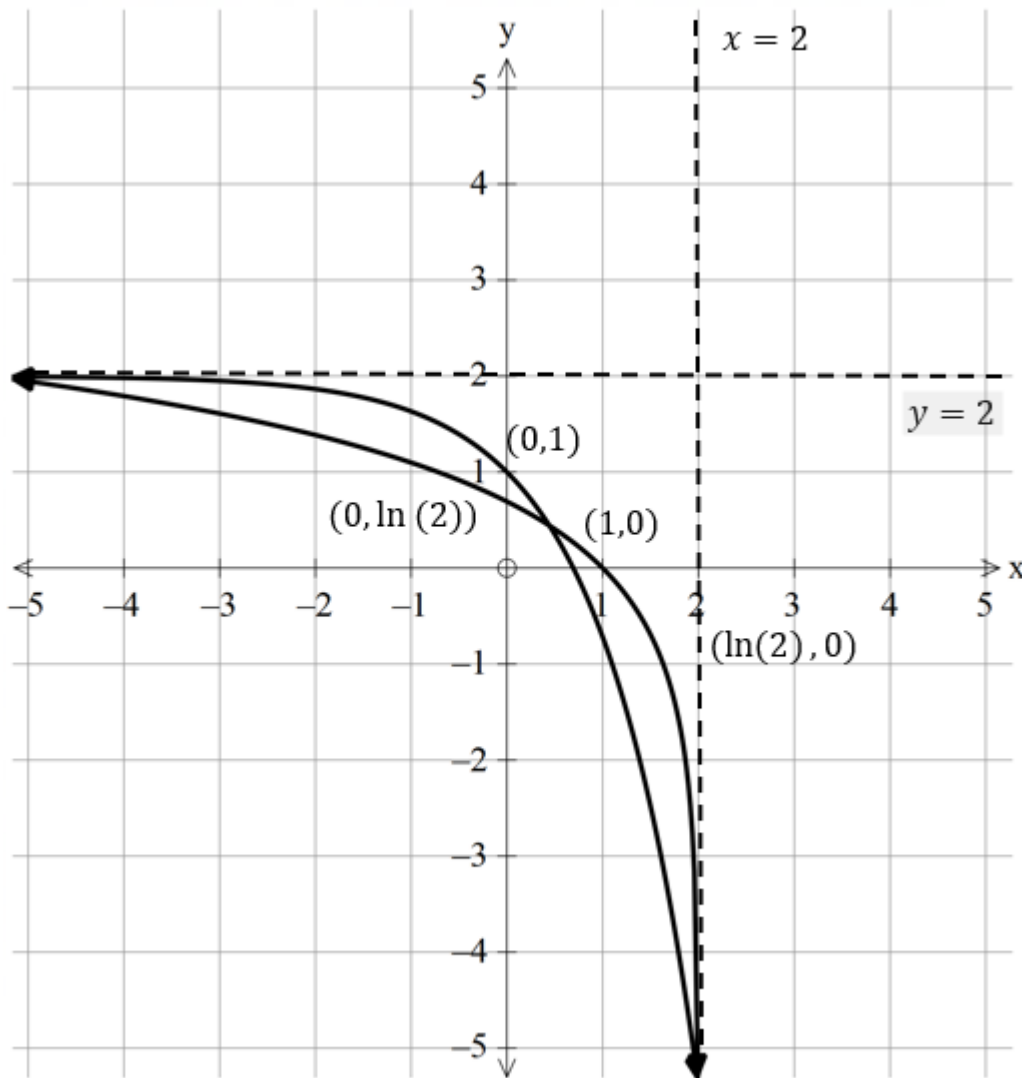
$$e^x = 2 - y$$

$$y = 2 - e^x$$

$f^{-1}(x) = g(x) = 2 - e^x$ 1 mark

Domain $f^{-1}(x) = \text{range of } f(x)$
 $= R$ 1 mark

c.



1 mark asymptotes
 1 mark $(1, 0)$ and $(0, 1)$
 1 mark $(\ln(2), 0)$ and $(0, \ln(2))$
 1 mark shape

Question 3

a. $2\log_e(x) - \log_e(x + 10) = \log_e\left(\frac{1}{2}\right)$
 $\log_e x^2 - \log_e(x + 10) = \log_e\left(\frac{1}{2}\right)$
 $\log_e\left(\frac{x^2}{x+10}\right) = \log_e\left(\frac{1}{2}\right)$ 1 mark
 $\frac{x^2}{x+10} = \frac{1}{2}$
 $2x^2 = x + 10$
 $2x^2 - x - 10 = 0$ 1 mark
 $(2x - 5)(x + 2) = 0$
 $x = -2, \frac{5}{2}$ 1 mark

As $x > 0$
 $x = \frac{5}{2}$ 1 mark

b. $\log_2(4 - x) - \log_2(2 - x) = 2$
 $\log_2\left(\frac{4-x}{2-x}\right) = 2$ 1 mark
 $2^2 = \frac{4-x}{2-x}$
 $8 - 4x = 4 - x$
 $4 = 3x$
 $x = \frac{4}{3}$ 1 mark

Since $x < 2$, this solution fits the domain

Question 4

a. $f(u) \times f(-u) = (e^{2u} - e^{-u})(e^{-2u} - e^u)$ 1 mark
 $= e^0 - e^{3u} - e^{-3u} + e^0$
 $= 2 - e^{3u} - e^{-3u}$ 1 mark

b. $f(x) = e^{2x} - e^{-x}$
 $= e^x(e^x - e^{-2x})$ 1 mark
 $= e^x\left(e^x - \frac{1}{e^{2x}}\right)$
 $= e^x\left(\frac{e^{3x}-1}{e^{2x}}\right)$ 1 mark

c. Let $x = \log_e \sqrt{\frac{y}{2}}$ 1 mark

$$e^x = \sqrt{\frac{y}{2}}$$

$$e^{2x} = \frac{y}{2}$$

$$y = 2e^{2x}$$

$$h(x) = 2e^{2x} \quad \text{1 mark}$$

$$f(x) + g(x) = 2e^{2x}$$

$$g(x) = 2e^{2x} - f(x) \quad \text{1 mark}$$

$$g(x) = 2e^{2x} - (e^{2x} - e^{-x})$$

$$= 2e^{2x} - e^{2x} + e^{-x}$$

$$= e^{2x} + e^{-x} \quad \text{1 mark}$$