Neap:

Trial Examination 2019

VCE Mathematical Methods Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 19 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2019 VCE Mathematical Methods Units 3&4 Written Examination 2.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

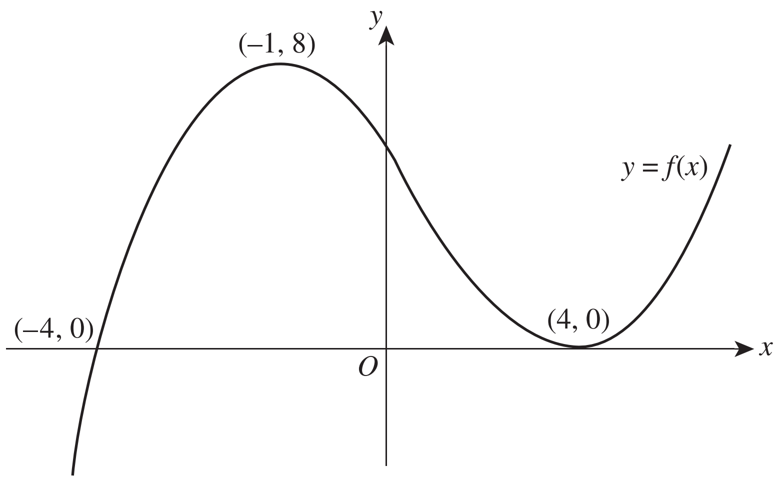
Question 1

At the point (2, 1) on the graph of the function with the rule $y = (x - 2)^3 + 1$,

A. the gradient is undefined.
B. there is a minimum turning point.
C. there is a maximum turning point.
D. there is a stationary point of inflection.
E. the graph is not smooth.

Question 2

Part of the graph of the polynomial f is shown below.



$f'(x) < 0$ for

A. $x \in (-\infty, -4)$
B. $x \in (-\infty, -1) \cup (4, \infty)$
C. $x \in (0, 8)$
D. $x \in (-1, 4)$
E. $x \in (-\infty, -4) \cup (4, \infty)$

Question 3

Consider a tangent to the graph of $y = x^3$ at the point (1, 1).

The coordinates of the y-intercept of the tangent are

A. (0, 0)

B. (0, –4)

C. (0, –3)

D. (0, –2)

E. (0, –1)

Question 4

A box contains three red marbles and two green marbles. Two marbles are drawn at random from the box without replacement.

The probability that the marbles are the same colour is

A. $\frac{1}{10}$

B. $\frac{3}{10}$

C. $\frac{2}{5}$

D. $\frac{3}{5}$

E. $\frac{13}{25}$

Question 5

The average rate of change of $f(x) = \sqrt{3 - x}$ from $x = –6$ to $x = 2$ is

A. –4

B. $-\frac{1}{4}$

C. $-\frac{1}{3}$

D. 2

E. $\frac{13}{6}$

Question 6

The area of the region enclosed by the graph of $y = x(x-1)(x+3)$ and the x-axis is

A. $\frac{7}{12}$

B. 3

C. $\frac{32}{3}$

D. $\frac{45}{4}$

E. $\frac{71}{6}$

Question 7

The transformation $T : R^2 \to R^2$ with the rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$ maps the graph of $y = x^3$ onto the graph of

A. $y = 6 - 3(x-1)^3$

B. $y = 3(x-1)^3 + 6$

C. $y = \frac{1}{3}(1-x)^3 - \frac{2}{3}$

D. $y = 3(1-x)^3 + 2$

E. $y = 6 - 3(x+1)^3$

Question 8

If $\int_{-2}^{4} f(x)dx = 5$, then $\int_{-2}^{4} (x - 2f(x))dx$ is equal to

A. –10
B. –4
C. 0
D. 5
E. 6

Question 9

The linear function $f : [-1, 3] \to R$, $f(x) = mx + 2$ has a range of $[-7, 5]$.

The value of m is

A. –3
B. –1
C. 1
D. 3
E. 9

Question 10

The area of the region bounded by the graph of $y = f(x)$ and the x-axis where $f(x) = x^2(p - x)(x - q)$ and $0 < p < q$ is given by the integral

A. $\int_0^q f(x)dx$

B. $\int_p^q f(x)dx$

C. $\int_0^p f(x)dx + \int_p^q f(x)dx$

D. $\int_0^p f(x)dx - \int_p^q f(x)dx$

E. $\int_p^0 f(x)dx + \int_p^q f(x)dx$

Question 11

The function f satisfies the functional equation $f(f(x)) = x$ for the maximal domain of f.

The rule for the function f is

A. $f(x) = 2\sqrt{x}$

B. $f(x) = 3x + 2$

C. $f(x) = e^x$

D. $f(x) = 2 + \dfrac{3}{x - 2}$

E. $f(x) = \log_e x$

Question 12

The graph of $y = \log_e(x + k^2)$ will have a positive x-intercept when

A. $k > 0$

B. $k < 0$

C. $-1 < k < 1$

D. $k > 1$

E. $k < -1$

Question 13

The simultaneous equations $mx - y = a$ and $2x - ay = -4m$ have an infinite set of solutions when

A. $a = -2$ and $m = -1$.

B. $a = -2$ and $m \neq -1$.

C. $a \neq -2$ and $m = -1$.

D. $a \neq -2$ and $m \neq -1$.

E. $a = 2$ and $m = 1$.

Question 14

A survey showed that out of a random sample of 40 customers that entered a shop, 10 made a purchase.

If this information is used to construct a 95% confidence interval for the proportion of all customers who made a purchase, then the margin of error will be

A. 0.025

B. 0.0685

C. 0.1342

D. 0.25

E. 0.2644

Question 15

The random variable X has a normal distribution with a mean of μ and a standard deviation of σ.

Given that $\Pr(X > 2\mu) = \frac{1}{2}\Pr(X < \mu)$, then σ is approximately equal to

A. 0.5μ

B. 0.67μ

C. μ

D. 1.48μ

E. 2μ

Question 16

The average value of the function with the rule $f(x) = \sin(2x)$ over the interval $\left[0, \frac{\pi}{4}\right]$ is

A. $\frac{1}{2}$

B. $\frac{\pi}{2}$

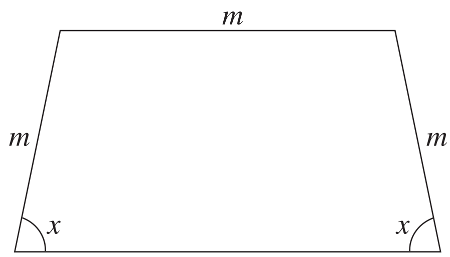
C. $\frac{2}{\pi}$

D. $\frac{\pi}{4}$

E. $\frac{4}{\pi}$

Question 17

A trapezium has side lengths of *m* as indicated on the diagram below.



The maximum area of the trapezium is

A. m

B. $\frac{3m^2\sqrt{3}}{4}$

C. $\frac{(\sqrt{3}+2)m^2}{4}$

D. $\frac{(\sqrt{2}+1)m^2}{2}$

E. $\sqrt{2}m^2$

Question 18

Let $f(x) = e^x$ and $g(x) = \sin(x)$.

The range of the composite function $f(g(x))$ is given by

A. R

B. R^+

C. $[-1, 1]$

D. $(0, e]$

E. $\left[\frac{1}{e}, e\right]$

Question 19

Let $f(x) = x^5 - x^3 + kx$.

The minimum value of *k* for which the function $f(x)$ has an inverse function is

A. 0

B. $\frac{1}{4}$

C. $\frac{9}{20}$

D. 1

E. 3

Question 20

Consider the discrete probability distribution with random variable X shown in the table below.

x	0	1	2	3
$\Pr(X = x)$	a^2	b	a^2	$2a^2$

The minimum value of $a + b$ is

A. $-\frac{1}{2}$

B. 0

C. $\frac{1}{4}$

D. $\frac{1}{2}$

E. 1

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (8 marks)

Let $f : [0, \pi] \to R, f(x) = -\sqrt{2}\cos(4x)$.

a. Find the period and range of f. 2 marks

b. State the rule and domain for the derivative function f'. 2 marks

c. A tangent to the curve $y = f(x)$ has the equation $y = 4x - \frac{3\pi - 4}{4}$.

Find the coordinates where this tangent touches the curve $y = f(x)$. 2 marks

d. The transformation $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ is applied to $f(x)$, producing a resulting image equation of $y = \cos(x)$.

Find the values of a and b. 2 marks

Question 2 (11 marks)

The number of hamburgers sold at a fast-food restaurant on any given day is normally distributed with a mean of 120 and a standard deviation of 15.

a. Find the probability that on any given day more than 150 hamburgers are sold. Give your answer correct to four decimal places. 1 mark

b. If the restaurant sells at least x hamburgers on at least 75% of days, find the value of x to the nearest whole number. 1 mark

It is known that 70% of customers select a hamburger with cheese. A random sample of 100 customers was taken.

c. **i.** What is the probability, correct to four decimal places, that more than 75% of customers selected a hamburger with cheese? Do **not** use a normal approximation. 2 marks

ii. What is the answer to **part c.i.** if a normal approximation is used? Give your answer correct to four decimal places. 2 marks

The staff at the restaurant consists of four females and two males. Two of the staff are randomly rostered on for customer service.

d. Complete the following probability distribution for the proportion of female staff that are rostered on for customer service. 2 marks

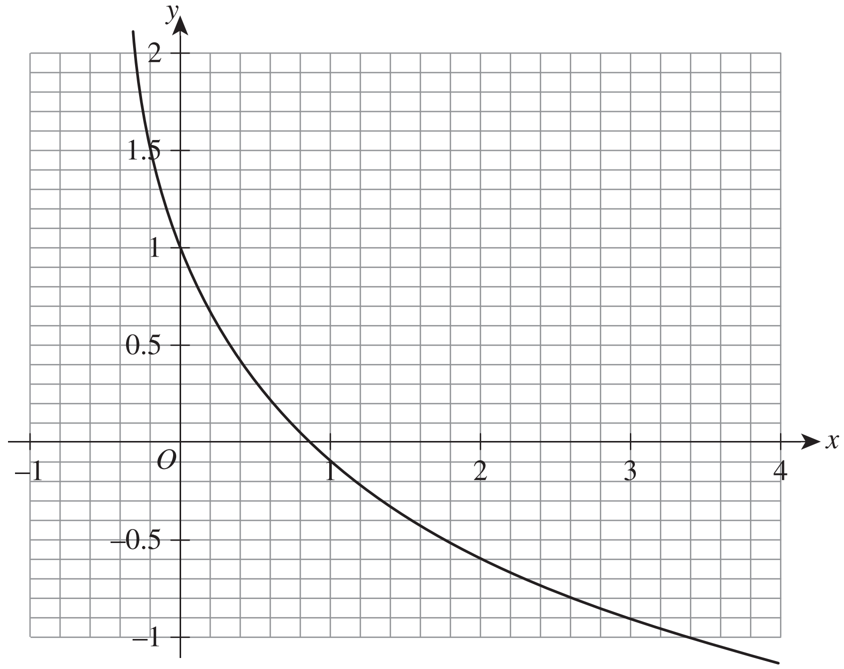
Proportion of female customer service staff ($\hat{p}$)			
Pr($\hat{P} = \hat{p}$)			

e. A customer orders a hamburger with cheese from a female customer service staff member.

Assuming the staffing has not changed, and each member of the customer service staff is equally likely to serve a customer, find the probability that the next customer who orders a hamburger will order a hamburger without cheese from a male customer service staff member. 3 marks

Question 3 (12 marks)

The graph of $y = f(x)$ where $f : S \to R$, $f(x) = 1 - \log_e(2x + 1)$ is sketched on the graph below.



a. State the maximal domain, S. 1 mark

b. Show that the rule for the inverse function can be written as $f^{-1}(x) = \frac{1}{2}(e^{1-x} - 1)$. 2 marks

c. **i.** Sketch the graph of $y = f^{-1}(x)$ on the set of axes above, indicating all intercepts and equations of asymptotes. 2 marks

ii. Hence find the area bounded by the cartesian axes and the graph of $y = f(x)$. 3 marks

d. Find the coordinates of the point of intersection of f and f^{-1} correct to two decimal places. 1 mark

e. Find the acute angle between the tangents to the graphs of f and f^{-1} at the point of intersection. Give your answer to the nearest degree. 3 marks

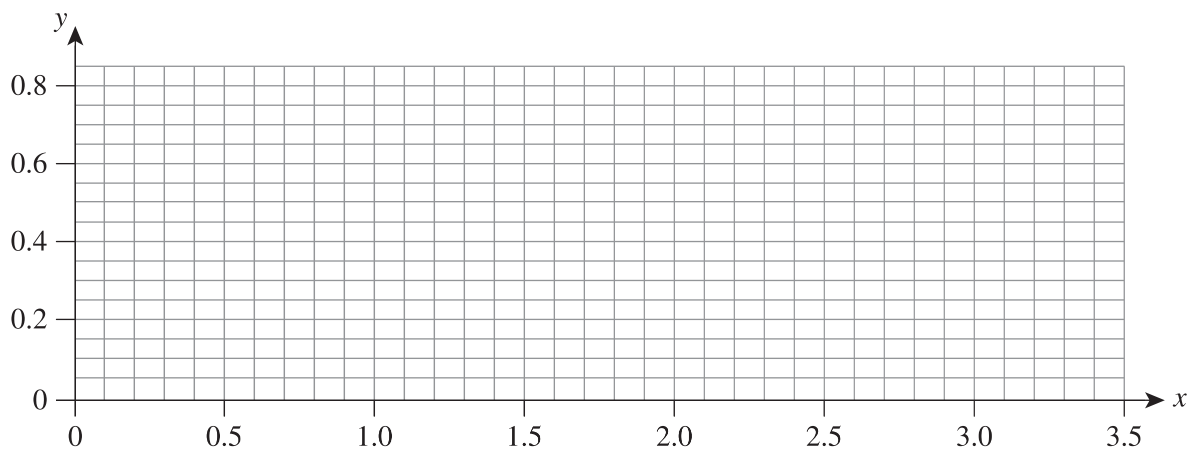
Question 4 (14 marks)

Max is learning to juggle. The time that he can juggle before dropping a ball, t, in minutes, can be modelled by the probability density function $f(t)$.

$$f(t) = \begin{cases} t & 0 \le t \le \frac{2}{3} \\ k(t-3) & \frac{2}{3} < t \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

a. Show that $k = -\frac{2}{7}$. 3 marks

b. Sketch the graph of f on the axes provided below. 2 marks



c. Find the probability that, given Max juggles for longer than 30 seconds, he juggles for at least 2 minutes. 3 marks

d. Find a such that $\Pr(T > a) = 0.6$. 2 marks

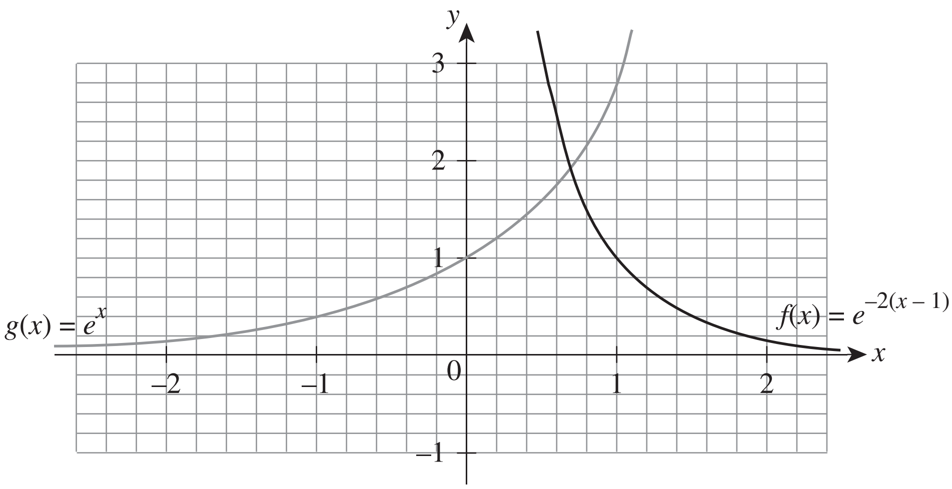
e. An individual attempt where Max juggles for at least two minutes is classified as 'successful' by Max. Assume that the amount of time that Max juggles in any given attempt is independent of any other attempt.

i. Show that the number of attempts that Max would need to have to ensure that he had at least a 70% chance of two 'successful' attempts can be found by solving the equation $n < \frac{9}{5} \times \left(\frac{7}{6}\right)^n - 6$. 3 marks

ii. Hence, or otherwise, find the minimum number of attempts required for Max to ensure that he had at least a 70% chance of two 'successful' attempts. 1 mark

Question 5 (15 marks)

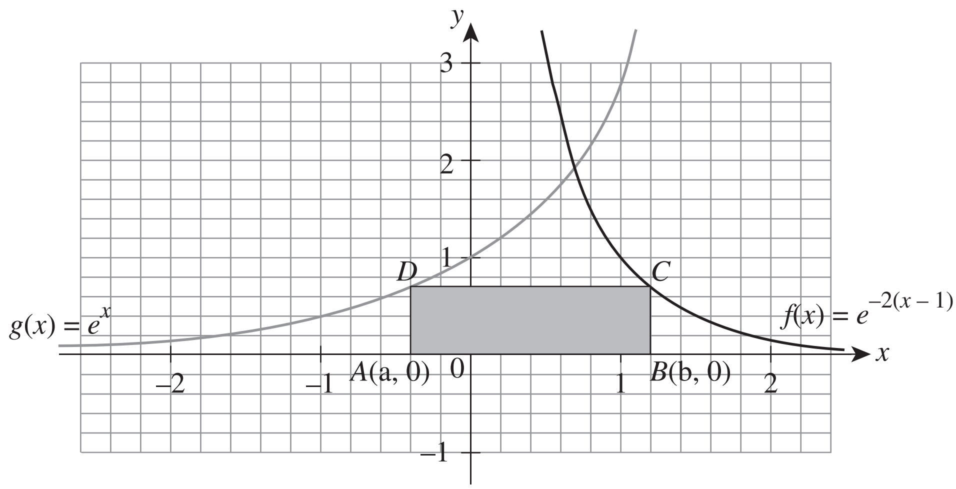
Let $f : R \to R, f(x) = e^{-2(x-1)}$ and $g : R \to R, g(x) = e^{x}$. Part of the graphs of $y = f(x)$ and $y = g(x)$ are shown below.



a. Find the coordinates of the point of intersection of f and g. 1 mark

b. Find the area bounded by the graphs of $y = f(x)$, $y = g(x)$ and the line $y = 1$. 3 marks

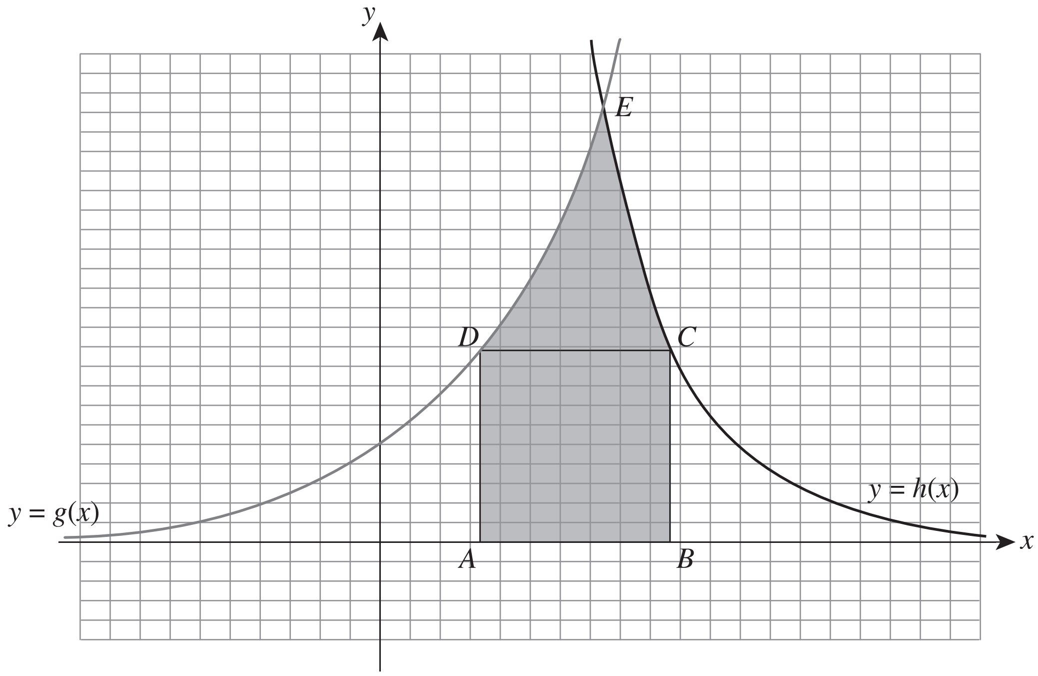
The rectangle $ABCD$ is formed as shown below, with points A and B on the x-axis and points C and D on the graphs of $y = f(x)$ and $y = g(x)$ respectively.



c. **i.** Show that the area of the rectangle $ABCD$ can be expressed as $(3b-2)e^{2-2b}$. 2 marks

ii. Hence, or otherwise, find the maximum area of the rectangle $ABCD$. 2 marks

Let $h : R \to R,\ h(x) = e^{-2(x-p)}$, where $p \in R$. Part of the graphs of $y = g(x)$ and $y = h(x)$ are shown below. The rectangle *ABCD* is also illustrated and the point *E* is defined as the intersection of g and h.



d. **i.** Find the value of p for which the maximum area of the rectangle *ABCD* is obtained when *ABCD* is a square. 3 marks

ii. Find the shaded area of *CDE* when the maximum area of the rectangle *ABCD* is obtained when *ABCD* is a square. 4 marks

END OF QUESTION AND ANSWER BOOKLET