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Mathematical Methods

2020

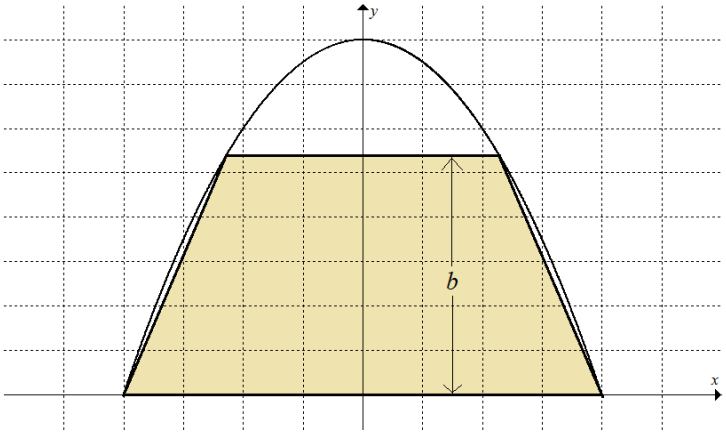
Trial Examination 1
(1 hour)

Instructions

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

The following diagram shows a trapezium (height $= b$) bounded by a parabola and the x-axis.
All corners of the trapezium are on the parabola.
The turning point of the parabola is $(0, 4)$, and the x-intercepts are $(-2, 0)$ and $(2, 0)$.



a. Write down the equation of the parabola in turning point form. 1 mark

b. Express the area of the trapezium in terms of b. 3 marks

c. Determine the value of b for maximum area of the trapezium. 2 marks

Question 2

Consider polynomial $P(x) = x^6 + 64$.

a i. By long division find the quotient and the remainder when $P(x)$ is divided by $x^2 + 4$. 2 marks

a ii. Hence or otherwise express $P(x)$ as the product of 3 quadratic functions of x. 2 marks

b. Show that the turning points of the three quadratic functions form the vertices of an equilateral triangle.

2 marks

Question 3

The curve $y = 2x^3 - e^x$ undergoes transformation T given by $\begin{bmatrix} x' \\ y' \end{bmatrix} = T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ where $\begin{bmatrix} x' \\ y' \end{bmatrix}$ is the image of $\begin{bmatrix} x \\ y \end{bmatrix}$.

Define $\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{2}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x' \\ y' \end{bmatrix}\right) + \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Find the relationship between X and Y (i.e. an equation satisfied by X and Y). 3 marks

Question 4

a. Solve $5e^{-4x} + 2e^{-2x} = 3$ for x. 2 marks

b. Given $\log_3 5 + \log_4 x = 1$, express $\log_e x$ in the form $\frac{(\log_{10} a)(\log_{10} b)}{(\log_{10} c)(\log_{10} d)}$ where $a, b, c, d \in R \setminus \{0\}$.

2 marks

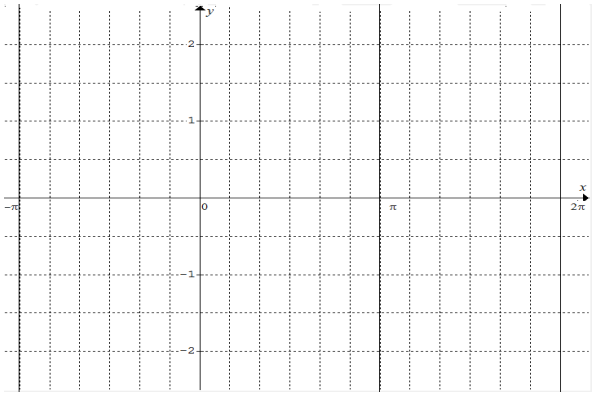
c. Given (e, e) and (e^2, e^2) are points on $y = Ae^{kx}$ where $k, A \in R \setminus \{0\}$, find the values of k and A.

2 marks

Question 5

Consider $f(x) = 2\sin x + \cos\left(x + \frac{5\pi}{6}\right)$.

a. Sketch accurately the graphs of $y = 2\sin x$ and $y = \cos\left(x + \frac{5\pi}{6}\right)$ for $-\pi \le x \le 2\pi$ on the same set of axes below. 2 marks



b. Use the graphs to find the solutions to $2\sin x + \cos\left(x + \frac{5\pi}{6}\right) = 0$ in the interval $-\pi \le x \le 2\pi$. 1 mark

c. Write a general solution to $2\sin x + \cos\left(x + \frac{5\pi}{6}\right) = 0$. 1 mark

Question 6

Consider $f(x) = \frac{\cos x}{\sin x}$, $x \in (0, \pi)$.

a. Find the derivative of $f(x)$. 1 mark

b. Hence or otherwise evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin^2 x}\,dx$. 2 marks

Question 7

Consider events A and B such that $\Pr(A)=\frac{3}{5}$ and $\Pr(B' \cap A)=\frac{1}{4}$.

a. Evaluate $\Pr(A \cap B)$. 1 mark

b. Given $\Pr(A \mid B)=\frac{3}{5}$, evaluate $\Pr(A \cup B)$. 1 mark

c. Are A' and B' independent? Explain. 1 mark

Question 8

Two tellers A and B in a bank serve customers.
On a particular day the proportion of 4 random customers choosing the queue served by Teller A waiting longer than 5 minutes is 0.36 and the proportion of 144 random customers choosing the queue served by Teller B waiting longer than 5 minutes is 0.64.

Assume both queues have the same length when a customer enters the bank.

a. Which queue should a customer choose knowing these statistics?
Explain by quoting the approximate 95% confidence intervals. 2 marks

b. The probabilities that a customer going to Teller A and Teller B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively, and the probability of not doing banking on the day after entering the bank due to long queues is $\frac{1}{6}$.

Using 0.36 for A and 0.64 for B as point estimates, determine the probability to 1 decimal place that the customer spends more than 5 minutes in waiting. 1 mark

Question 9

Consider $y = x^{\frac{m}{n}}$ and $y = x^{\frac{m+2}{n+2}}$ where m is a positive even integer, and n a positive odd integer, and $m > n$.

a. Show that $x^{\frac{m+2}{n+2}} > x^{\frac{m}{n}}$ for $x \in (-1, 0)$ or $x \in (0, 1)$. 3 marks

b i. Find the area A of the regions bounded by $y = x^{\frac{m}{n}}$ and $y = x^{\frac{m+2}{n+2}}$ in terms of m and n. 2 marks

b ii. Show that $A \to 0$ as $n \to \infty$. 1 mark

End of Exam