Name: ANSWERS.

MM12 Assignment 5 – Applications of Calculus and Antidifferentiation

Section A: Short Answer

Total 21 marks

Unless otherwise stated, exact answers should be given

1. Find the gradient function $f'(x)$, given $f(x) = (3x^2 + 2)(x^2 + 3x)$

$$f(x) = 3x^4 + 9x^3 + 2x^2 + 6x$$

$$f'(x) = 12x^3 + 27x^2 + 4x + 6$$

(2 marks)

2. Use the quotient rule to show that the derivative of $f(x) = \frac{3-2x}{3x+1}$ is $f'(x) = \frac{-11}{(3x+1)^2}$

Let $u = 3 - 2x$ $\quad v = 3x + 1$

$u' = -2$ $\quad v' = 3$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{-2(3x+1) - 3(3-2x)}{(3x+1)^2}$$

$$= \frac{\cancel{-6x} - 2 - 9 + \cancel{6x}}{(3x+1)^2}$$

$$= \frac{-11}{(3x+1)^2}$$

(3 marks)

3. Find the equation of the tangent to the curve $y = 2x^3 - x + 4$ at the point (-1, 3).

$$\frac{dy}{dx} = 6x^2 - 1$$

when $x = -1$ $\quad \frac{dy}{dx} = 5$

$$M_T = 5$$

Equation of tangent

$$y - 3 = 5(x+1)$$

$$y = 5x + 8$$

(3 marks)

4. The graph of $y = 2x^3 + ax^2 + b$ has a stationary point at (1, –7). Find the values of a and b.

$$\frac{dy}{dx} = 6x^2 + 2ax$$

when $x = 1$ $\quad \frac{dy}{dx} = 0$

$$6(1)^2 + 2a(1) = 0$$

$$a = -3$$

$$y = 2x^3 + ax^2 + b$$

$$y = 2x^3 - 3x^2 + b$$

sub. $(1, -7)$

$$-7 = 2(1)^3 - 3(1)^2 + b$$

$$b = -6$$

(3 marks)

5. Find the following antiderivatives

(a) $\int(6x^2 + 4x)dx$

$= \frac{6x^3}{3} + \frac{4x^2}{2} + c$

$= 2x^3 + 2x^2 + c$

(b) $\int \frac{6}{x^2} + 3\,dx$

$= \int 6x^{-2} + 3\,dx$

$= \frac{6x^{-1}}{-1} + 3x + c$

$= -\frac{6}{x} + 3x + c$

(4 marks)

6. Find the value of the definite integral $\int_{-1}^{3}(2x^2 - x - 3)dx$

$\int_{-1}^{3} (2x^2 - x - 3)\,dx$

$= \left[\frac{2x^3}{3} - \frac{x^2}{2} - 3x\right]_{-1}^{3}$

$= \left(\frac{2\times 27}{3} - \frac{9}{2} - 9\right) - \left(\frac{-2}{3} - \frac{1}{2} + 3\right)$

$= 8/3$

(3 marks)

7. It is known that $f'(x) = x^3 - 2x^2$ and $f(0) = 6$. Find $f(x)$.

$f(x) = \int(x^3 - 2x^2)\,dx$

$f(x) = \frac{x^4}{4} - \frac{2x^3}{3} + c$

when $x = 0$ $y = 6$ $\Rightarrow c = 6$

$f(x) = \frac{x^4}{4} - \frac{2x^3}{3} + 6$

(3 marks)

Name: ______________________

Section B: Multiple Choice Calculators are allowed **Time allowed: 25 minutes**

Circle Correct Response

1. The x-coordinates of the stationary points of the curve $f(x) = \frac{1}{3}x^3 + 4x^2 + 7x - 66$ are:

A $x = 3$
B $x = 1,\ x = 6$
C $x = 1,\ x = 7$
D $x = 7,\ x = 8$
E $x = -7,\ x = -1$

$f'(x) = x^2 + 8x + 7$

$f'(x) = 0 \Rightarrow x^2 + 8x + 7 = 0$

$x = -7, -1$

2. The curve with equation $y = \frac{7}{2x^2} + 5x$ has gradient -51 when x equals

A -4

B $\frac{1}{2}$

C $-\frac{1}{4}$

D 2

E $\frac{1}{4}$

$y = \frac{7}{2}x^{-2} + 5x$

$\frac{dy}{dx} = -7x^{-3} + 5$

$-7x^{-3} + 5 = -51$

$x =$

3. Given that $g'(x) = 2x^2 + 3$ and $g(0) = 2$, then $g(x)$ equals

A $4x$

B $2x^2 - 1$

C $2x^3 + 3x + 2$

D $\frac{2x^3}{3} + 3x + 2$

E $4x + 2$

$g(x) = \int 2x^2 + 3 \, dx$

$g(x) = \frac{2x^3}{3} + 3x + c$

sub $(0,2)$

$c = 2$

4. A particle moves in a straight line so that its position x cm from a fixed point O at time t seconds ($t \geq 0$) is given by $x = -t^3 + 6t^2 - 24t$. The particle's acceleration at $t = 3$ is

A -6 cm/s^2

B -26 cm/s^2

C -18 cm/s^2

D 54 cm/s^2

E 0 cm/s^2

$v = -3t^2 + 12t - 24$

$a = -6t + 12$

when $t = 3$ $\quad a = -6$

5. Which of the following sets describes the conditions for which the gradient of the graph of $y = f(x)$ is positive? (Coordinates given are those of intercepts and turning points.)

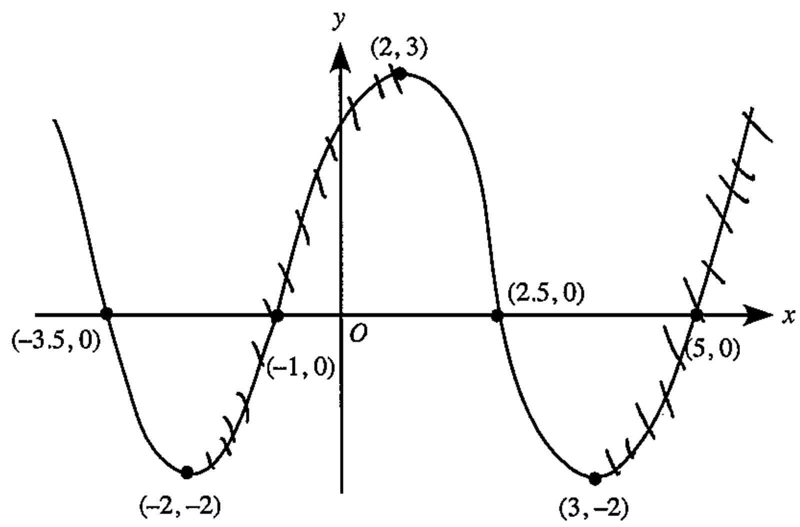
A $(-\infty, -3.5) \cup (-2, 2) \cup (5, \infty)$

B $(-2, 2) \cup (3, \infty)$

C $(-\infty, 3.5) \cup (5, 0)$

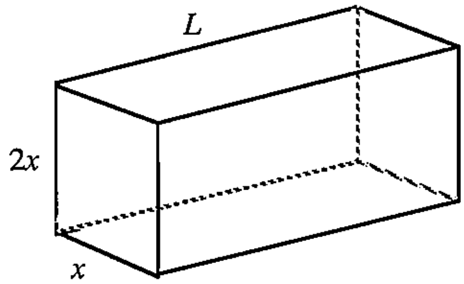
D $(-3.5, -1) \cup (2, \infty)$

E $(0, 2) \cup (5, \infty)$



SECTION C: ANALYSIS **Calculators allowed**
Unless otherwise stated, exact answers should be given

1. Wire is used to make the frame of a container in the shape of a cuboid as shown in the diagram. The total length of wire used to make the frame was 18 m.



(All lengths are in metres.)

a Show that $L = \frac{9-6x}{2}$.

$18 = 4L + 12x$

$4L = 18 - 12x$

$L = \frac{18-12x}{4}$

$L = \frac{9-6x}{2}$

(3 marks)

b Show that the volume can be expressed as $V = 9x^2 - 6x^3$

$V = L \times 2x \times x$

$= 2x^2\left(\frac{9-6x}{2}\right)$

$= 9x^2 - 6x^3$

(1 mark)

c Find the maximum volume of the container and the corresponding dimensions. You are not required to justify it is a maximum.

$V = 9x^2 - 6x^3$

$\frac{dV}{dx} = 18x - 18x^2$

Max/Min when $\frac{dV}{dx} = 0$

$18x - 18x^2 = 0$

$18x(1-x) = 0$

$x = 0$ or $x = 1$
(min) (max)

$V = 9(1)^2 - 6(1)^3$
$= 3 m^3$

$L = 3/2$ length
$x = 1$ width
$2x = 2$ height

(3 marks)

2. A particle moves in a straight line and starts from a position one metre **left** of a fixed point O on the line.

Its velocity v m/s^2 is given by $v = 15t - 9t^2$ for any time t seconds, $t \geq 0$.

a At what times is its velocity equal to zero?

$15t - 9t^2 = 0$

$t = 0$ and $t = 5/3$ seconds.

(2 marks)

b What is the acceleration of the particle when $t = 3$?

$a = 15 - 18t$

when $t = 3$ $a = -39$ m/s^2

(2 marks)

c What is its displacement from O at any time t seconds?

$x = \int 15t - 9t^2 \, dt$

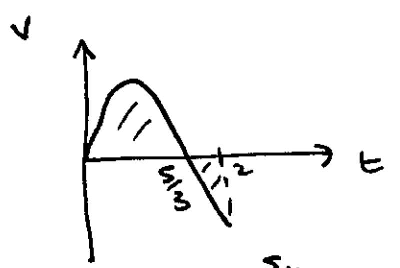
$= \frac{15t^2}{2} - 3t^3 + c$

$t = 0$, $x = -1 \Rightarrow c = -1$

$x = \frac{15t^2}{2} - 3t^3 - 1$

(3 marks)

d Find the total distance covered by the particle in the first 2 seconds.



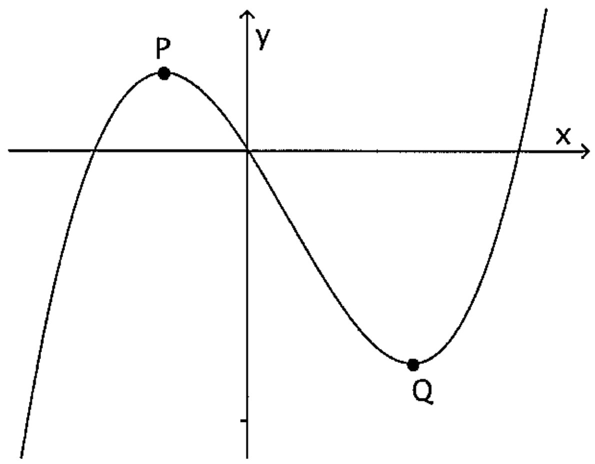
Distance travelled = area under v/t graph

$d = \int_0^{5/3} 15t - 9t^2 \, dt - \int_{5/3}^{2} 15t - 9t^2 \, dt$

(2 marks)

$= 71/9$ m.

3. The graph of $f(x) = x^3 - 2x^2 - 3x$ is shown below. The tangent lines to the curve at the points P and Q are parallel to the x-axis.



a Find the derivative $f'(x)$

$$f'(x) = 3x^2 - 4x - 3$$

(1 mark)

b Find the x values of the coordinates of points P and Q.

$$3x^2 - 4x - 3 = 0$$

$$x = \frac{\sqrt{13}+2}{3}, \quad \frac{-\sqrt{13}+2}{3}$$

Q P

(2 marks)

c Find the equation of the **tangent** line to the curve at point (1, -4).

$$y_T = -4x$$

* Use CAS.
Menu, 4, 9

(1 mark)

d Find the equation of the **normal** to the curve at point (-1, 0).

$$y_N = -\frac{x}{4} - \frac{1}{4}$$

* Use CAS.
Menu 4, A

(1 mark)

e Write the integral expression that could be used to find the area bound by the curve and the x axis

$f(x) = x^3 - 2x^2 - 3x$

x intercepts $x = -1, 0, 3$

Area $= \int_{-1}^{0} f(x)\,dx - \int_{0}^{3} f(x)\,dx$

(2 marks)

f Hence, find the area bound by the curve and the x-axis

Area $= \frac{71}{6}$ sq. units.

(1 mark)

End of Section C
Analysis Mark = /24