VCE METHODS EXAM 1 2019 (NOV)

Question 1

a) i) $f(x) = (3x-1)^{-1}$

$$f'(x) = -(3x-1)^{-2} \times 3$$

$$= \boxed{\frac{-3}{(3x-1)^2}}$$

ii) $$\int \frac{1}{3x-1}\,dx = \frac{1}{3}\log_e|3x-1| + c$$

as $x > \frac{1}{3}$ $$= \boxed{\frac{1}{3}\log_e(3x-1) + c}$$

b) $$g(x) = \frac{\sin(\pi x)}{x+1}$$

$$g'(x) = \frac{\pi\cos(\pi x)(x+1) - \sin(\pi x)}{(x+1)^2}$$

$$g'(1) = \frac{\pi\cos(\pi)\cdot 2 - \sin(\pi)}{2^2}$$

$$= \frac{-2\pi}{4}$$

$$= \boxed{-\frac{\pi}{2}}$$

QUESTION 2

a) $f(x) = \frac{1}{3x-1}$

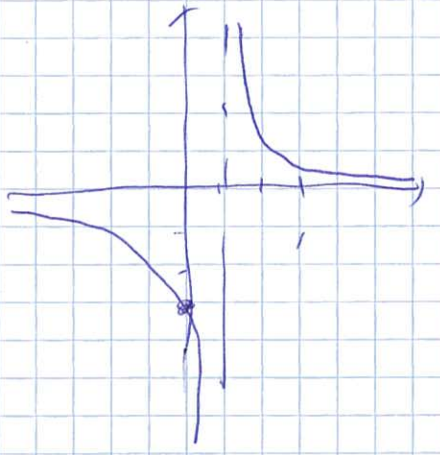
$x = \frac{1}{3y-1}$

$x(3y-1) = 1$

$3xy - x = 1$

$3xy = x+1$

$y = \frac{x+1}{3x} \Rightarrow f^{-1}(x) = \frac{x+1}{3x}$



range $f(x) = \mathbb{R} \setminus \{0\}$

b) domain $f^{-1}(x)$: $\boxed{x \in (-\infty, 0) \cup (0, \infty)}$

or: $\boxed{x \in \mathbb{R} \setminus \{0\}}$

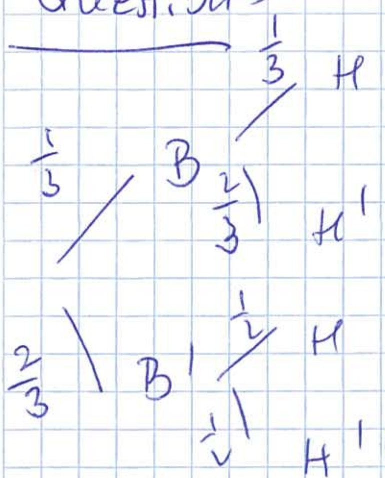
c) $\frac{1}{3x-1} \to \frac{x+1}{3x}$

$\frac{1}{3(x-\frac{1}{3})} \to \frac{1}{3} + \frac{1}{3x}$

$\boxed{c = -\frac{1}{3}}$

$\boxed{d = \frac{1}{3}}$

Question 3



a)

BH or B'H

$$\frac{1}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{2}$$

$$Pr(H) = \boxed{\frac{4}{9}}$$

b) $$Pr(B' | H) = \frac{Pr(B' \cap H)}{Pr(H)}$$

$$= \frac{\frac{1}{3}}{\frac{4}{9}} = \boxed{\frac{3}{4}}$$

QUESTION 4

a) $1 - \cos\left(\frac{x}{2}\right) = \cos\left(\frac{x}{2}\right)$ $\qquad x \in [-2\pi, \pi]$

$$2\cos\left(\frac{x}{2}\right) = 1$$

$$\cos\left(\frac{x}{2}\right) = \frac{1}{2}$$

$$\frac{x}{2} = \frac{\pi}{3}, \frac{5\pi}{3}\ {-4\pi} \qquad \text{period} = 4\pi$$

$$x = \frac{2\pi}{3}, \frac{10\pi}{3}$$

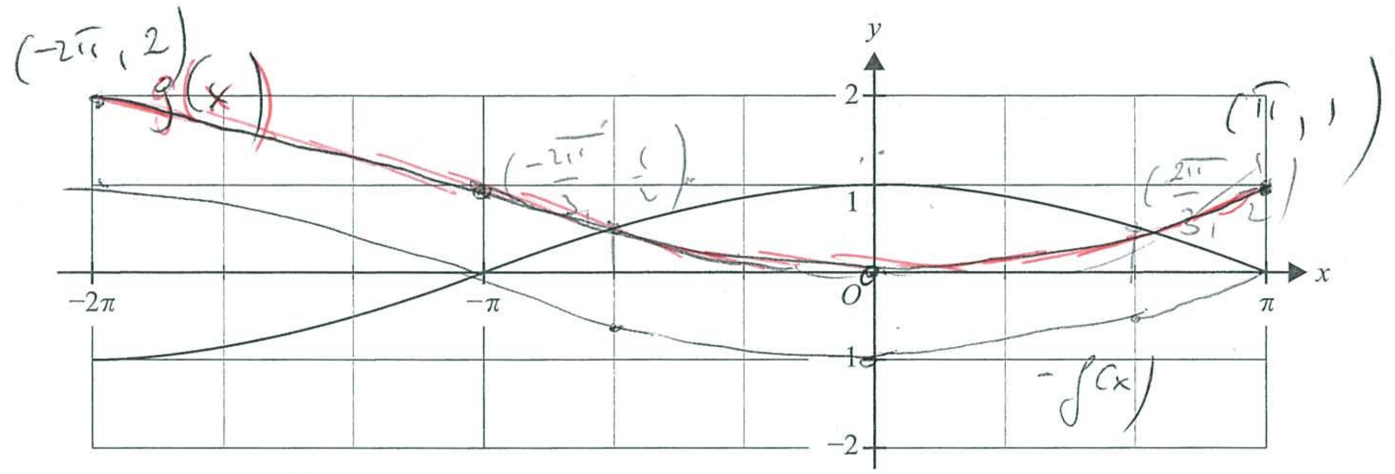
$$\boxed{x = \frac{-2\pi}{3}, \frac{2\pi}{3}}$$

b) next page.

Question 4 (4 marks)

a. Solve $1-\cos\left(\frac{x}{2}\right)=\cos\left(\frac{x}{2}\right)$ for $x \in [-2\pi, \pi]$. 2 marks

b. The function $f: [-2\pi, \pi] \to R,\ f(x)=\cos\left(\frac{x}{2}\right)$ is shown on the axes below.



Let $g: [-2\pi, \pi] \to R,\ g(x)=1-f(x)$.

Sketch the graph of g on the axes above. Label all points of intersection of the graphs of f and g, and the endpoints of g, with their coordinates. 2 marks

$g(x) = 1 - \cos\left(\frac{x}{2}\right)$

Question 5

a) i) $f(x) = \frac{2}{(x-1)^2} + 1$

$f(-1) = \boxed{\frac{3}{2}}$

ii) NEXT PAGE

b) Area $= \int_{-1}^{0} \frac{2}{(x-1)^2} + 1 \, dx$

$= 2 \int_{-1}^{0} (x-1)^{-2} + 1 \, dx$

$= \left[2 \frac{(x-1)^{-1}}{-1} + x \right]_{-1}^{0}$

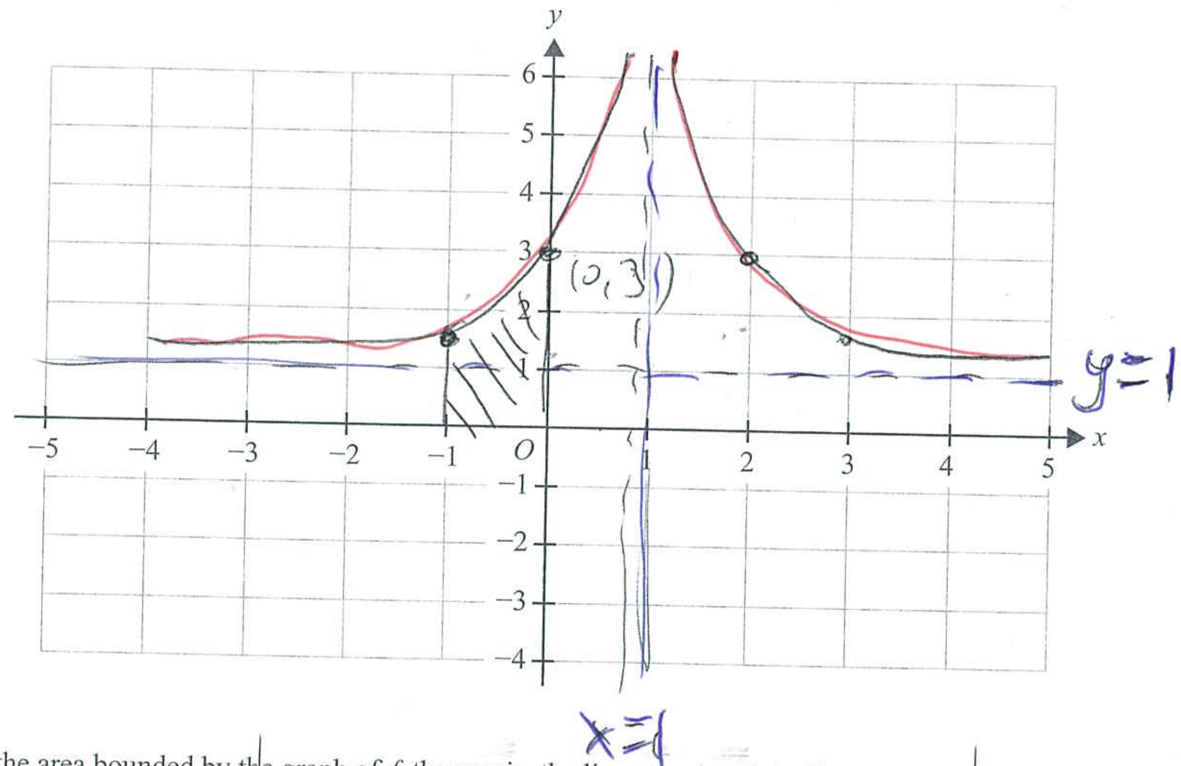
$= \boxed{2}$

Question 5 (5 marks)

Let $f: R \setminus \{1\} \to R,\ f(x) = \dfrac{2}{(x-1)^2} + 1$.

a. **i.** Evaluate $f(-1)$. 1 mark

ii. Sketch the graph of f on the axes below, labelling all asymptotes with their equations. 2 marks



b. Find the area bounded by the graph of f, the x-axis, the line $x = -1$ and the line $x = 0$. 2 marks

QUESTION 6

$n = 41$

a) $\boxed{p = \frac{8}{41}}$

b) $p = \frac{1}{6}$

$n = 12$

$P\left(p \in \frac{1}{6}\right)$

$$\boxed{\begin{aligned} a &= \frac{17}{6} \\ b &= \frac{5}{6} \\ n &= 11 \end{aligned}}$$

Question 7

a) $P(x, \sqrt{1-x^2})$ $\quad B(x, 0)$

$$PB = \sqrt{(x-x)^2 + (0-\sqrt{1-x^2})^2}$$

$$= \boxed{\sqrt{1-x^2}}$$

b) $$A_\Delta = \frac{1}{2}\underbrace{(x+1)}_{u}\underbrace{\sqrt{1-x^2}}_{v}$$

$$\frac{dA}{dx} = \frac{1}{2}\sqrt{1-x^2} + \frac{1}{\cancel{2}}(x+1)\cdot\frac{-\cancel{2}x}{2\sqrt{1-x^2}}$$

$$= \frac{1}{2}\sqrt{1-x^2} - \frac{x(x+1)}{2\sqrt{1-x^2}}$$

$$= \frac{1-x^2-x^2-x}{2\sqrt{1-x^2}}$$

$$= \frac{-2x^2-x+1}{2\sqrt{1-x^2}} = 0$$

$$2x^2+x-1 = 0. \qquad (x+1)(2x-1)=0$$

$$x = \frac{1}{2}.$$

$$A = \frac{1}{2}\left(\frac{1}{2}+1\right)\sqrt{1-\frac{1}{4}}$$

$$= \frac{1}{2}\cdot\frac{3}{2}\cdot\sqrt{\frac{3}{4}} = \frac{3}{4}\cdot\frac{1}{2}\sqrt{3} = \boxed{\frac{3\sqrt{3}}{8}}$$

QUESTION 8

a) $f(x) = ax^2(x+1)(x-1)$

$f(x) = ax^2(x^2-1)$

$1 = a \cdot \frac{1}{2}\left(\frac{1}{2}-1\right)$

$2 = a\left(-\frac{1}{2}\right)$

$a = -4$

$$\boxed{f(x) = -4x^2(x^2-1)}$$

b) $g(x) = -4x^2(x^2-1)$

$= 4x^2(1-x^2)$

$h(x) = \log_e[4x^2(1-x)(1+x)] - \log_e[x^2(x+1)]$

$x \neq 0 \quad 1-x^2 > 0 \quad x+1 > 0$

$$\boxed{D = (-1,0) \cup (0,1)}$$

c) range:

$$\boxed{(-\infty, \log_e(4)) \cup (\log_e(4), 3\log_e(2))}$$

Question 9

$f(x) = 3 + 2x - x^2$

$g(x) = e^x$

a) $g(f(x)) = \boxed{e^{3+2x-x^2}}$

b) $[g(f(x))]' = (-2x+2)\, e^{3+2x-x^2}$

$-2x + 2 < 0$

$-2x < -2$

$\boxed{x > 1}$

c) $f(g(x)) = \boxed{3 + 2e^x - e^{2x}}$

d) $-e^{2x} + 2e^x + 3 = 0$

$e^{2x} - 2e^x - 3 = 0$

let $a = e^x$

$a^2 - 2a - 3 = 0$

$(a-3)(a+1) = 0$

$a = 3$

$e^x = 3$

$\boxed{x = \log_e 3}$

e)

$$f(g(x)) = 3 + 2e^{x} - e^{2x}$$

$$(f(g(x)))' = 2e^{x} - 2e^{2x} = 0$$

$$2e^{x} - 2e^{x} \cdot e^{x} = 0$$

$$2e^{x}(1 - e^{x}) = 0$$

$$e^{x} = 1$$

$$x = 0$$

$$f(g(x))(0) = 3 + 2 - 1$$
$$= 4$$

$$\boxed{(0, 4)}$$

f)

$$e^{3+2x-x^2} \neq 3 + 2e^{x} - e^{2x} = 0$$

$$e^{3+2x-x^2} = e^{2x} - 2e^{x} - 3$$

$$\frac{e^{3} \cdot e^{2x}}{e^{x^2}} = e^{2x} - 2e^{x} - 3$$

one solution

$$e^{3} \cdot e^{2x} = e^{x^2}(e^{2x} - 2e^{x} - 3)$$