Neap:

Trial Examination 2019

VCE Mathematical Methods Units 3&4

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	D
2	D
3	D
4	C
5	B
6	E
7	E
8	B
9	A
10	E
11	D
12	C
13	A
14	C
15	D
16	C
17	B
18	E
19	C
20	A

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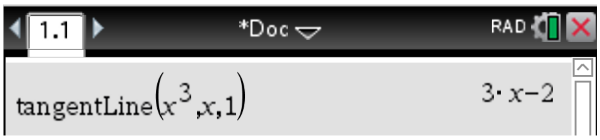
Question 1 **D**

It is a cubic graph with a stationary point of inflection.

Question 2 **D**

The gradient is negative for $x \in (-1, 4)$.

Question 3 **D**



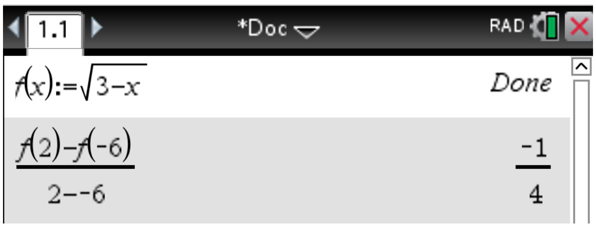
tangent: $y = 3x - 2$

y-intercept: $(0, -2)$

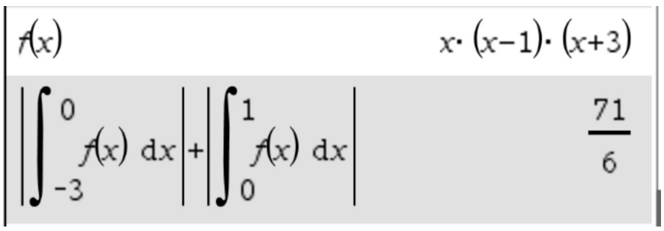
Question 4 **C**

$$\begin{aligned}\Pr(\text{same}) &= \Pr(RR) + \Pr(GG)\\ &= \frac{3}{5}\times\frac{2}{4}+\frac{2}{5}\times\frac{1}{4}\\ &= \frac{2}{5}\end{aligned}$$

Question 5 **B**



Question 6 **E**



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Question 7 E

$x' = -(x+1) \Rightarrow x = -1 - x'$

$y' = 3(y+2) \Rightarrow y = \frac{y'}{3} - 2$

$$y = x^3$$

$$\frac{y'}{3} - 2 = (-1 - x')^3$$

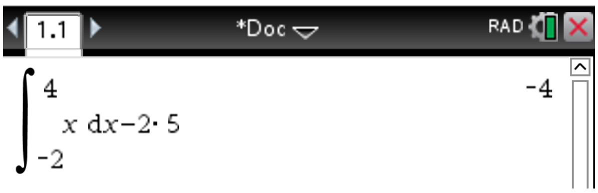
$$y' = -3(1 + x')^3 + 6$$

$$= 6 - 3(x' + 1)^3$$

Question 8 B

$$\int_{-2}^{4} (x - 2f(x))dx = \int_{-2}^{4} (x)dx - 2\int_{-2}^{4} f(x)dx$$

$$= \int_{-2}^{4} (x)dx - 2 \times 5$$



Question 9 A

There are two possible combinations to check:

Option 1 (correct):

$$m = \frac{-7-5}{3-(-1)}$$

$$= -3$$

Check: $y - 5 = -3(x - (-1))$

$$y = -3x + 2$$

Option 2 (incorrect):

$$m = \frac{-7-5}{-1-3}$$

$$= 3$$

Check: $y - (-7) = 3(x - (-1))$

$$y = 3x - 4$$

A quick sketch to scale would also be of benefit in determining the correct option.

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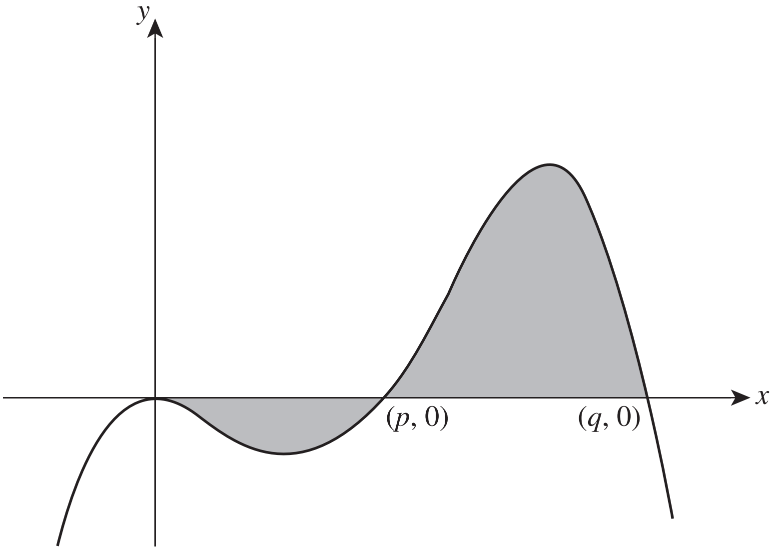
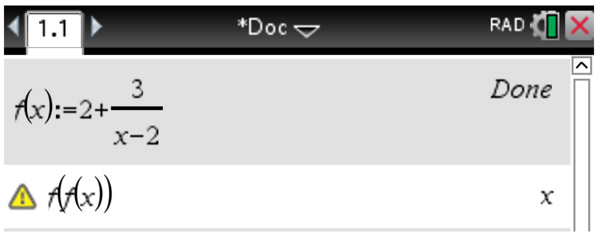
Question 10 **E**



$$\text{area} = -\int_0^p f(x)dx + \int_p^q f(x)dx$$

$$= \int_p^0 f(x)dx + \int_p^q f(x)dx$$

Question 11 **D**



Question 12 **C**

For x-intercept, let $y = 0$.

$$\log_e(x + k^2) = 0$$

$$x + k^2 = 1$$

$$x = 1 - k^2$$

For positive x-intercept:

$$1 - k^2 > 0$$

$$-1 < k < 1$$

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Question 13 A

$y = mx - a$ **(eq. 1)**

$y = \frac{2}{a}x + \frac{4m}{a}$ **(eq. 2)**

For infinite solutions, $m_1 = m_2$ and $c_1 = c_2$.

$$\text{solve}\left(\begin{cases}\frac{2}{a}=m\\ -a=\frac{4\cdot m}{a}\end{cases},\{a,m\}\right) \quad a=-2 \text{ and } m=-1$$

Question 14 C

zInterval_1Prop 10,40,0.95: *stat.results*

"Title"	"1−Prop z Interval"
"CLower"	0.11581
"CUpper"	0.38419
"p̂"	0.25
"ME"	0.13419
"n"	40.

Question 15 D

$$\Pr(X > \mu) = \frac{1}{2}$$

$$\Rightarrow \Pr(X > 2\mu) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Let $\Pr(Z > z) = \frac{1}{4}$.

$$\text{invNorm}(0.75,0,1) \quad 0.67449$$

$z = 0.6477\ldots$

$$z = \frac{x-\mu}{\sigma} = \frac{2\mu-\mu}{\sigma} = \frac{\mu}{\sigma}$$

$$\frac{\mu}{\sigma} = 0.6744$$

$$\sigma \approx 1.48\mu$$

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Question 16 **C**

$$\frac{1}{\frac{\pi}{4}} \cdot \int_0^{\frac{\pi}{4}} \sin(2 \cdot x)\, dx \qquad \frac{2}{\pi}$$

Question 17 **B**

$$\text{area} = \frac{1}{2}(a + b)h$$
$$= \frac{1}{2}(m + (m + 2m\cos(x))) \times m\sin(x)$$
$$= \frac{1}{2}(2m + 2m\cos(x)) \times m\sin(x)$$
$$= m^2 \sin(x)(\cos(x) + 1)$$

$$a(x) \qquad m^2 \cdot \sin(x) \cdot (\cos(x)+1)$$

$$\text{solve}\left(\frac{d}{dx}(a(x))=0, x\right)|0<x<\pi \qquad 0<x<\pi \text{ and } m=0 \text{ or } x=\frac{\pi}{3}$$

$$a\left(\frac{\pi}{3}\right) \qquad \frac{3 \cdot m^2 \cdot \sqrt{3}}{4}$$

Question 18 **E**

range $\sin(x) = [-1, 1]$

range $e^{\sin(x)} = [e^{-1}, e^{1}] = \left[\frac{1}{e}, e\right]$

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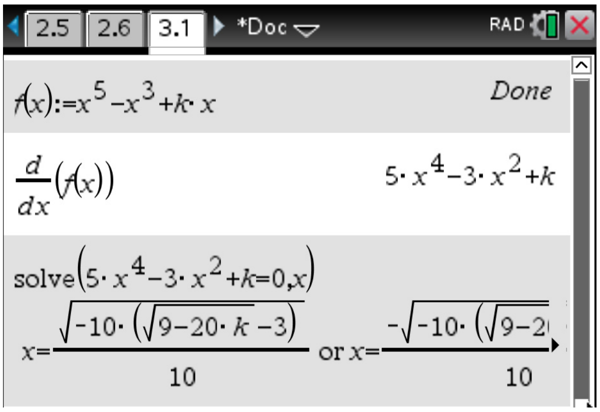
Question 19 **C**

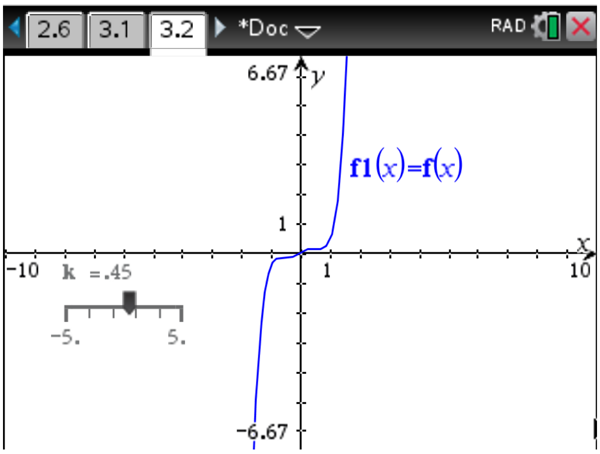
The inverse occurs if $f(x)$ is a one-to-one function (no turning points).



$$9 - 20k = 0$$

$$k = \frac{9}{20}$$

Sliders are also useful to confirm properties of the graph.



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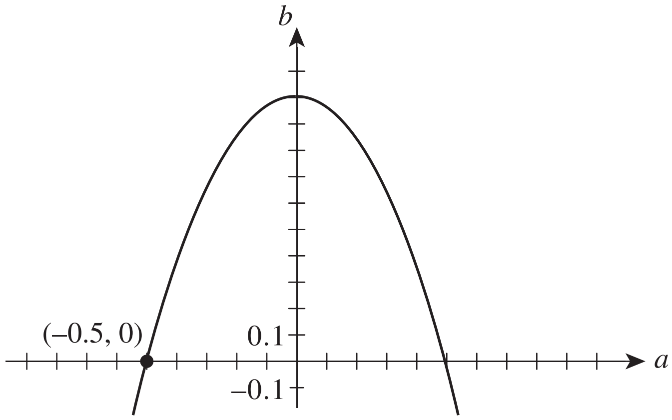
Question 20 A

$a^2 + b + a^2 + 2a^2 = 1$ for a probability distribution.

$4a^2 + b = 1$

$b = 1 - 4a^2$

A sketch of the graph of $b = 1 - 4a^2$ gives information about the relationship. Given that $b > 0$, the minimum value of $a + b$ must occur at the negative x-intercept, where $a = -\frac{1}{2}$ and $b = 0$. Therefore the minimum value of $a + b = -\frac{1}{2}$.



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SECTION B

Question 1 (8 marks)

a. period $= \frac{2\pi}{4} = \frac{\pi}{2}$ A1

range $= [-\sqrt{2}, \sqrt{2}]$ A1

b.

$f(x)$	$-\sqrt{2}\cdot\cos(4\cdot x)$
$\frac{d}{dx}(f(x))$	$4\cdot\sqrt{2}\cdot\sin(4\cdot x)$

$f'(x) = 4\sqrt{2}\sin(4x)$ A1

domain $= (0, \pi)$ A1

c. Let $f'(x) = 4$ (gradient of tangent).

$\text{solve}(4\cdot\sqrt{2}\cdot\sin(4\cdot x)=4,x)|0<x<\pi$ M1

$x=\frac{\pi}{16}$ or $x=\frac{3\cdot\pi}{16}$ or $x=\frac{9\cdot\pi}{16}$ or $x=\frac{11\cdot\pi}{16}$

Test values:

$\text{tangentLine}\left(f(x),x,\frac{\pi}{16}\right)$	$4\cdot x-\frac{\pi+4}{4}$
$\text{tangentLine}\left(f(x),x,\frac{3\cdot\pi}{16}\right)$	$4\cdot x-\frac{3\cdot\pi-4}{4}$

$f\left(\frac{3\cdot\pi}{16}\right)$	1

coordinates $= \left(\frac{3\pi}{16}, 1\right)$ A1

d. A dilation factor of 4 from the y-axis, a dilation factor of $\frac{1}{\sqrt{2}}$ from the x-axis and reflection in the x-axis are required.

$a = 4$ A1

$b = -\frac{1}{\sqrt{2}}$ A1

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Question 2 (11 marks)

a. $X \sim N(120, 15^2)$

4.2	4.3	5.1	*Doc	RAD
normCdf(150,∞,120,15)				0.02275

$\Pr(X > 150) = 0.0228$ A1

b. $\Pr(X > a) = 0.75$

$\Pr(X < a) = 0.25$

invNorm(0.25,120,15)	109.883

$x \approx 110$ hamburgers A1

c. **i.** $Y \sim \text{Bi}(100, 0.7)$

$\Pr(Y > 75) = \Pr(Y \geq 76)$ M1

binomCdf(100,0.7,76,100)	0.11357

$\Pr(Y > 75) = 0.1136$ A1

ii. $n = 100,\ p = 0.7$

$$\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$$

$$= \sqrt{\frac{0.7(1-0.7)}{100}}$$

$$= \frac{\sqrt{21}}{100}$$ M1

$$\hat{P} \sim N\left(0.7, \left(\frac{\sqrt{21}}{100}\right)^2\right)$$

$\text{normCdf}\left(0.75,\infty,0.7,\frac{\sqrt{21}}{100}\right)$	0.137617

$\Pr(\hat{P} > 0.75) = 0.1376$ A1

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d. $\Pr(\hat{P} = 0) = \Pr(\text{two males}) = \frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$

$\Pr(\hat{P} = 1) = \Pr(\text{two females}) = \frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$

$\Pr\left(\hat{P} = \frac{1}{2}\right) = 1 - (\hat{P} = 0) - (\hat{P} = 1) = 1 - \frac{1}{15} - \frac{2}{5} = \frac{8}{15}$

Proportion of female customer service staff ($\hat{p}$)	0	$\frac{1}{2}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{1}{15}$	$\frac{8}{15}$	$\frac{2}{5}$

first row all correct A1
second row all correct A1

e. $\Pr(\text{M/F staffing} \mid \text{at least one female}) = \Pr\left(\hat{P} = \frac{1}{2} \,\middle|\, \hat{P} \geq \frac{1}{2}\right)$

$$= \frac{\Pr\left(\hat{P} = \frac{1}{2}\right)}{\Pr\left(\hat{P} \geq \frac{1}{2}\right)}$$

$$= \frac{\frac{8}{15}}{\frac{8}{15} + \frac{2}{5}}$$

$$= \frac{4}{7}$$ M1

$\Pr(\text{served by male} \mid \text{staffing is M/F}) = \frac{1}{2}$

$\Pr(\text{customer orders without cheese}) = 1 - 0.7 = \frac{3}{10}$ M1

$\Pr(\text{customer orders without cheese from male}) = \frac{4}{7} \times \frac{1}{2} \times \frac{3}{10}$

$$= \frac{3}{35}$$ A1

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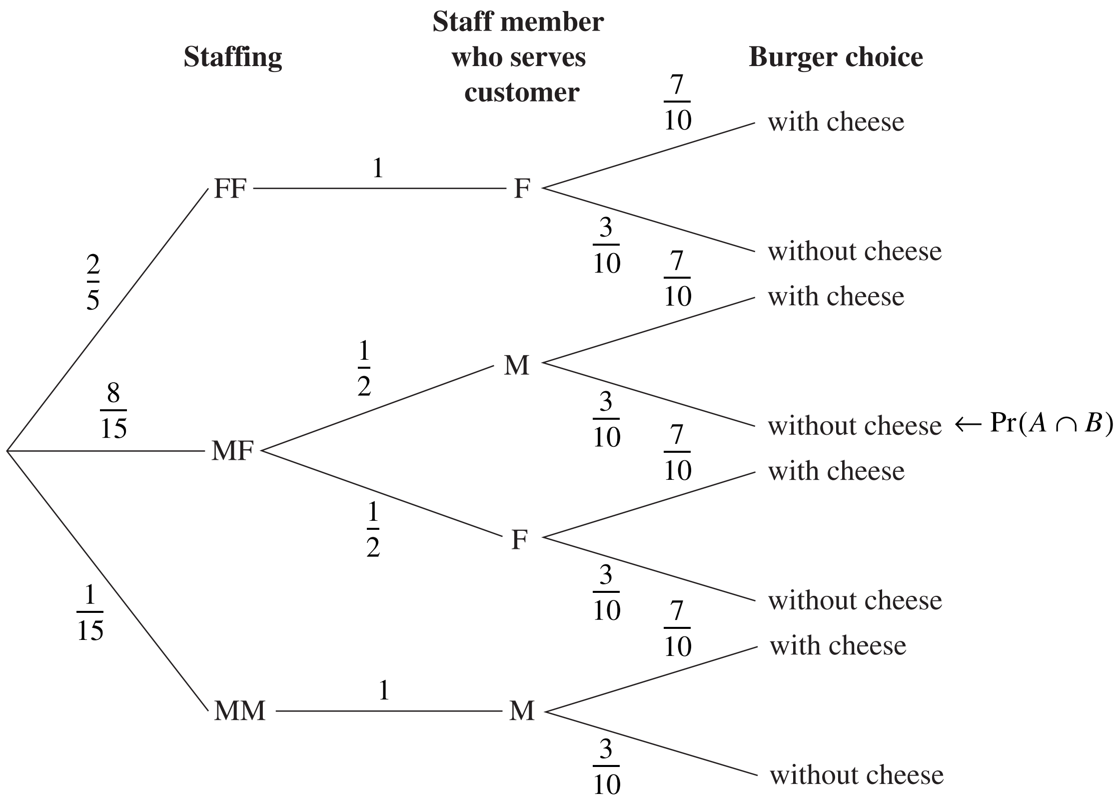
Alternative solution, using a tree diagram:

Let event A = customer orders burger with no cheese from male.

Let event B = at least one female is working.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$



M1

$$\Pr(A \cap B) = \frac{8}{15} \times \frac{1}{2} \times \frac{3}{10}$$

M1

$$= \frac{2}{25}$$

$$\Pr(B) = \frac{14}{15}$$

$$\therefore \Pr(A|B) = \frac{\frac{2}{25}}{\frac{14}{15}}$$

$$= \frac{3}{35}$$

A1

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Question 3 (12 marks)

a. $S = \left(-\frac{1}{2}, \infty\right)$ A1

b. Let $y = 1 - \log_e(2x + 1)$.

For inverse, swap x, y.

$x = 1 - \log_e(2y + 1)$ M1

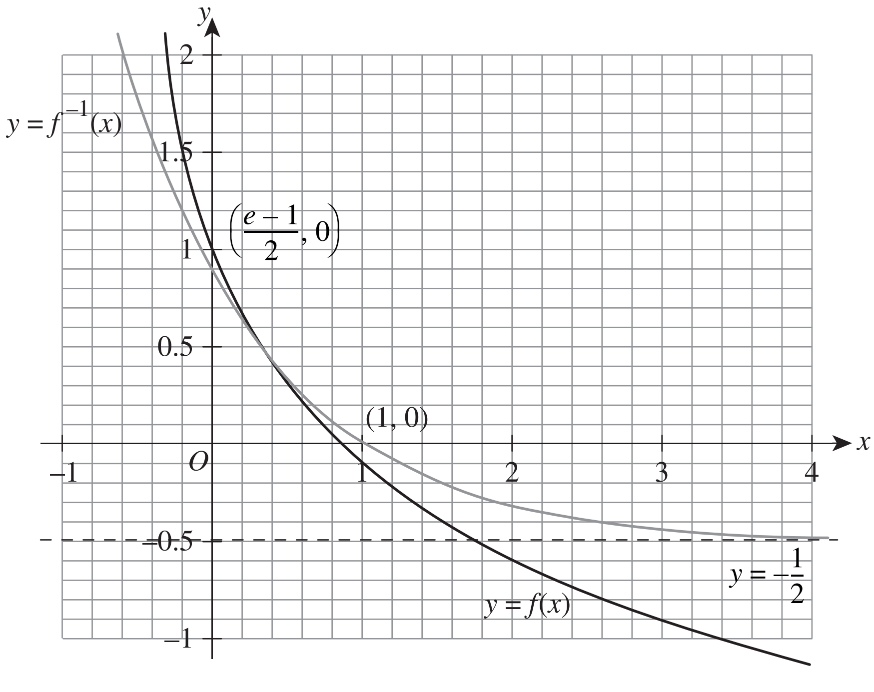
$\log_e(2y + 1) = 1 - x$

$2y + 1 = e^{1-x}$

$y = \frac{1}{2}(e^{1-x} - 1)$ A1

$f^{-1}(x) = \frac{1}{2}(e^{1-x} - 1)$

c. **i.**



correct intercepts A1
correct asymptote and shape A1

ii. The area bounded by the cartesian axes and the graph of $y = f(x)$ is equal to the area bounded by the cartesian axes and the graph of $y = f^{-1}(x)$.

$$\text{area} = \int_0^1 \left(\frac{1}{2}(e^{1-x} - 1)\right) dx$$ A1

$$= \left[\frac{-(xe^{1-x} + e)e^{-x}}{2}\right]_0^1$$ M1

$$= \frac{e-2}{2}$$ A1

d.

$$\text{solve}\left(1-\ln(2\cdot x+1)=\frac{1}{2}\cdot\left(e^{1-x}-1\right),x\right) \quad x=0.405795$$

$$\text{solve}\left(1-\ln(2\cdot x+1)=x,x\right) \quad x=0.405795$$

point of intersection = (0.41, 0.41) A1

e. The gradient of the tangent to $f(x)$ at the point of intersection is equal to –1.104…

The gradient of the tangent to $f^{-1}(x)$ at the point of intersection is equal to –0.9057… A1

$m = \tan(\theta) \Rightarrow \theta = \tan^{-1}(m)$ M1

$$\frac{d}{dx}\left(1-\ln(2\cdot x+1)\right)|x=0.405795 \quad -1.104$$

$$\frac{d}{dx}\left(\frac{1}{2}\cdot\left(e^{1-x}-1\right)\right)|x=0.405795 \quad -0.905795$$

$$\tan^{-1}(-1.1040025612859)-\tan^{-1}(-0.90579507\ldots \quad -5.65974$$

acute angle $\approx 6°$ (to the nearest degree) A1

Question 4 (14 marks)

a.

$$\int_0^{\frac{2}{3}}(t)\,dt + \int_{\frac{2}{3}}^{3} k(t-3)\,dt = 1$$ M1

$$\left[\frac{t^2}{2}\right]_0^{\frac{2}{3}} + k\left[\frac{t^2}{2}-3t\right]_{\frac{2}{3}}^{3} = 1$$ M1

$$\frac{2}{9} - 0 + k\left(\left(\frac{9}{2}-9\right)-\left(\frac{2}{9}-2\right)\right) = 1$$

$$\frac{2}{9} + k\left(-\frac{9}{2}+\frac{16}{9}\right) = 1$$

$$\frac{2}{9} - \frac{49k}{18} = 1$$

$$-\frac{49k}{18} = \frac{7}{9}$$

$$k = \frac{7}{9} \times -\frac{18}{49}$$

$$= -\frac{2}{7} \text{ as required}$$ A1

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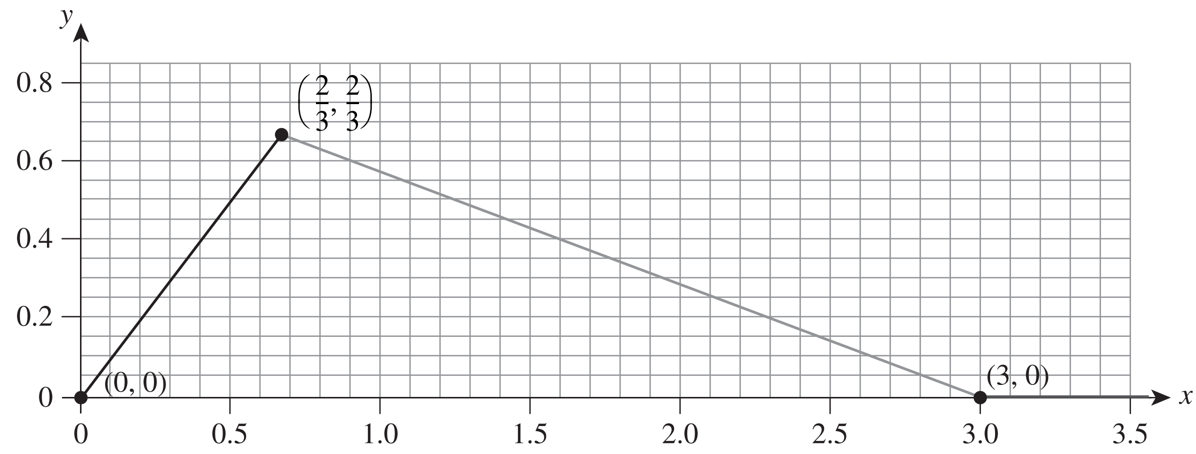
b.



correct coordinates A1
correct line segments A1

c. $\Pr(T > 30 \text{ secs}) = \Pr\left(T > \frac{1}{2}\text{mins}\right)$

$$\Pr\left(T > \frac{1}{2}\right) = 1 - \Pr\left(T \le \frac{1}{2}\right)$$

$$= 1 - \int_0^{\frac{1}{2}} (t)dt$$

$$= 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$ A1

$$\Pr\left(T \ge 2 | T > \frac{1}{2}\right) = \frac{\Pr\left(T \ge 2 \cap T > \frac{1}{2}\right)}{\Pr\left(T > \frac{1}{2}\right)}$$

$$= \frac{\Pr(T \ge 2)}{\Pr\left(T > \frac{1}{2}\right)}$$ M1

$$\Pr(T \ge 2) = \int_2^3 -\frac{2}{7}(t-3)dt = \frac{1}{7}$$

$$\Pr\left(T > \frac{1}{2}\right) = \frac{7}{8}$$

$$\Pr\left(T \ge 2 | T > \frac{1}{2}\right) = \frac{\frac{1}{7}}{\frac{7}{8}}$$

$$= \frac{8}{49}$$ A1

d. $\Pr(T > a) = \frac{3}{5}$

$\int_a^3 -\frac{2}{7}(t-3)\,dt = \frac{3}{5}$ for $\frac{2}{3} \le a \le 3$ M1

$a = \frac{15 - \sqrt{105}}{5}$ A1

e. **i.** $\Pr(T \ge 2) = \frac{1}{7}$ (from **part c.**)

$A \sim \text{Bi}\left(n, \frac{1}{7}\right)$ M1

$\Pr(A \ge 2) \ge \frac{7}{10} \Rightarrow \Pr(A = 0) + \Pr(A = 1) < \frac{3}{10}$

${}^nC_0 \times \left(\frac{1}{7}\right)^0 \times \left(\frac{6}{7}\right)^n + {}^nC_1 \times \left(\frac{1}{7}\right)^1 \times \left(\frac{6}{7}\right)^{n-1} < \frac{3}{10}$ M1

$$\left(\frac{6}{7}\right)^n + n \times \left(\frac{1}{7}\right) \times \left(\frac{6}{7}\right)^{n-1} < \frac{3}{10}$$

$$\left(\frac{6}{7}\right)^n + n \times \left(\frac{1}{7}\right) \times \left(\frac{6}{7}\right)^n \times \left(\frac{7}{6}\right) = \frac{3}{10}$$

$$\left(\frac{6}{7}\right)^n + \frac{n}{6} \times \left(\frac{6}{7}\right)^n < \frac{3}{10}$$

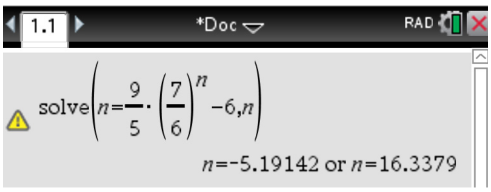
$$\left(\frac{6}{7}\right)^n\left(1 + \frac{n}{6}\right) < \frac{3}{10}$$

$$\left(1 + \frac{n}{6}\right) < \frac{3}{10} \times \left(\frac{7}{6}\right)^n$$

$$\frac{n}{6} < \frac{3}{10} \times \left(\frac{7}{6}\right)^n - 1$$

$$n < \frac{9}{5} \times \left(\frac{7}{6}\right)^n - 6$$ A1

ii.



$n = 17$ attempts A1

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Question 5 (15 marks)

a.

$f(x) := e^{-2\cdot(x-1)}$	*Done*
$g(x) := e^{x}$	*Done*
$\text{solve}(f(x)=g(x),x)$	$x=\frac{2}{3}$
$f\left(\frac{2}{3}\right)$	$e^{\frac{2}{3}}$

point of intersection = $\left(\frac{2}{3}, e^{\frac{2}{3}}\right)$ A1

b.

$\text{solve}(f(x)=1,x)$	$x=1$
$\text{solve}(g(x)=1,x)$	$x=0$

M1

area = $\int_0^{\frac{2}{3}} (g(x)-1)\,dx + \int_{\frac{2}{3}}^{1} (f(x)-1)\,dx$ M1

$\int_0^{\frac{2}{3}} (g(x)-1)\,dx + \int_{\frac{2}{3}}^{1} (f(x)-1)\,dx$	$\frac{3\cdot e^{\frac{2}{3}}}{2}-\frac{5}{2}$

area = $\frac{3e^{\frac{2}{3}}-5}{2}$ A1

c. i. coordinates: $D(a, e^{a})$ and $C(b, e^{-2(b-1)})$

C and D have the same y-coordinate $\Rightarrow a = -2(b-1) = 2-2b$ M1

area = base × height

$= (b-a)\times e^{a}$

$= (b-(2-2b))e^{2-b}$

$= (3b-2)e^{2-2b}$ A1

ii.

$a(b):=(3\cdot b-2)\cdot \mathbf{e}^{2-2\cdot b}$	*Done*
$\frac{d}{db}(a(b))$	$-(6\cdot b-7)\cdot \mathbf{e}^{2-2\cdot b}$
$\text{solve}\left(-(6\cdot b-7)\cdot \mathbf{e}^{2-2\cdot b}=0,b\right)$	$b=\frac{7}{6}$

$b = \frac{7}{6}$ gives the maximum area. A1

$a\left(\frac{7}{6}\right)$	$\frac{3\cdot \mathbf{e}^{\frac{-1}{3}}}{2}$

maximum area $= \frac{3}{2e^{\frac{1}{3}}}$ A1

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d. **i.** coordinates: $D(a, e^{a})$ and $C(b, e^{-2(b-p)})$

C and D have the same y-coordinate $\Rightarrow a = -2(b-p) = 2p - 2b$

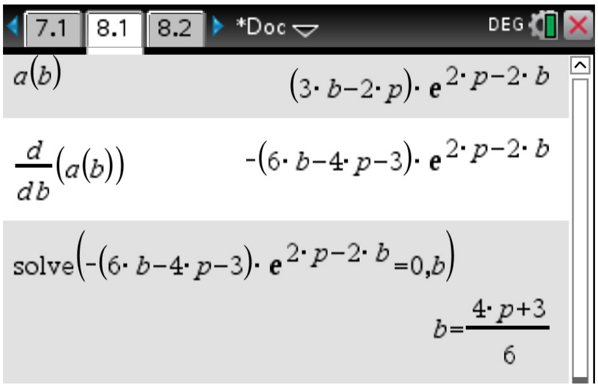
area = base × height

$= (b-a) \times e^{a}$

$= (b-(2p-2b))e^{2-b}$

$= (3b-2p)e^{2-2b}$

Let $A(b) = (3b-2p)e^{2-2b}$. M1



However, $a = 2p - 2b \Rightarrow b = p - \frac{a}{2}$.

$a = 2p - \left(\frac{4p+3}{6}\right)$

$= \frac{2p-3}{3}$

base = $b - a$

$= \frac{4p+3}{6} - \frac{2p-3}{3}$

$= \frac{3}{2}$ M1

base = height = $\frac{3}{2}$

$\therefore e^{a} = \frac{3}{2} \Rightarrow a = \log_e\left(\frac{3}{2}\right)$

Also, $b - a = e^{a}$ and $b = p - \frac{a}{2}$.

$\Rightarrow p - \frac{a}{2} - a = e^{a}$

$p = \frac{3}{2}a + e^{a}$

$= \frac{3}{2}\log_e\left(\frac{3}{2}\right) + \frac{3}{2}$ A1

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ii. $a = \log_e\left(\frac{3}{2}\right)$

$b - a = \frac{3}{2} \Rightarrow b = \frac{3}{2} + \frac{3}{2}\log_e\left(\frac{3}{2}\right)$

$p = \frac{3}{2}\log_e\left(\frac{3}{2}\right) + \frac{3}{2}$ M1

$g(x) = e^x$ and $h(x) = e^{-2(x-p)}$.

Solve $g(x) = h(x)$ for $p = \frac{3}{2}\log_e\left(\frac{3}{2}\right) + \frac{3}{2}$.

$x = \log_e\left(\frac{3}{2}\right) + 1$ A1

p	$\frac{3\cdot\ln\left(\frac{3}{2}\right)}{2}+\frac{3}{2}$
$\text{solve}(g(x)=h(x),x)$	$x=\ln(3)-\ln(2)+1$
a	$\ln\left(\frac{3}{2}\right)$
b	$\frac{2\cdot\ln(3)-2\cdot\ln(2)+3}{2}$

area square $ABCD = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Shaded area CDE is the sum of two integrals – square $ABCD$ as follows: M1

$$\int_{a}^{\ln\left(\frac{3}{2}\right)+1} g(x)\,dx + \int_{\ln\left(\frac{3}{2}\right)+1}^{b} h(x)\,dx - \frac{9}{4}$$

$$\frac{9\cdot e}{4} - \frac{9}{2}$$

area $CDE = \frac{9e}{4} - \frac{9}{2}$ A1

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