

The Mathematical Association of Victoria

Trial Examination 2020

MATHEMATICAL METHODS

Trial Written Examination 2 - SOLUTIONS

SECTION A: Multiple Choice

Question	Answer	Question	Answer
1	B	11	C
2	B	12	D
3	A	13	B
4	D	14	E
5	E	15	A
6	C	16	D
7	C	17	E
8	E	18	B
9	C	19	B
10	A	20	A

Question 1

Answer B

$$y = \frac{a}{x-5} + 6$$

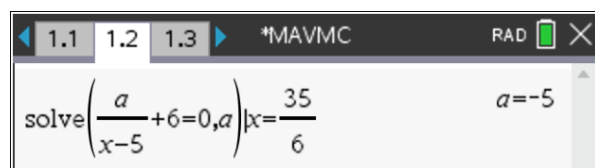
$$\frac{a}{\frac{35}{6} - 5} + 6 = 0$$

$$\frac{6a}{35 - 30} = -6$$

$$a = -5$$

$$y = \frac{-5}{x-5} + 6$$

$$y = \frac{5}{5-x} + 6$$

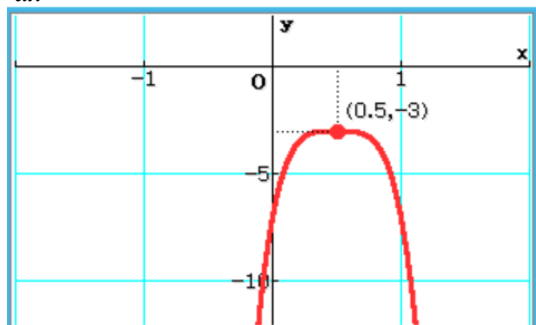


The screenshot shows a calculator interface with the following text: "1.1 1.2 1.3 *MAVMC RAD X solve(a/(x-5) + 6 = 0, a) | x = 35/6 a = -5".

Question 2**Answer B**

The graph of $y = -4(2x - 1)^4 - 3$ is quartic reflected in the x -axis.

$\frac{dy}{dx} = -32(2x - 1)^3 = 0$ gives stationary point at $x = \frac{1}{2}$ but **not** a point of inflexion



At the point $\left(\frac{1}{2}, -3\right)$ the gradient is zero.

Question 3**Answer A**

Solving $\cos(2x) = \cos(x)$

`solve(cos(2*x)=cos(x), x)`

$$\left\{ x = 2 \cdot \pi \cdot \text{constn}(1), x = \frac{2 \cdot \pi \cdot \text{constn}(2)}{3} \right\}$$

We read this as $x = 2\pi k, x = \frac{2\pi k}{3}$ where $k \in \mathbb{Z}$

Option A : $x = \frac{2\pi k}{3}$ where $k \in \mathbb{Z}$, also includes the set of solutions $x = 2\pi k$

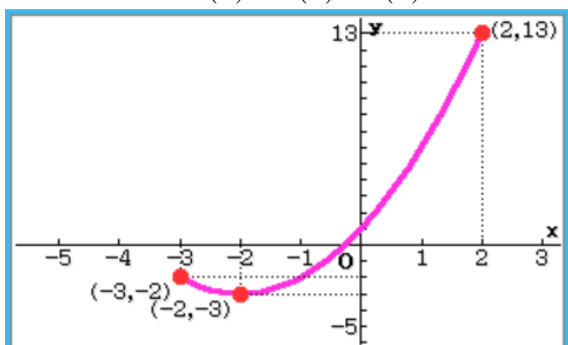
x	y1	y2
1	6.2832	2.0944
2	12.566	4.1888
3	18.850	6.2832
4	25.133	8.3776
5	31.416	10.472
6	37.699	12.566
7	43.982	14.661
8	50.265	16.755
9	56.549	18.850
10	62.832	20.944
11	69.115	23.038
12	75.398	25.133
13	81.681	27.227
14	87.965	29.322
15	94.248	31.416
16	100.53	33.510
17	106.81	35.605
18	113.10	37.699

x	y1	y2
1	6.2832	2.0944
2	12.566	4.1888
3	18.850	6.2832
4	25.133	8.3776
5	31.416	10.472
6	37.699	12.566

Question 4**Answer D**

$$f: [-3, 4) \rightarrow \mathbb{R}, f(x) = 2x + 1 \text{ and } g: [-4, 2] \rightarrow \mathbb{R}, g(x) = x^2 + 2x.$$

The domain of $h(x) = f(x) + g(x)$ is the intersection of the domains of f and $g = [-3, 2]$



The range of $h(x) = f(x) + g(x)$ is $[-3, 13]$

Question 5**Answer E**

$$y = \frac{1}{2x} \text{ to be transformed to } y_T = -\frac{3}{x-1} + 6.$$

Step 1: to change $y = \frac{1}{2x}$ to $y = \frac{1}{x}$ we can dilate from the x -axis by a factor of 2.

$$y = \frac{1}{2x} \Rightarrow y_1 = \frac{2}{2x} \Rightarrow y_1 = \frac{1}{x}$$

Step 2: to change $y_1 = \frac{1}{x}$ to $y_2 = \frac{3}{x}$ we can dilate from the y -axis by a factor of 3.

$$y_1 = \frac{1}{\left(\frac{x}{3}\right)} \Rightarrow y_2 = \frac{3}{x}$$

Step 3: to change $y_2 = \frac{3}{x}$ to $y_3 = -\frac{3}{x}$ we reflect in the x -axis

Step 4: to change $y_3 = -\frac{3}{x}$ to $y_4 = -\frac{3}{x-1} + 6$ we translate in the positive direction of the x -axis by 1 unit and the y -axis by 6 units.

This is the required image graph: $y_T = -\frac{3}{x-1} + 6$

Question 6**Answer C**

$$2x + ky = a$$

$$kx + 3y = 7$$

Method 1 (Using ratios)

The simultaneous equations will have no solutions when $\frac{k}{2} = \frac{3}{k} \neq \frac{7}{a}$ or $\frac{2}{k} = \frac{k}{3} \neq \frac{a}{7}$

$$\frac{k}{2} = \frac{3}{k}$$

$$k^2 = 6$$

$$k = \pm\sqrt{6}$$

When $k = \sqrt{6}$

$$\frac{\sqrt{6}}{2} \neq \frac{7}{a}$$

$$a \neq \frac{14}{\sqrt{6}}$$

When $k = -\sqrt{6}$

$$\frac{-\sqrt{6}}{2} \neq \frac{7}{a}$$

$$a \neq -\frac{14}{\sqrt{6}}$$

$$k = \sqrt{6} \text{ and } a \in R \setminus \left\{ \frac{14}{\sqrt{6}} \right\} \text{ or } k = -\sqrt{6} \text{ and } a \in R \setminus \left\{ -\frac{14}{\sqrt{6}} \right\}$$

Method 2 (using gradient and intercept)

Simultaneous equations will have no solutions when the gradients are equal and the y -intercepts are different.

$$2x + ky = a, y = -\frac{2}{k}x + \frac{a}{k},$$

$$kx + 3y = 7, y = -\frac{k}{3}x + \frac{7}{3}$$

$$m_1 = m_2$$

$$-\frac{2}{k} = -\frac{k}{3}$$

$$k^2 = 6$$

$$k = \pm\sqrt{6}$$

$$c_1 \neq c_2$$

$$\frac{a}{k} \neq \frac{7}{3}, a \neq \frac{7k}{3}$$

$$\text{If } k = \sqrt{6}, a \neq \frac{7\sqrt{6}}{3} \text{ i.e. } a \neq \frac{14}{\sqrt{6}}$$

$$\text{If } k = -\sqrt{6}, a \neq \frac{-7\sqrt{6}}{3} \text{ i.e. } a \neq \frac{-14}{\sqrt{6}}$$

Question 7**Answer C**

$$f(x) = 2 \log_e(1-4x) + 1$$

$$\text{Let } y = 2 \log_e(1-4x) + 1$$

Inverse swap x and y

$$x = 2 \log_e(1-4y) + 1$$

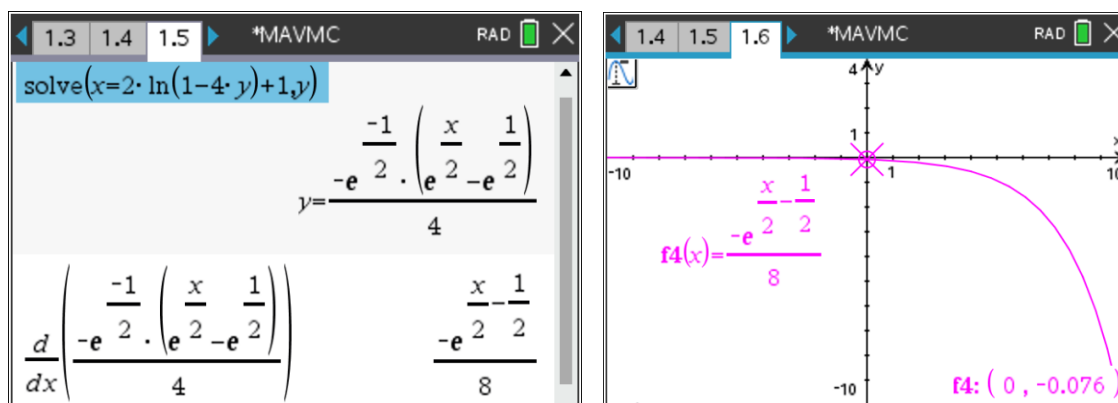
$$\log_e(1-4y) = \frac{x-1}{2}$$

$$1-4y = e^{\frac{x-1}{2}}$$

$$y = f^{-1}(x) = \frac{1}{4} \left(1 - e^{\frac{x-1}{2}} \right)$$

$$\frac{dy}{dx} = -\frac{1}{8} e^{\frac{x-1}{2}}$$

$$\frac{dy}{dx} = -\frac{1}{8} e^{\frac{x-1}{2}} \text{ has an asymptote with equation } y = 0 \text{ and a } y\text{-axis intercept at } -\frac{1}{8} e^{-\frac{1}{2}}.$$

**Question 8****Answer E**

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$x' = y + 2$$

$$y' = x - 3$$

$$y' = x'^{\frac{1}{3}} + 1$$

$$x - 3 = (y + 2)^{\frac{1}{3}} + 1$$

$$y = (x - 4)^3 - 2$$

$$f(x) = a(x - b)^3 + c$$

$$a = 1, b = 4, c = -2$$

OR

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$x' = y + 2$$

$$y' = x - 3$$

$$y = x' - 2 \text{ and } x = y' + 3$$

$$y = a(x - b)^3 + c \text{ and so } x' - 2 = a(y' + 3 - b)^3 + c$$

$$\frac{x' - 2 - c}{a} = (y' + 3 - b)^3 \text{ and so } (y' + 3 - b)^3 = \frac{x' - 2 - c}{a}$$

$$\text{Now } y' = x'^{\frac{1}{3}} + 1 \text{ and so } a = 1, c = -2, 3 - b = -1$$

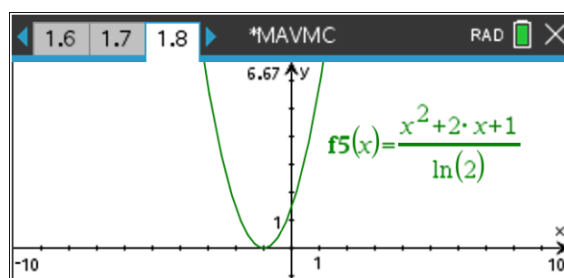
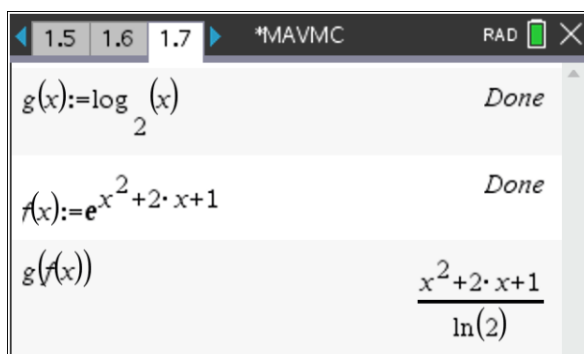
$$a = 1, b = 4, c = -2$$

Question 9**Answer C**

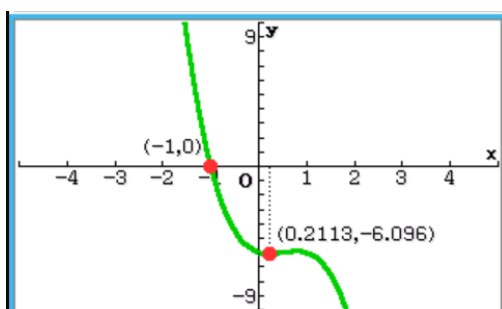
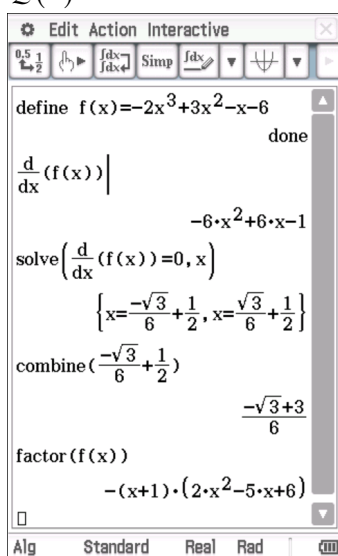
$$f(x) = e^{x^2 + 2x + 1} \text{ and } g(x) = \log_2(x)$$

$$g(f(x)) = \log_2\left(e^{(x+1)^2}\right)$$

$$g(f(x)) = \frac{(x+1)^2}{\log_e(2)}, \text{ range is } [0, \infty)$$

**Question 10****Answer A**

$$Q(x) = -2x^3 + 3x^2 - x - 6$$



One of the two stationary points of the graph is at $x = \frac{3 - \sqrt{3}}{6}$.

Question 11**Answer C**

$$\text{Let } y = f(\log_e(2x))$$

$$\text{Using the Chain rule } \frac{dy}{dx} = \frac{1}{x} f'(\log_e(2x))$$

Question 12**Answer D**

$$f(x) = ax^6 + bx^5 + x^4 - 3$$

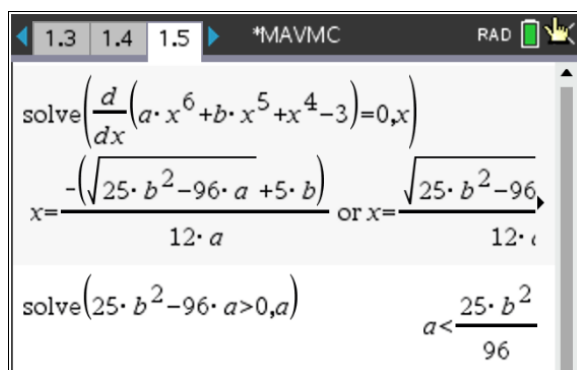
$$f'(x) = 6ax^5 + 5bx^4 + 4x^3$$

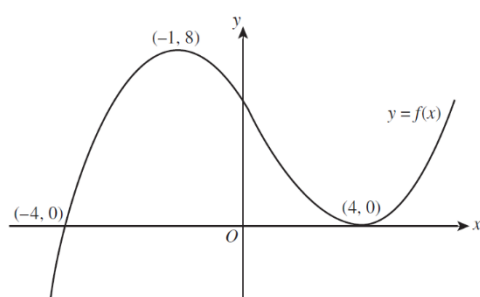
$$x^3(6ax^2 + 5bx + 4) = 0$$

$$x = 0, x = \frac{-5b \pm \sqrt{25b^2 - 96a}}{12a}$$

Two more stationary points when $25b^2 - 96a > 0$

$$a < \frac{25b^2}{96}$$



Question 13**Answer B**

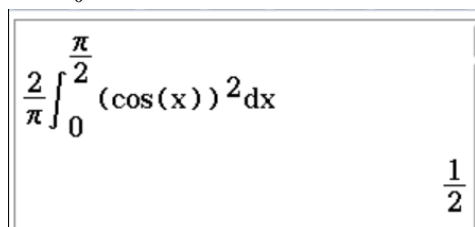
Options A, C, D and E are correct.

It is **incorrect** to state that the graph of the derivative of f is strictly increasing for $x \in (-\infty, -1] \cup [4, \infty)$.These values relate to the graph of f , **not** the graph of the derivative of f .**Question 14****Answer E**

$$\text{Average value} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$\text{Average value of } y = \cos^2(x) \text{ over the interval } \left[0, \frac{\pi}{2}\right] \text{ is } = \frac{1}{\frac{\pi}{2} - 0} \int_0^{\frac{\pi}{2}} (\cos^2(x)) dx.$$

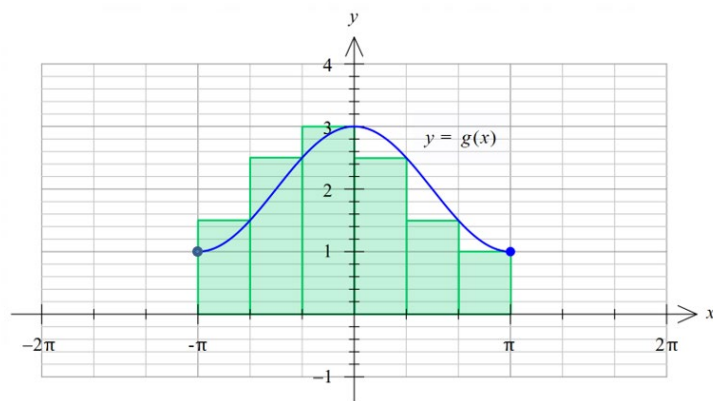
$$= \frac{2}{\pi} \int_0^{\frac{\pi}{2}} (\cos^2(x)) dx = \frac{1}{2}$$



Question 15**Answer A**

Given $\int_{-1}^6 f(x) dx = 3$, swap limits to get $\int_{-1}^6 -1 + 2f(x) dx$

$$\begin{aligned} \int_{-1}^6 -1 + 2f(x) dx &= \int_{-1}^6 -1 dx + 2 \int_{-1}^6 f(x) dx \\ &= -7 + 2 \int_{-1}^6 f(x) dx = -7 + 2 \times 3 = -1 \end{aligned}$$

Question 16**Answer D**

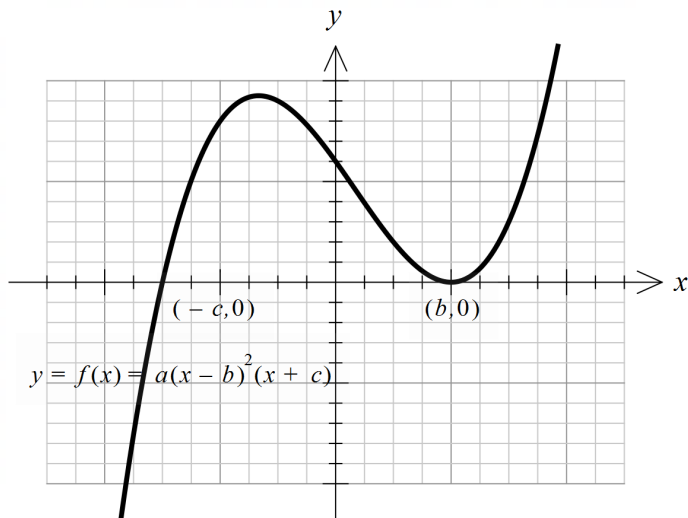
$$\begin{aligned} \text{Area} &= \frac{\pi}{3} \left(g\left(-\frac{2\pi}{3}\right) + g\left(-\frac{\pi}{3}\right) + g(0) + g\left(\frac{\pi}{3}\right) + g\left(\frac{2\pi}{3}\right) + g(\pi) \right) \\ &= \frac{\pi}{3} \left(2g\left(-\frac{2\pi}{3}\right) + 2g\left(-\frac{\pi}{3}\right) + g(0) + g(\pi) \right) \end{aligned}$$

Question 17**Answer E**

$f(x) = a(x-b)^2(x+c)$ where a , b and c are positive real constants.

The graph has x -intercepts at $(-c, 0)$ and $(b, 0)$.

$$\text{Area} = \int_{-c}^b f(x) dx$$

**Question 18****Answer B**

$$X \sim \text{Bi}(n, 0.7)$$

$$\Pr(X > 20) > 0.95$$

$$\Pr(21 \leq X \leq 36) = 0.953 \text{ correct to three decimal places}$$

More than 35.

1.9	1.10	1.11	*MAVMC	RAD	X
binomCdf(35,0.7,21,35)		0.926931			
binomCdf(36,0.7,21,36)		0.952962			

Question 19**Answer B**

$$X \sim N(40, 9)$$

$$Z \sim N(0, 1)$$

$$\Pr(-3 < Z < 1) = \Pr(-1 < Z < 3)$$

$$\Pr(40 - 3 < X < 40 + 3 \times 3)$$

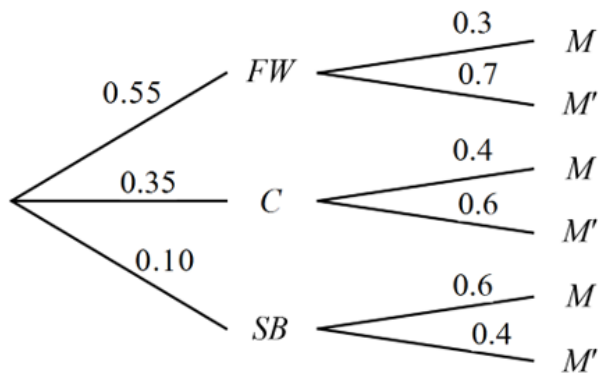
$$= \Pr(37 < X < 49)$$

```
normCdf(-3, 1, 1, 0)
          0.839994848
normCdf(37, 49, 3, 40)
          0.839994848
```

Question 20**Answer A**

Let FW be flat white, C cappuccino, SB short black and M muffin.

$$\Pr(SB | M) = \frac{0.1 \times 0.6}{0.1 \times 0.6 + 0.35 \times 0.4 + 0.55 \times 0.3} = \frac{12}{73}$$



```
1.1 1.2 1.3 *MAVMC RAD X
exact( (0.1 * 0.6) / (0.1 * 0.6 + 0.35 * 0.4 + 0.55 * 0.3) ) 12/73
```

SECTION B**Question 1**

$$f(x) = a(x-b)^3(x-c)$$

a. $f'(x) = a(x-b)^2(4x-b-3c)$ **1A in fully factorised form**

b. $f(x) = a(x-b)^3(x-c)$

Substituting $a = -\frac{1}{2}$, $b = -3$, $c = 1$ gives $f(x) = -\frac{1}{2}(x+3)^3(x-1)$.

By letting $y = 0$, we get x -intercepts at $x = 1$ and $x = -3$.

By using $f'(x)$ from **part a** we get $f'(x) = -\frac{1}{2}(x+3)^2(4x+3-3) = -\frac{1}{2}(x+3)^2(4x)$.

It follows that if $f'(x) = -\frac{1}{2}(x+3)^2(4x)$ we get

$$f'(0) = -\frac{1}{2}(0+3)^2 \times 0 = 0 \text{ as required} \quad \mathbf{1M \text{ Verify (2 parts)}}$$

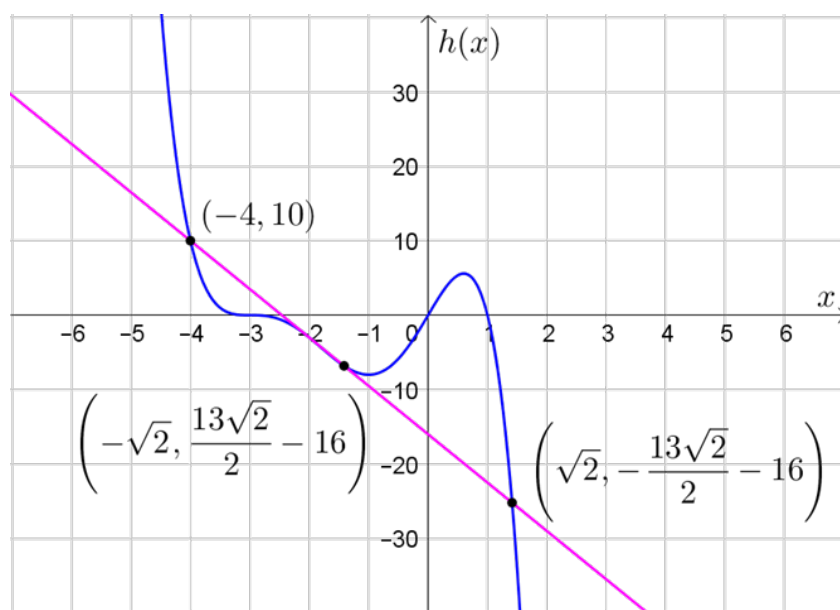
c. Stationary points are

$\left(0, \frac{27}{2}\right)$: a local maximum turning point **1A**

$(-3, 0)$: a stationary point of inflexion **1A**

d. $(-4, 10)$, $\left(-\sqrt{2}, \frac{13\sqrt{2}}{2} - 16\right)$, $\left(\sqrt{2}, -\frac{13\sqrt{2}}{2} - 16\right)$ **1A**

Correctly graphed **1A**



Edit Action Interactive

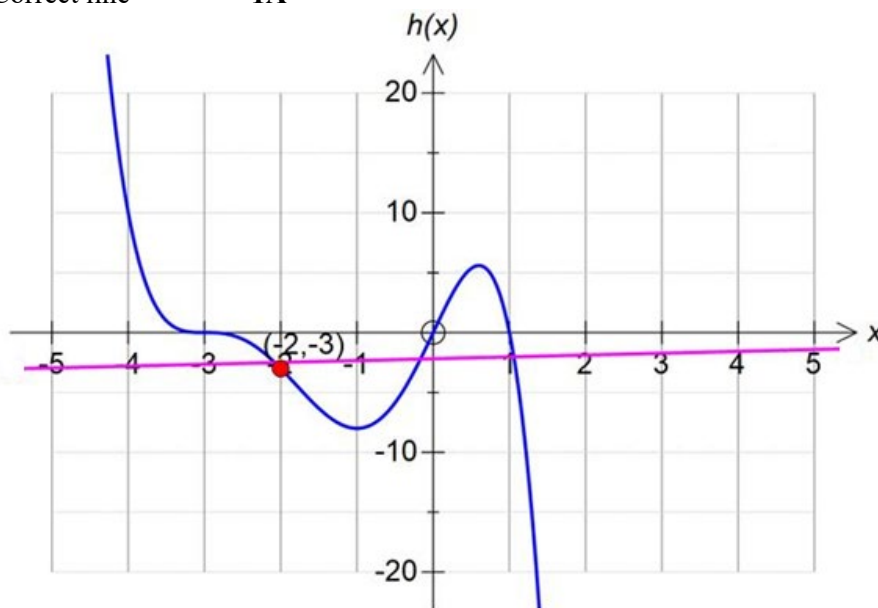
$\frac{0.5}{2}$ $\frac{1}{2}$ $\int dx$ $\int dx$ $\int dx$ $\int dx$ $\int dx$ $\int dx$ $\int dx$ $\int dx$

```

define h(x)=- $\frac{x}{2}$ (x+3)3(x-1)
done
tanLine(h(x), x, -2)
 $-\frac{13 \cdot x}{2} - 16$ 
solve(h(x)= $-\frac{13 \cdot x}{2} - 16$ , x)
{x=-4, x=-2, x=- $\sqrt{2}$ , x= $\sqrt{2}$ }
h(-4)
10
simplify(h(- $\sqrt{2}$ ))
 $\frac{13 \cdot \sqrt{2}}{2} - 16$ 
simplify(h( $\sqrt{2}$ ))
 $-\frac{13 \cdot \sqrt{2}}{2} - 16$ 
    
```

Alg Standard Real Rad

e. Correct line **1A**



f. $(-2, -3)$, $(-0.21, -2.72)$, $(1.07, -2.53)$ correct to two decimal places

2A

$$\text{solve}\left(h(x) = \frac{2 \cdot x}{13} - \frac{35}{13}, x\right)$$

$$\{x = -2, x = -0.2071888265, x = 1.070070347\}$$

$h(-2)$	-3
$h(-0.2071)$	-2.723074425
$h(1.07007)$	-2.52766751

g.i. Area = $\int_{-2}^{-0.21} \left(\frac{2}{13}x - \frac{35}{13}\right) - h(x) dx + \int_{-0.21}^{1.07} h(x) - \left(\frac{2}{13}x - \frac{35}{13}\right) dx$

2A

ii. Area = 12.4 square units, correct to one decimal place.

1A

h.i. Point on curve $\left(-\frac{1}{2}, -\frac{375}{64}\right)$

1A

Gradient of tangent = $-\frac{13}{2}$.

Equation of the line parallel to the tangent going through point $\left(-\frac{1}{2}, -\frac{375}{64}\right)$ is

$$y = -\frac{13}{2}x - \frac{583}{64}$$

1A



$$\text{tanLine}(h(x), x, -2)$$

$$\frac{-13 \cdot x}{2} - 16$$

$$\text{solve}\left(y + \frac{375}{64} = -\frac{13}{2} \cdot \left(x + \frac{1}{2}\right), y\right)$$

$$\left\{y = -\frac{13 \cdot x}{2} - \frac{583}{64}\right\}$$

ii. Let A be a point on the tangent and B be a point on the parallel line.

Choose a point on one of the lines and let the point on the other line, $y = f(x)$, be $(a, f(a))$.

$$A\left(a, -\frac{13a}{2} - 16\right) \text{ and } B\left(-\frac{1}{2}, -\frac{375}{64}\right)$$

$$d(AB) = \sqrt{\left(-\frac{375}{64} - \left(-\frac{13a}{2} - 16\right)\right)^2 + \left(-\frac{1}{2} - a\right)^2} \quad \mathbf{1M}$$

Find the minimum distance.

$$d = 1.05 \text{ correct to two decimal places} \quad \mathbf{1A}$$

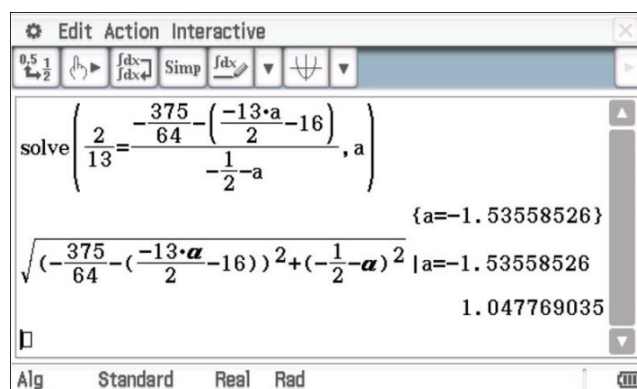
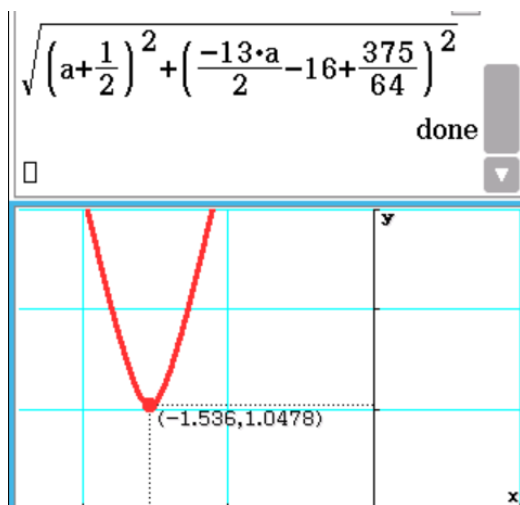
OR

Use the perpendicular line. $m_p = \frac{2}{13}$

$$\frac{-\frac{375}{64} - \left(-\frac{13a}{2} - 16\right)}{-\frac{1}{2} - a} = \frac{2}{13}, \quad a = -1.53\dots \quad \mathbf{1M}$$

$$d(AB) = \sqrt{\left(-\frac{375}{64} - \left(-\frac{13 \times -1.53\dots}{2} - 16\right)\right)^2 + \left(-\frac{1}{2} - (-1.53\dots)\right)^2}$$

$$d = 1.05 \text{ correct to two decimal places} \quad \mathbf{1A}$$



Question 2

$$\text{a. } C(t) = \frac{1000}{1 + e^{-\frac{1}{5}(t-10)}} = \frac{1000}{1 + \frac{1}{e^{\frac{1}{5}(t-10)}}} = \frac{1000e^{\frac{1}{5}(t-10)}}{e^{\frac{1}{5}(t-10)} + 1}$$

$$\text{As required } C(t) = \frac{1000e^{\frac{1}{5}(t-10)}}{1 + e^{\frac{1}{5}(t-10)}} \quad \mathbf{1M \text{ Show that}}$$

$$\text{b. } C(t) = \frac{1000e^{\frac{1}{5}(t-10)}}{1 + e^{\frac{1}{5}(t-10)}}$$

Using the quotient rule

$$C'(t) = \frac{\left(1 + e^{\frac{1}{5}(t-10)}\right) 200e^{\frac{1}{5}(t-10)} - 1000e^{\frac{1}{5}(t-10)} \left(\frac{1}{5}e^{\frac{1}{5}(t-10)}\right)}{\left(1 + e^{\frac{1}{5}(t-10)}\right)^2}$$

Simplify

$$C'(t) = \frac{\left(200e^{\frac{1}{5}(t-10)}\right) \left(1 + e^{\frac{1}{5}(t-10)} - e^{\frac{1}{5}(t-10)}\right)}{\left(1 + e^{\frac{1}{5}(t-10)}\right)^2}$$

$$C'(t) = \frac{200e^{\frac{1}{5}(t-10)}}{\left(1 + e^{\frac{1}{5}(t-10)}\right)^2} \quad \mathbf{1M \text{ Show that}}$$

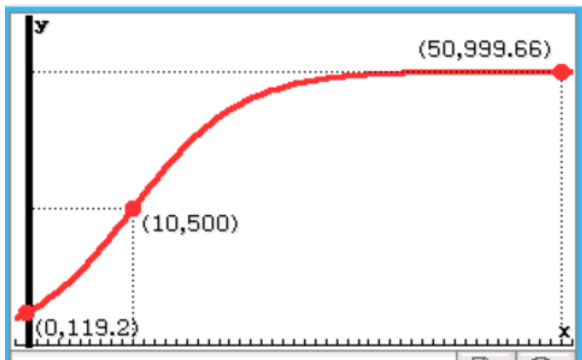
The screenshot shows a CAS interface with the following text:

```
define f(t) = 1000e^{0.2(t-10)} / (1 + e^{0.2(t-10)})
done
d/dt (f(t))
200 * e^{t-10/5} / (e^{t-10/5} + 1)^2
```

c. Let $C'(t) = 0$ for stationary points.

Let $200e^{\frac{1}{5}(t-10)} = 0$ gives no solutions.
So no stationary points.

1M Show that

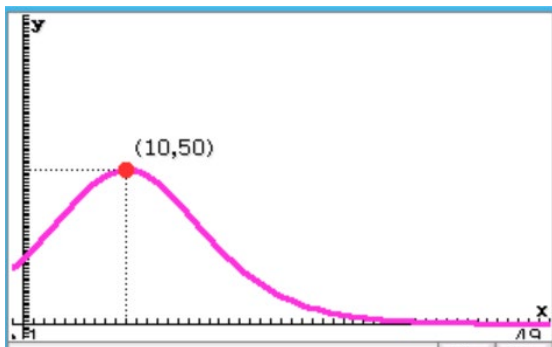


```

define f(t) = 1000e^{0.2(t-10)} / (1 + e^{0.2(t-10)})
done
solve(d/dt (f(t)) = 0, t)
No Solution
    
```

d. The gradient is at a maximum at $t = 10$.

1A



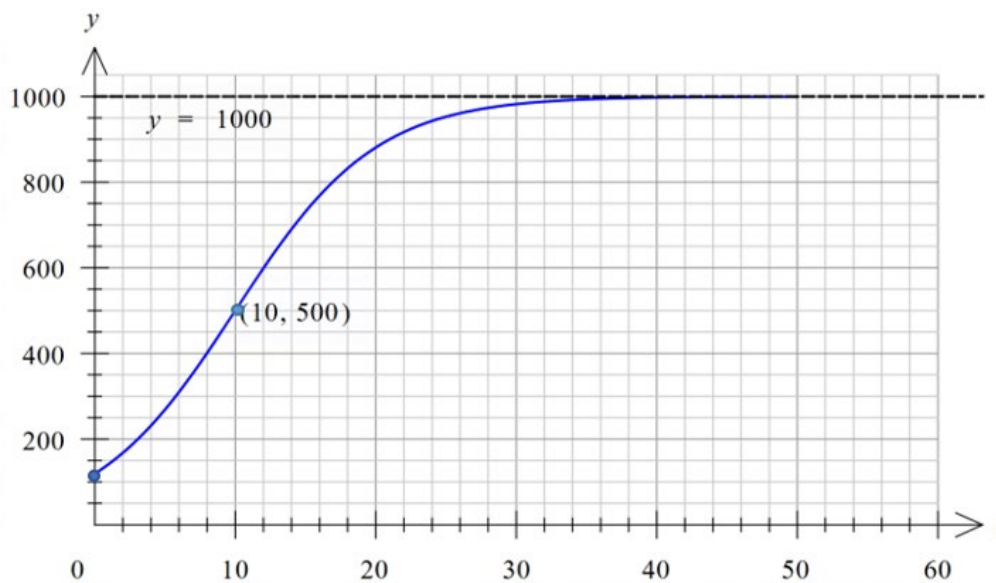
e. Gradient = 50 at $t = 10$.

1A

f. Correct dilations 1A, Correct translation 1A

- Dilate from the t -axis by a factor of 1000
- Dilate from the y -axis by a factor of 5
- Translate in the positive t direction by 10 units

g. Correct shape and correct coordinate 1A, Asymptote 1A



h. Will never reach 1000 confirmed cases as there is a horizontal asymptote at $C(t) = 1000$. 1A

i. Solve $C'(t) = \frac{1}{10}$, $t = 48$ correct to the nearest integer

$$\text{Average rate of change per day for the 48}^{\text{th}} \text{ day} = \frac{C(48) - C(47)}{48 - 47} = 0.11... \quad \mathbf{1M}$$

$$\text{Average rate of change per day for the 49}^{\text{th}} \text{ day} = \frac{C(49) - C(48)}{49 - 48} = 0.09...$$

Day 59

1A

The screenshot shows a TI-84 Plus calculator interface. At the top, there is a menu bar with 'Edit', 'Action', and 'Interactive'. Below the menu bar, there are several icons for calculator functions. The main display area shows the following text:

```
define c(t) = 1000 / (1 + e^(-1/5 * (t - 10)))
solve(d(c(t)) = 1/10, t)
{t = -27.99950854, t = 47.99950854}
```

Below the main display area, there are two rows of calculations:

$$\frac{c(48) - c(47)}{48 - 47} = 0.1106782524$$

$$\frac{c(49) - c(48)}{49 - 48} = 0.09063394209$$

Question 3

a. $X \sim N(62.9, 1.6^2)$

$$\Pr(X < 61) = 0.1175 \text{ correct to four decimal places} \quad \mathbf{1A}$$

The screenshot shows a TI-84 Plus calculator interface. The top of the screen displays '1.1', '1.2', '1.3', '*MAVEA', 'RAD', and a battery icon. The main display area shows the following text:

```
normCdf(-∞, 61, 62.9, 1.6) 0.117515
```

b. $\Pr(X > 58 | X < 61)$ **1M**

$$= \frac{\Pr(58 < X < 61)}{\Pr(X < 61)}$$

$$= 0.991 \text{ correct to three decimal places} \quad \mathbf{1A}$$

*MAVEA	
normCdf($-\infty, 61, 62.9, 1.6$)	0.117515
<u>normCdf($58, 61, 62.9, 1.6$)</u>	0.99066
0.11751528796724	

c. $Y \sim \text{Bi}(30, 0.1175\dots)$ **1A**

$\Pr(6 \leq Y \leq 30) = 0.1330$ correct to four decimal places **1A**

d. $E(Y) = np = 3.525$ correct to three decimal places **1A**

$\text{sd}(Y) = \sqrt{np(1-p)} = 1.764$ correct to three decimal places **1A**

*MAVEA	
$30 \cdot 0.11751528796724$	3.52546
$\sqrt{30 \cdot 0.11751528796724 \cdot (1 - 0.1175152879\dots)}$	1.76385

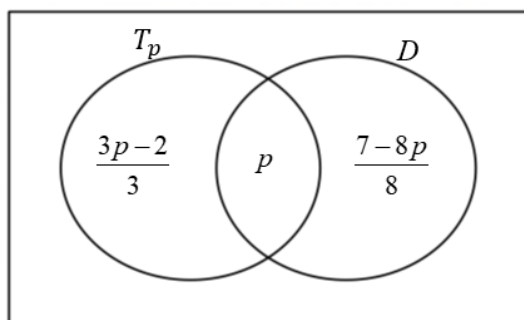
e. She could buy 2 packets on one day and none on the other 6 days or she could buy one packet on two days and none on the other days. **1M**

$$\binom{7}{1}(0.05)(0.7)^6 + \binom{7}{2}(0.2)^2(0.7)^5$$

= 0.1824 correct to four decimal places **1A**

*MAVEA	
$7 \cdot 0.05 \cdot (0.7)^6 + nCr(7, 2) \cdot (0.2)^2 \cdot (0.7)^5$	0.182356

f. Let T_p be the probability he plays with toilet paper and D the probability he eats his dinner.



$$\Pr(T_p) = \left(p + \frac{3p-2}{3} \right), \quad \Pr(D) = p + \frac{7-8p}{8}$$

For independent events $\Pr(T_p \cap D) = \Pr(T_p) \times \Pr(D)$.

$$\text{Solve } \left(p + \frac{3p-2}{3}\right) \left(p + \frac{7-8p}{8}\right) = p \text{ for } p, p = \frac{7}{9} \quad \mathbf{1M}$$

$$\Pr(T_p' \cap D') = 1 - \left(p + \frac{3p-2}{3} + \frac{7-8p}{8}\right) = \frac{1}{72} \quad \mathbf{1A}$$

Calculator screenshot showing the solution to the equation $\left(p + \frac{3p-2}{3}\right) \left(p + \frac{7-8p}{8}\right) = p$ for p , resulting in $p = \frac{7}{9}$. Below, the expression $1 - \left(p + \frac{3p-2}{3} + \frac{7-8p}{8}\right)$ is evaluated at $p = \frac{7}{9}$, resulting in $\frac{1}{72}$.

Question 4

a. $w_2 = 5 \sin\left(2t + \frac{\pi}{2}\right) + 6$

Amplitude is 5, Period $\frac{2\pi}{2} = \pi$ $\mathbf{1A}$

b. $w_2 = 5 \cos(2t) + 6$ $\mathbf{1A}$

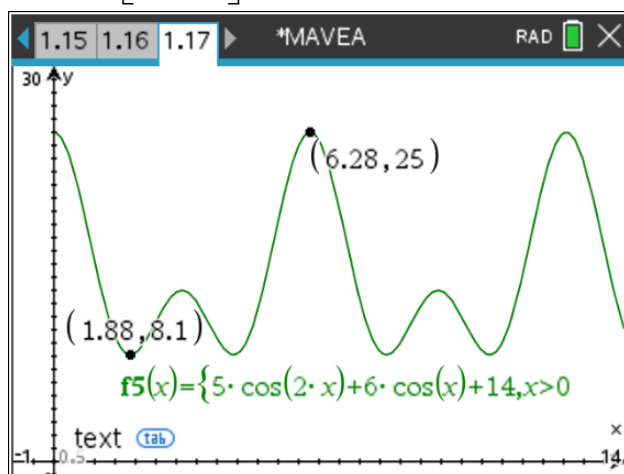
Calculator screenshot showing the expression $5 \cdot \sin\left(2 \cdot t + \frac{\pi}{2}\right) + 6$ and its simplified form $5 \cdot \cos(2 \cdot t) + 6$.

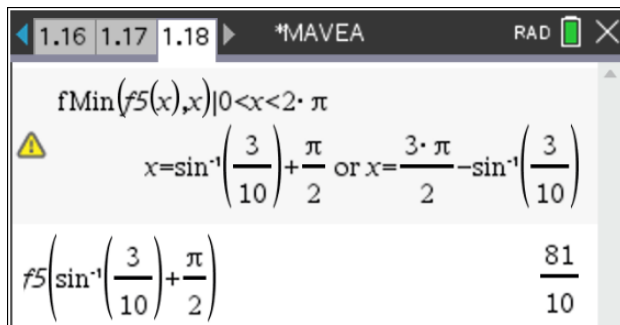
c. $w_r = w_1 + w_2 = 6 \cos(t) + 8 + 5 \cos(2t) + 6$

$w_r = 6 \cos(t) + 5 \cos(2t) + 14$ $\mathbf{1A}$

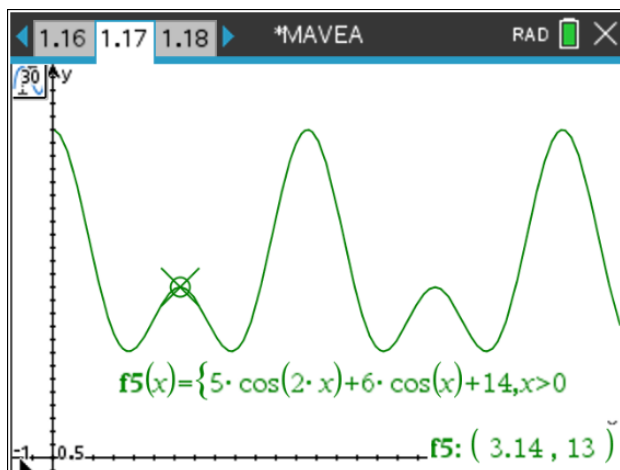
d. Period = 2π $\mathbf{1A}$

Range is $\left[\frac{81}{10}, 25\right]$ $\mathbf{1A}$





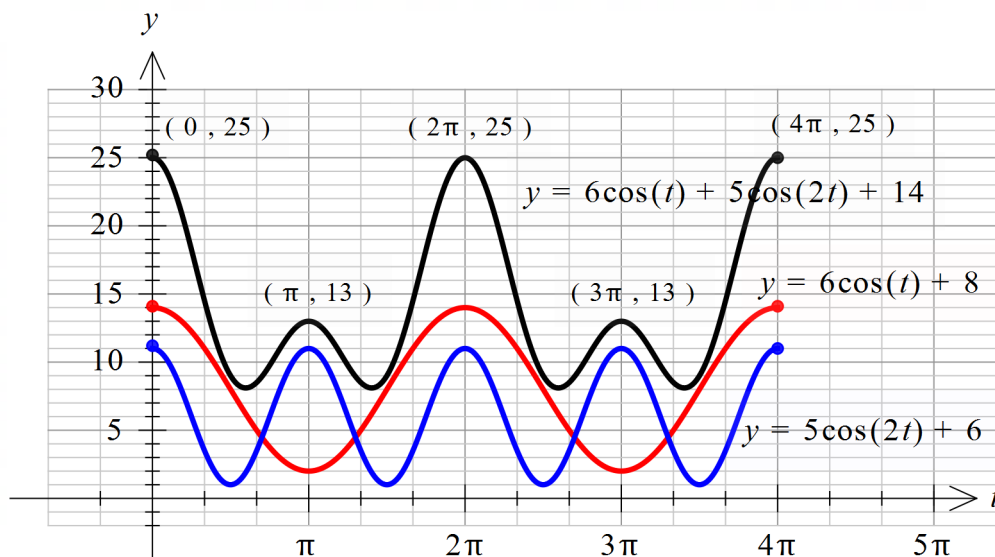
e. π and 3π 1A



f. w_1 and w_2 labelled correctly 1A

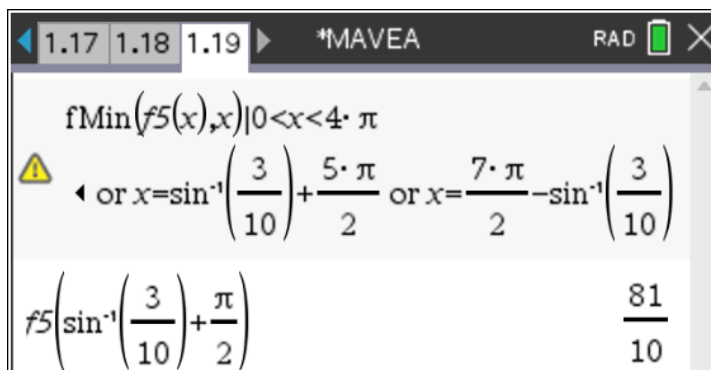
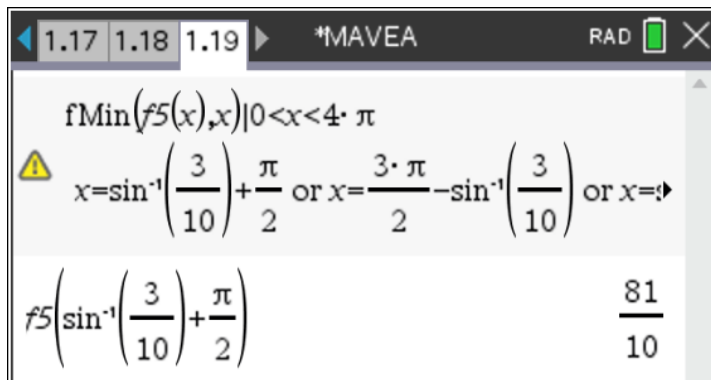
w_r drawn correctly 1A

Correct coordinates 1A



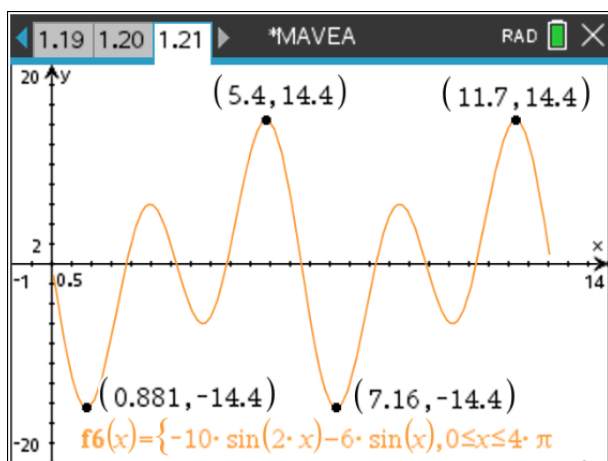
$$\mathbf{g.} \left(\sin^{-1}\left(\frac{3}{10}\right) + \frac{\pi}{2}, \frac{81}{10} \right), \left(\frac{3\pi}{2} - \sin^{-1}\left(\frac{3}{10}\right), \frac{81}{10} \right)$$

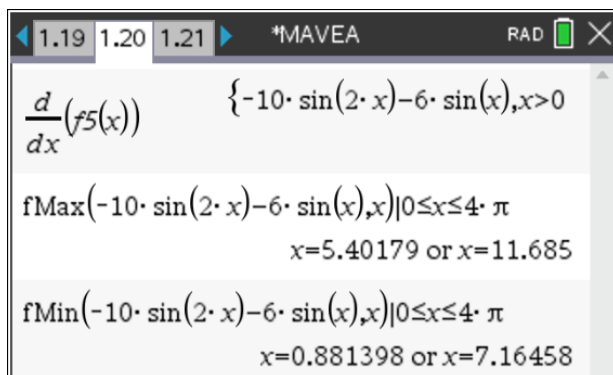
$$\left(\sin^{-1}\left(\frac{3}{10}\right) + \frac{5\pi}{2}, \frac{81}{10} \right), \left(\frac{7\pi}{2} - \sin^{-1}\left(\frac{3}{10}\right), \frac{81}{10} \right)$$

1A

h. Sketch the graph of the derivative and find the coordinates of the maximum and minimum turning points or find the derivative and use the maximum and minimum functions. **1M**

$t = 0.9, 5.4, 7.2, 11.7$

1A

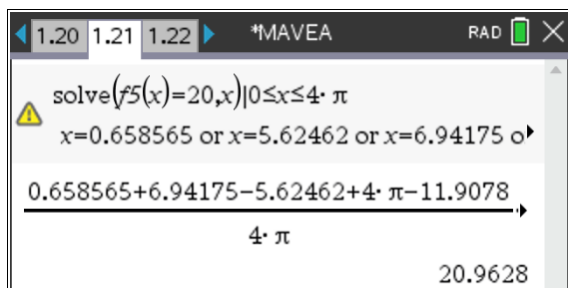


$\frac{d}{dx}(f5(x)) \quad \{-10 \cdot \sin(2 \cdot x) - 6 \cdot \sin(x), x > 0$
 $f\text{Max}(-10 \cdot \sin(2 \cdot x) - 6 \cdot \sin(x), x) | 0 \leq x \leq 4 \cdot \pi$
 $x = 5.40179 \text{ or } x = 11.685$
 $f\text{Min}(-10 \cdot \sin(2 \cdot x) - 6 \cdot \sin(x), x) | 0 \leq x \leq 4 \cdot \pi$
 $x = 0.881398 \text{ or } x = 7.16458$

i. $w_r(t) = 20$

$$\frac{0.658\dots + (6.941\dots - 5.624\dots) + 4\pi - 11.907\dots}{4\pi} \times 100 \quad \mathbf{1M}$$

$$= 20.96\% \quad \mathbf{1A}$$



$\text{solve}(f5(x)=20, x) | 0 \leq x \leq 4 \cdot \pi$
 $x = 0.658565 \text{ or } x = 5.62462 \text{ or } x = 6.94175$
 $\frac{0.658565 + 6.94175 - 5.62462 + 4 \cdot \pi - 11.9078}{4 \cdot \pi}$
 20.9628

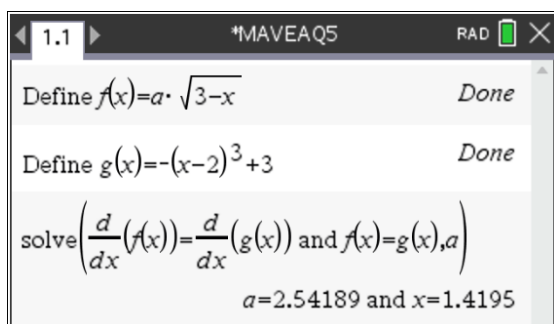
Question 5

$f(x) = a\sqrt{3-x}$ and $g(x) = -(x-2)^3 + 3$, where $a \in \mathbb{R} \setminus \{0\}$

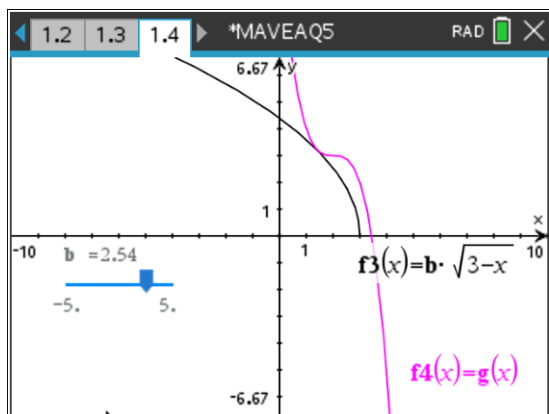
a. Solve $f(x) = g(x)$ and $f'(x) = g'(x)$ for a . $\mathbf{1M}$

There will be one solution if the curves touch and their gradients will also be equal at the point of intersection. This occurs when $x = 1.419\dots$

$a = 2.542$ correct to three decimal places $\mathbf{1A}$



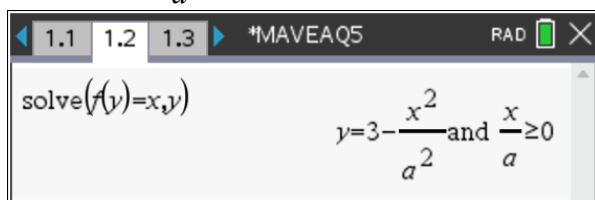
$\text{Define } f(x) = a \cdot \sqrt{3-x} \quad \text{Done}$
 $\text{Define } g(x) = -(x-2)^3 + 3 \quad \text{Done}$
 $\text{solve}\left(\frac{d}{dx}(f(x)) = \frac{d}{dx}(g(x)) \text{ and } f(x) = g(x), a\right)$
 $a = 2.54189 \text{ and } x = 1.4195$



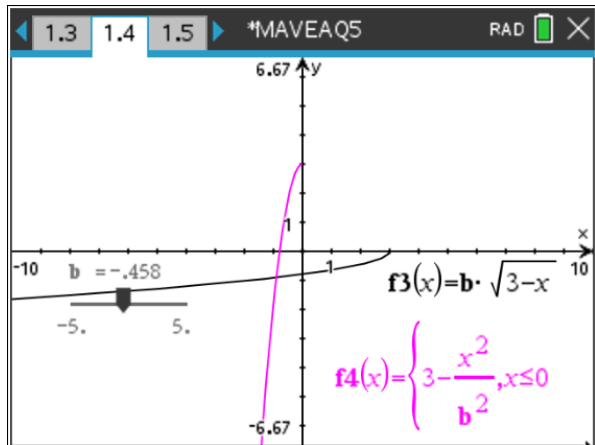
b. Let $y = a\sqrt{3-x}$
 Inverse swap x and y and solve for y .

$$x = a\sqrt{3-y}$$

$$f^{-1}(x) = 3 - \frac{x^2}{a^2} \text{ and } x \leq 0 \quad \mathbf{1A}$$

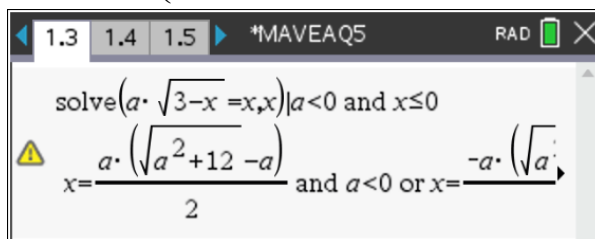


Note: use a different variable name in the graphing section.



c. Solve $f(x) = x$ for x .

$$\text{Intersection} \left(\frac{-a^2 + a\sqrt{a^2+12}}{2}, \frac{-a^2 + a\sqrt{a^2+12}}{2} \right) \quad \mathbf{1A}$$



d. Substitute $(0, 3)$ into $f(x) = a\sqrt{3-x}$ and solve for a .

Since there are three points of intersections, one point of intersection is on the line $y = x$ and the other two are at $(0, 3)$ and $(3, 0)$.

$$3 = a\sqrt{3}$$

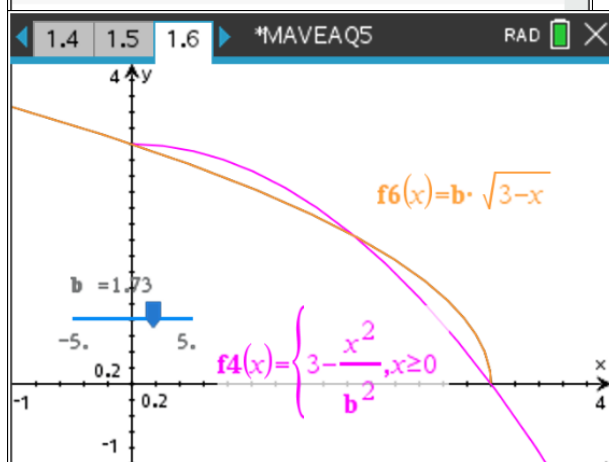
$$a = \sqrt{3}$$

1A

1.4 1.5 1.6 *MAVEAQ5 RAD X

$$\text{solve}\left(a \cdot \sqrt{3-x} = 3 - \frac{x^2}{a^2}, x \mid a = \sqrt{3} \text{ and } x \geq 0\right)$$

$$x=0 \text{ or } x = \frac{3 \cdot (\sqrt{5} - 1)}{2} \text{ or } x=3$$



1.4 1.5 1.6 *MAVEAQ5 RAD X

$$\text{solve}\left(a \cdot \sqrt{3-x} = 3 - \frac{x^2}{a^2}, x \mid a = \sqrt{3} \text{ and } x \geq 0\right)$$

$$x=0 \text{ or } x = \frac{3 \cdot (\sqrt{5} - 1)}{2} \text{ or } x=3$$

e. Solve $f(x) = f^{-1}(x)$ and $f'(x) = f^{-1}'(x)$ for a . 1M

There will be one solution if the curves touch and their gradients will also be equal at the point of intersection. This occurs when $a = 2$ and $x = 2$.

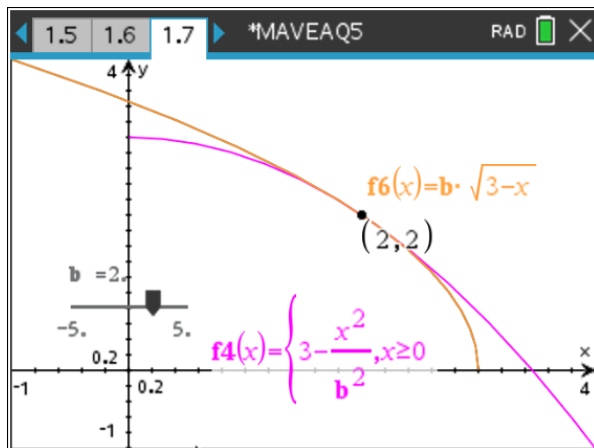
$$a \in [\sqrt{3}, 2)$$

1A

1.4 1.5 1.6 *MAVEAQ5 RAD X

$$\text{solve}\left(\frac{d}{dx}(a \cdot \sqrt{3-x}) = \frac{d}{dx}\left(3 - \frac{x^2}{a^2}\right) \text{ and } a \cdot \sqrt{3-x} = 3 - \frac{x^2}{a^2}\right)$$

$$a=2 \text{ and } x=2$$



END OF SOLUTIONS