TSSM
Creating VCE Success

2018 Trial Examination

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STUDENT NUMBER | | | | | | | | | Letter | |

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	4	4	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference book, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 21 pages

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions. Choose the response that is correct for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will not be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f: R \to R,\ f(x) = 1 - 2\sin(3x)$.

The period and range of this function are respectively

A. $3\pi\ \ and\ \ [-1, 3]$
B. $\frac{2\pi}{3}\ \ and\ \ [-1, 3]$
C. $2\pi\ \ and\ \ [-1, 1]$
D. $\frac{2\pi}{3}\ \ and\ \ (-1, 3)$
E. $\frac{2\pi}{3}\ \ and\ \ [-1, 1]$

Question 2

Let $f\ and\ g$ be functions such that $f(x) = x^2 - 1$ and $g(x) = \frac{3-x}{4}$

The value of $g\big(f(2)\big)$ is

A. 0
B. $-\frac{15}{16}$
C. 1
D. -1
E. $\frac{3}{4}$

Question 3

A bag contains five red balls and four blue balls. Two balls are drawn at random from the box without replacement.

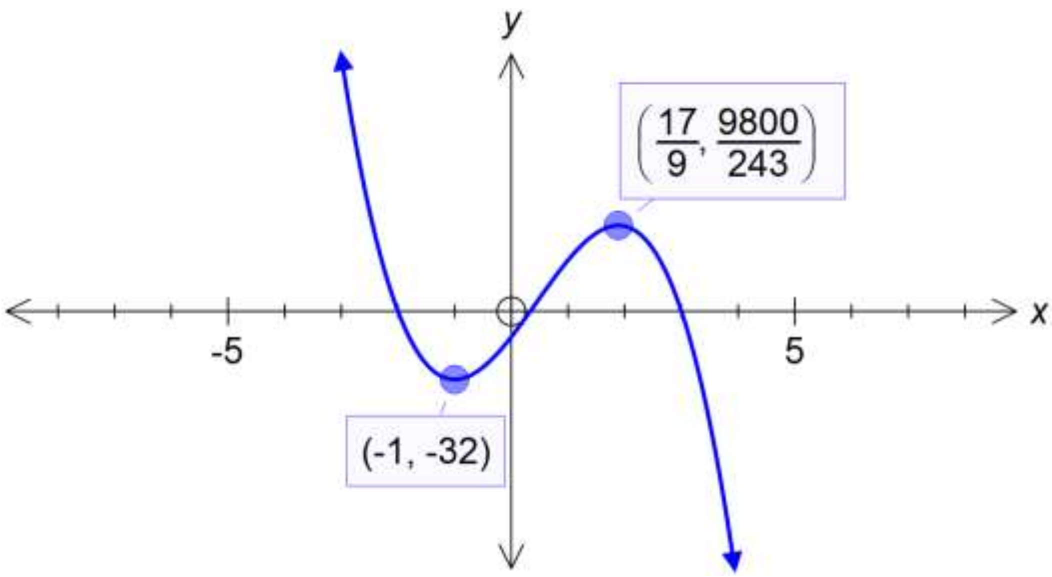
The probability of at least one ball being red is

A. $\frac{65}{81}$
B. $\frac{5}{9}$
C. $\frac{5}{6}$
D. $\frac{25}{81}$
E. $\frac{5}{8}$

SECTION A - continued

Question 4

Part of the graph of a cubic polynomial f and the coordinates of its stationary points are shown below.



$f'(x) < 0$ for the interval

A. R

B. $(-\infty, -2) \cup (3, \infty)$

C. $(-2, 3)$

D. $\left(-1, \frac{17}{9}\right)$

E. $(-\infty, -1) \cup \left(\frac{17}{9}, \infty\right)$

Question 5

If $y = b^{a-\frac{x}{2}} - 5$, where $b > 0$, then *x* is equal to

A. $\frac{1}{2}\left(a - log_b(y+5)\right)$

B. $2(log_b(y) + 5 - a)$

C. $2a - log_b(y+5)$

D. $2\left(a - log_b(y+5)\right)$

E. $2\left(b - log_a(y+5)\right)$

SECTION A - continued

TURN OVER

Question 6

The average rate of change of the function with the rule $f(x) = 5x + x^3$ over the interval [0, a], where $a > 0$, is 8.

The value of a is

A. $\sqrt{3}$
B. $-\sqrt{3}$
C. 2
D. 1
E. $\sqrt{5}$

Question 7

A transformation $T: R^2 \to R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{4} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ maps the graph of

$y = \sin\left(3\left(x - \frac{\pi}{2}\right)\right)$ onto the graph of

A. $y = 4\cos\left(3x + \frac{\pi}{2}\right)$
B. $y = -4\cos\left(3x + \frac{\pi}{2}\right)$
C. $y = \frac{1}{4}\sin\left(3x + \frac{\pi}{2}\right)$
D. $y = -4\sin\left(3x + \frac{\pi}{2}\right)$
E. $y = -\frac{1}{4}\sin\left(3x + \frac{\pi}{2}\right)$

Question 8

The sum of the solutions of $\cos(2x) = \frac{1}{2}$ over the interval $[-\pi, \pi]$ is

A. -1
B. 0
C. π
D. 2π
E. 2

SECTION A - continued

Question 9

Let $h: R \rightarrow R,\ h(x) = \frac{1}{(x-1)^2}$.

Which one of the following statements about h is true?

A. $\left(h(x)\right)^2 = h(x^2)$
B. $2h(x) = h(x^2)$
C. $h(x)h(-x) = h(x^2)$
D. $h(x) - h(-x) = h(2x)$
E. $h(x) - h(0) = h(x)$

Question 10

The random variable X has the following probability distribution, where

x	-1	0	1	2
$Pr(X = x)$	p	$1 - p$	$2p$	$1 - 2p$

The variance of X is

A. $p(7 - 9p)$
B. $7 - 9p^2$
C. $2p(7 - 9p)$
D. $p(5 - 7p)$
E. $\frac{p}{2}(5 - 7p)$

Question 11

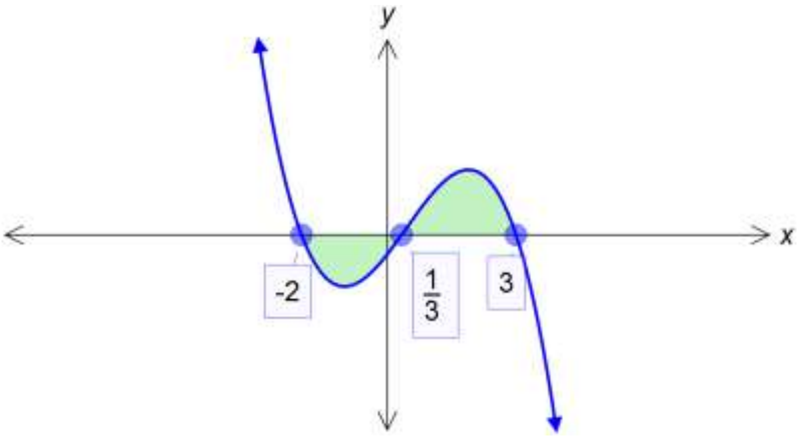
Let X be a discrete ransom variable with binomial distribution $X \sim Bi(20, p)$. If the mean is twice the standard deviation of this distribution, the value of p is

A. $\frac{2}{3}$
B. $\frac{1}{18}$
C. $\frac{1}{2}$
D. $\frac{1}{21}$
E. $\frac{1}{6}$

SECTION A - continued
TURN OVER

Question 12

The graph of a function f is shown below.



The unsigned area bound by the curve and the x-axis on the interval [-2, 3] is

A. $\int_{-2}^{3} f(x)\,dx$

B. $\int_{-2}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx$

C. $\int_{-2}^{\frac{1}{3}} f(x)\,dx + \int_{\frac{1}{3}}^{3} f(x)\,dx$

D. $\int_{\frac{1}{3}}^{3} f(x)\,dx - \int_{-2}^{\frac{1}{3}} f(x)\,dx$

E. $\int_{-2}^{\frac{1}{3}} f(x)\,dx + \int_{3}^{\frac{1}{3}} f(x)\,dx$

Question 13

A probability density function *f* is given by

$$f(x) = \begin{cases} k\sin(x) & 0 < x < \pi \\ 0 & elsewhere \end{cases}$$

The value of *k* is

A. $\frac{1}{2}$

B. $\frac{\pi}{2}$

C. $\frac{\pi-1}{2}$

D. 1

E. 2

SECTION A - continued

Question 14

The function $f: R \rightarrow R,\ f(x) = ax^2 - 3x + b$ has a local minimum at $= \frac{3}{4}$.

The values of a and b respectively could be

A. $-2, 4$
B. $2, 4$
C. $3, 6$
D. $-3, 6$
E. $1, 3$

Question 15

For random samples of four Australians, $\hat{P}$ is the random variable that represents the proportion who drive to work.

Given that $\Pr(\hat{P} = 0) = \frac{1}{625}$, then $\Pr(\hat{P} > 0.5)$, correct to four decimal places, is

A. 0.0256
B. 0.4096
C. 0.9728
D. 0.0272
E. 0.8192

Question 16

The equation $(2p - 1)x^2 - 2x = 1 - p$ has real roots when

A. $p \geq 0$
B. $-1 < p < 0$
C. $0 \leq p \leq \frac{3}{2}$
D. $p \geq \frac{3}{2}$
E. $-1 \leq p < 0$

SECTION A - continued
TURN OVER

Question 17

For a normal distribution $X \sim N(16, 4)$, $\Pr(X > a) = 0.8512$.

The value of a is closest to

A. 13.4
B. 13.5
C. 13.9
D. 18.1
E. 18.2

Question 18

For the function $f: [0, 6) \to R,\ f(x) = x^3 - 4x^2$, which of the following is **not** true?

A. f has a local minimum at $x = \frac{8}{3}$
B. f has an absolute maximum at $x = 6$
C. f does not have a point of inflection at $x = 0$
D. $f(x) < 0\ \ for\ \ 0 < x < 4$
E. $f'(x) > 0\ \ for\ \frac{8}{3} < x < 6$

Question 19

The simultaneous equations

$$mx + 4y = 0$$
$$2x - (2 - m)y = 0$$

where m is a real constant, have a unique solution if

A. $m \in \{0, 4\}$
B. $m \in R\backslash\{0\}$
C. $m \in R$
D. $m \in \{-2, 4\}$
E. $m \in R\backslash\{-2, 4\}$

Question 20

The average value of the function with rule $f(x) = x\sqrt{x}$ over the interval [0, 4] is

A. $\frac{16}{5}$
B. $\frac{64}{5}$
C. 2
D. $\frac{1}{2}$
E. $\frac{256}{4}$

END OF SECTION A

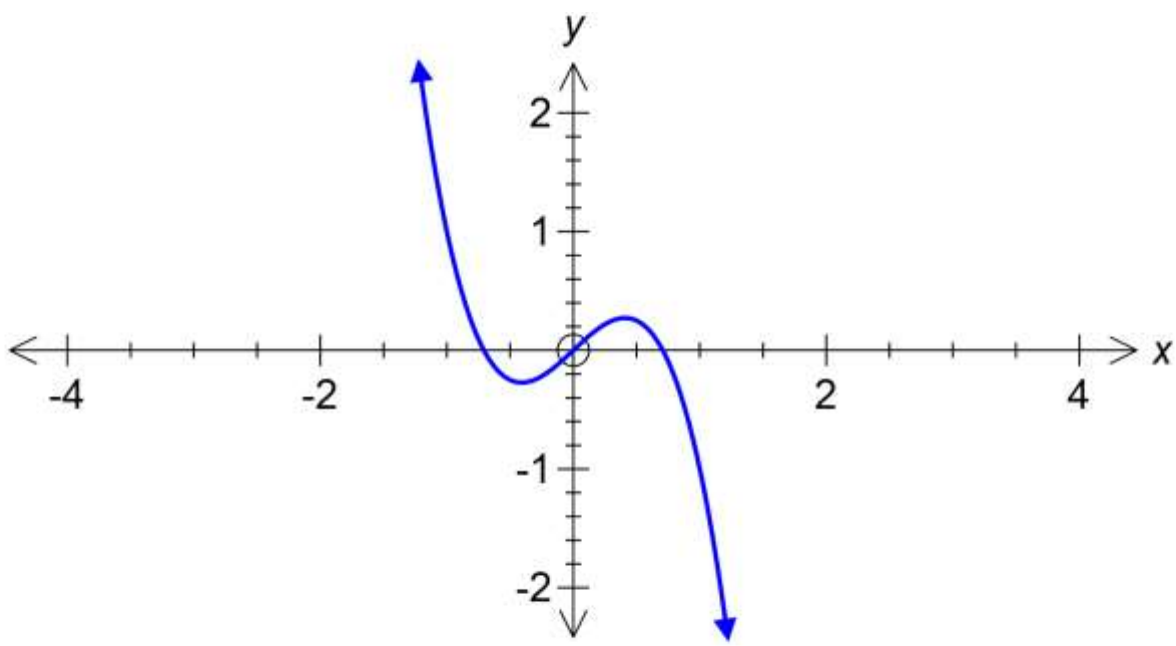
SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (14 marks)

Let $f: R \to R,\ f(x) = x - 2x^3$. Part of the graph of f is shown below.



a. Find the coordinates of the turning points. 3 marks

SECTION B – Question 1 - continued
TURN OVER

b. $A(-2, f(-2))$ and $B(2, f(2))$ are two points on the graph of f.

i. Find the equation of the straight line through A and B, in the form $ax + by = c$, where a, b and c are integers. 2 marks

ii. Find the distance AB. 1 mark

Let $g: R \to R,\ g(x) = x - kx^3,\ k \in R^+$.

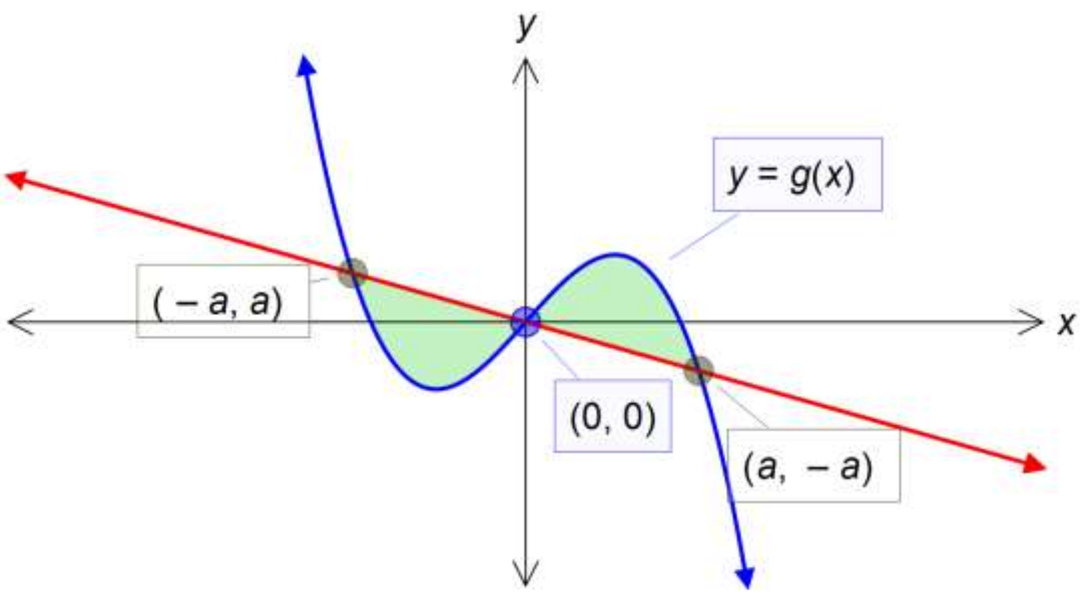
c. Let $C(-2, g(-2))$ and $D(2, g(2))$ be two points on the graph of g.

i. Find the equation of the straight line through C and D, in terms of k. 2 marks

SECTION B – Question 1 - continued

ii. The point (5, -1) lies on the straight line *CD*.
Find the value of *k*. 1 mark

d. The diagram below shows part of the graph of g, for $k > 0$. The line $y = -x$ intersects the graph of g at $(0, 0), (-a, a)$ and $(a, -a)$.



i. Find the value of *a* in terms of *k*. 2 marks

ii. Find the area of the shaded region in terms of *k*. 3 marks

SECTION B – continued
TURN OVER

Question 2 (16 marks)

The time Amaira takes to serve a customer at a shop varies from one customer to another. The continuous random variable *T*, which models the time, *t*, in minutes, that Amaira takes with each customer, has a probability density function, *f*, where

$$f(t) = \begin{cases} \frac{c}{2}t & 0 < t < 2 \\ \frac{c}{4}(10 - t) & 2 \le t < 6 \\ 0 & elsewhere \end{cases}$$

where c is a constant.

a. Show that the value of $c = \frac{1}{7}$. 3 marks

b. Find $\Pr(1 \le T \le 5)$. 2 marks

c. Find $\Pr(T \le 1 | T \le 5)$. 2 marks

SECTION B – Question 2 - continued

d. Find a such that $\Pr(T \leq a) = \frac{2}{7}$, correct to four decimal places. 3 marks

e. The probability that Amaira spends more than 3 minutes with a customer is $\frac{3}{7}$. Assume that the amount of time spent by Amaira with any customer is independent of the time spent on any other customer.

i. Find the probability that Amaira spends more than 3 minutes with at least four of the seven customers she serves on a particular day, correct to four decimal places. 2 marks

ii. Find the probability that Amaira spends more than 3 minutes with the first three customers on a particular day. 1 mark

SECTION B – Question 2 - continued
TURN OVER

f. The number of days in a month that Amaira does overtime at work approximately follows a normal distribution with a mean of 11 days and a standard deviation of b days.

The probability that Amaira works overtime at least 13 days in a particular month is 0.2312.

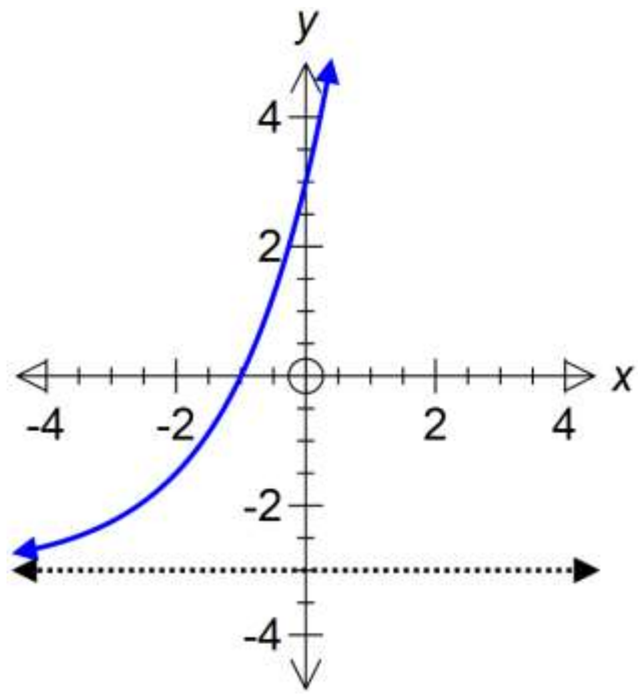
Find the value of b, correct to two decimal places.

3 marks

SECTION B – continued

Question 3 (17 marks)

Let $f: R \to R,\ f(x) = 3 \times 2^{x+1} - 3$. Part of the graph of *f* is shown below



a. The transformation $T: R^2 \to R^2,\ T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b \\ c \end{bmatrix}$ maps the graph of $y = 2^x$ onto the graph of *f*.

State the values of *a*, *b* and *c*. 3 marks

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b. Find the rule and domain for f^{-1}, the inverse of f. 2 marks

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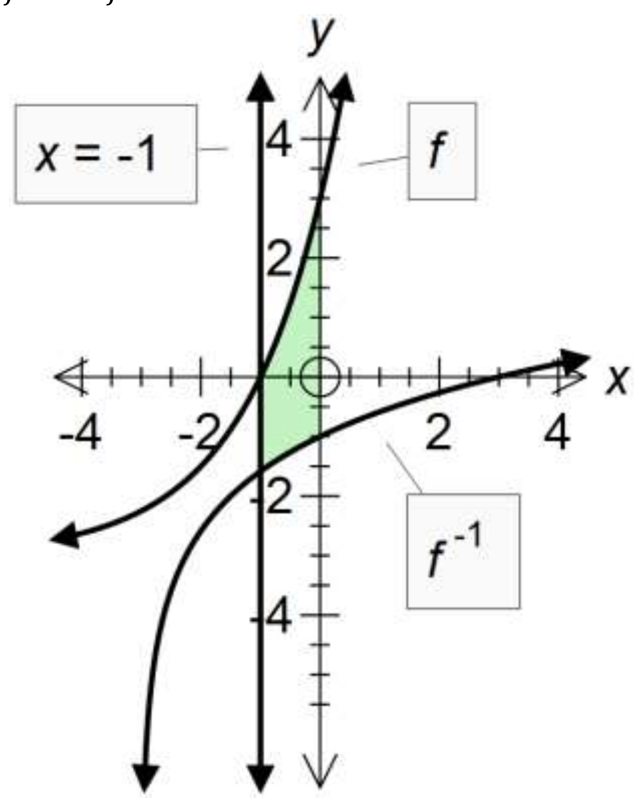
c. Parts of the graphs of f and f^{-1} are shown below.



Find the area bounded by the graphs of f and f^{-1} between $x = -1$ and $x = 0$, correct to two decimal places. 2 marks

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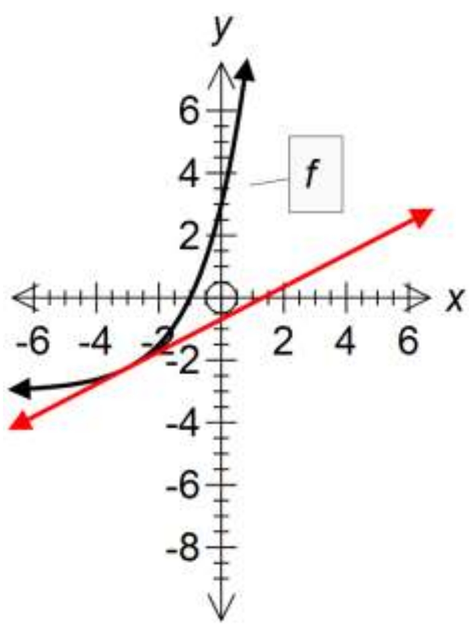
SECTION B – Question 3 - continued

d. Let $: R\backslash\{0\} \to R,\ g(x) = \frac{2}{x}$.

i. Explain why $g\big(f(x)\big)$ is not defined. 2 marks

ii. Find $f\big(g(x)\big)$ and state its domain. 2 marks

e. Part of the graph of f is shown below. The tangent to the graph of f at $(-3, f(-3))$ is also shown below.



i. Find the equation of the tangent shown above, with coefficients correct to two decimal places. 2 marks

SECTION B – Question 3 - continued
TURN OVER

ii. The tangent line shown above makes an angle of θ with the positive direction of x-axis. Find the value of θ, correct to one decimal place. 1 mark

iii. Another tangent line is drawn to the graph of f at the point $(k, f(k))$. Both tangent lines intersect at the point (0.57, -0.39).

Find the value of k. 3 marks

Question 4 (13 marks)

The height, h, of a point P above the ground on the Ferris wheel is modelled by

$h(t) = 45 - 30\cos\left(\frac{\pi t}{3}\right)$ where t is the time in minutes after the ride begins and h is measured in minutes.

a. State the maximum and minimum heights of P above the ground. 2 marks

SECTION B – Question 4 - continued

b. The ride finishes after two rotations of the Ferris wheel.

i. How long does the ride go for? 2 marks

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ii. After how many minutes is the height at its maximum point in the ride? 2 marks

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c.

i. Find the rate of change of h with respect to t. 1 mark

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ii. Hence, state the value(s) of t at which the rate of change of h is at its maximum. 2 marks

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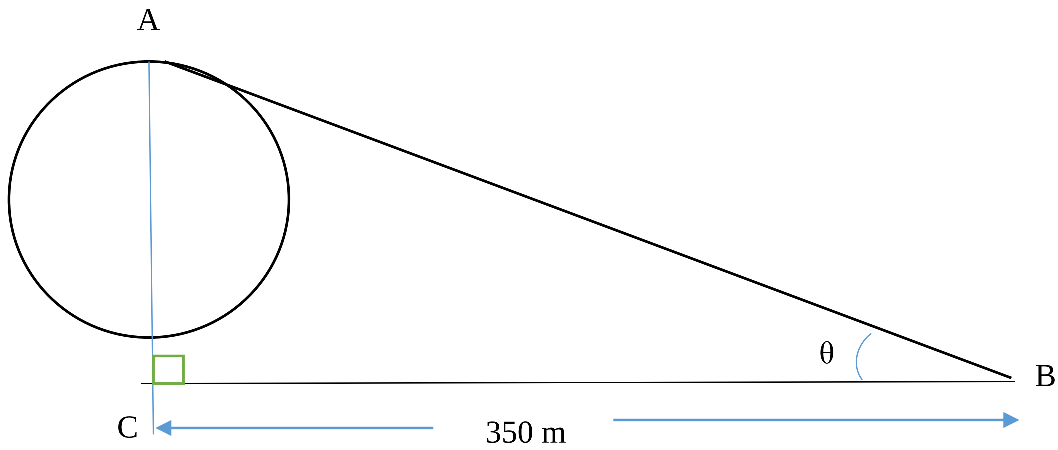
SECTION B – Question 4 - continued
TURN OVER

Sam is on the first round of his ride and when he is at A, the highest point on the Ferris wheel, he notices a boat at point *B*, as shown below.



d. Find θ in degrees, correct to two decimal places. 1 mark

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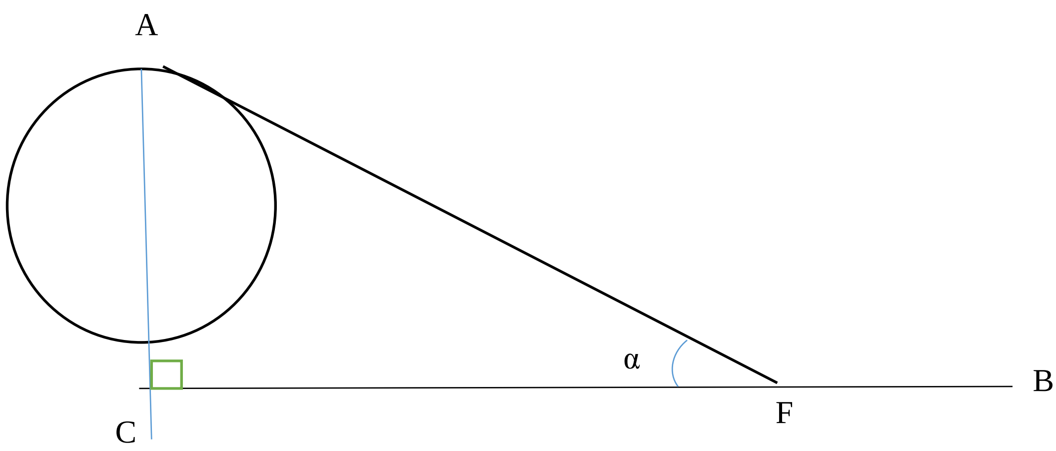
In the second round of the ride, Sam notices the boat at point F as shown below



SECTION B – Question 4 - continued

e. Find the distance CF in terms of α. 1 mark

f. If $\alpha - \theta = 14°$, find the distance CF, correct to two decimal places. 2 marks

END OF QUESTION AND ANSWER BOOK