The Mathematical Association of Victoria

Trial Exam 2019

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 10 pages,
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Differentiate $\frac{\cos(x)}{x}$. 2 marks

b. Find $f'(\pi)$ if $f(x) = 5x^2 \tan(3x)$. 2 marks

Question 2 (3 marks)

Solve $2e^x + 5 = 3e^{-x}$ for x.

Question 3 (5 marks)

Let $f:[1,\ 4) \to R,$ where $f(x) = (x-4)^2$ and $g: R \setminus \{0\} \to R,$ where $g(x) = \frac{1}{x} + 3$.

a. Explain why $g(f(x))$ exists. 1 mark

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b. Find the rule and domain for $g(f(x))$. 2 marks

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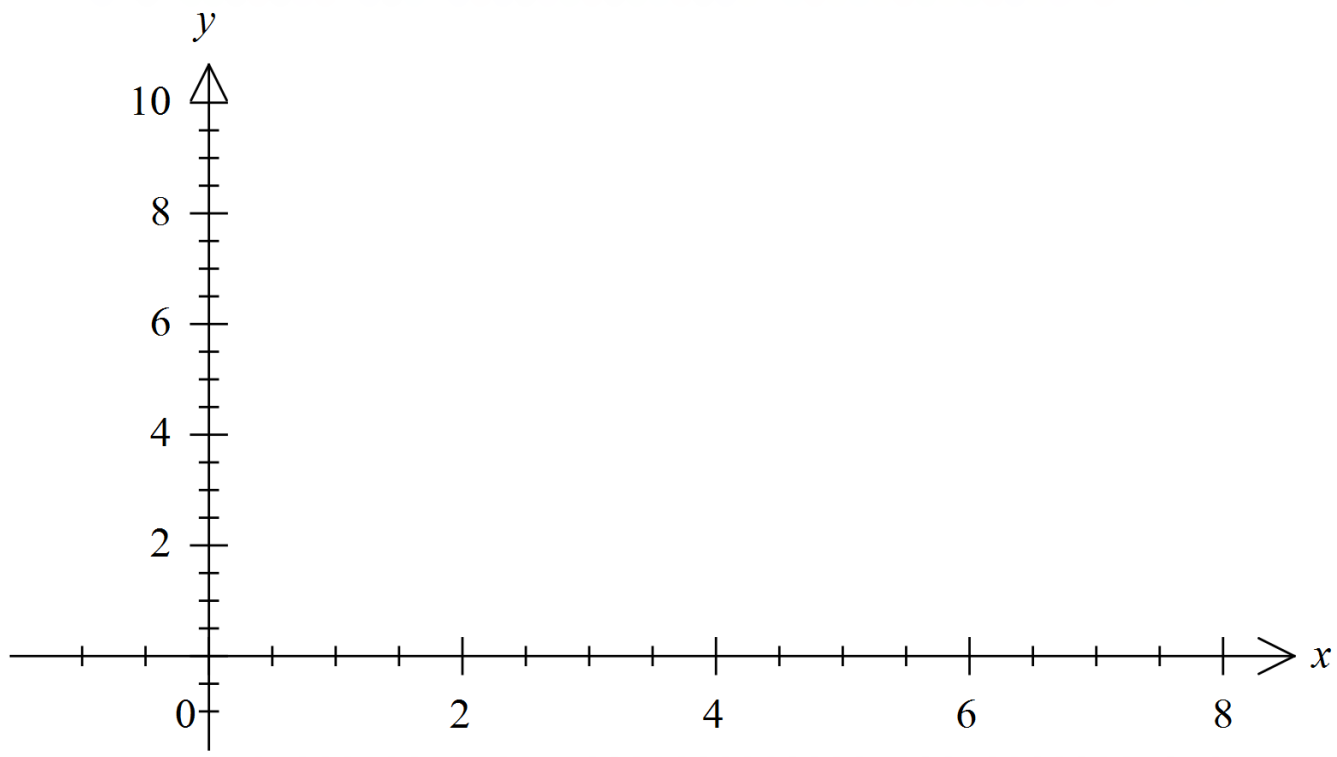
c. Sketch the graph of $y = g(f(x))$ on the set of axes below. Label the endpoint with its coordinates and any asymptotes with their equations. 2 marks



Question 4 (5 marks)

Let $h:[-1,\infty) \to R,$ where $h(x) = -\sqrt{x+1}$.

a. Define h^{-1}. 2 marks

b. Find the coordinates of the points where the graphs of h and h^{-1} intersect. 3 marks

TURN OVER

Question 5 (6 marks)

a. Differentiate $x\log_e(x) - x$. 1 mark

Let $g(x) = 2\log_e(x-1)$.

b.i. State the transformations required to map the graph with equation $f(x) = \log_e(x)$ to the graph of $g(x)$. 2 marks

ii. Using your answer to **part a.**, find the area bounded by the graph of g, the x-axis and the line $x = 3$. 3 marks

Question 6 (3 marks)

Wendy's mother has two boxes of chocolates. Box A has 6 white chocolates and 10 dark chocolates. Box B has 4 white chocolates and 5 dark chocolates. Wendy's mother allows her to randomly select a box and then take out two chocolates to eat.

a. What is the probability Wendy selects two white chocolates? 2 marks

Wendy selects two white chocolates.

b. What is the probability they are from Box A? 1 mark

TURN OVER

Question 7 (4 marks)

A probability density function, f has rule

$$f(x) = \begin{cases} \sqrt{x+1}, & -1 \le x < 0 \\ \dfrac{2}{x+2}, & 0 \le x \le a \\ 0, & \text{elsewhere} \end{cases}$$, where a is a real constant.

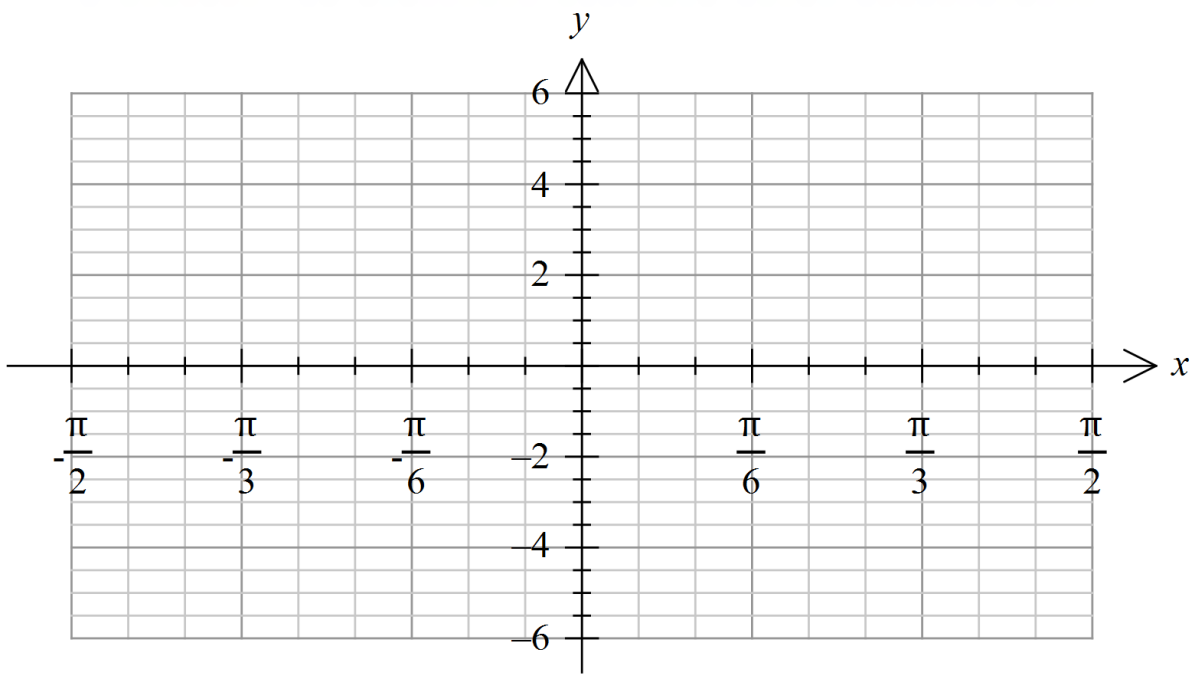
Find the value of a.

Question 8 (6 marks)

a. Solve the equation $\sqrt{3}\tan\left(2x-\frac{\pi}{2}\right)+2=1$ for $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. 3 marks

b. Sketch the graph of $y=\tan\left(2x-\frac{\pi}{2}\right)+\frac{1}{\sqrt{3}}$ for $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, on the grid below, labelling the asymptotes with their equations and the axial intercepts with their coordinates. 3 marks

Note: $\frac{1}{\sqrt{3}}\approx 0.6$.



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Question 9 (4 marks)

Let $f(x) = x^3 + 2x$.

a. Show that the equation of the tangent to the curve of f at $x = 0$ is $y = 2x$. 1 mark

Let $g : (0, \infty) \to R$, where $g(x) = \frac{1}{x} + 2$.

b. Write down an expression involving definite integrals which when evaluated will give the area bounded by the graphs of f and g and the tangent line in **part a**. Do not evaluate the expression. 3 marks

END OF QUESTION AND ANSWER BOOK