MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

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2018 Trial Examination

SOLUTIONS

Question 1

a. $f'(x) = \frac{(3+x)\times 2 - 2x\times 1}{(3+x)^2} = \frac{6}{(3+x)^2}$

2 marks

b. $g'(x) = 2(1-x^4)^1 \times -4x^3 = -8x^3(1-x^4)$
$g'(1) = 0$

2 marks

Question 2

a. $\frac{dy}{dx} = \cos(x) - x\sin(x)$

2 marks

b. $\int_0^{\pi}(x\sin(x)+1)dx = \int_0^{\pi}(\cos(x) - y + 1)\,dx = [\sin(x) - x\cos(x) + x]_0^{\pi}$
$(\sin(\pi) - \pi\cos(\pi) + \pi) - 0 = 2\pi$

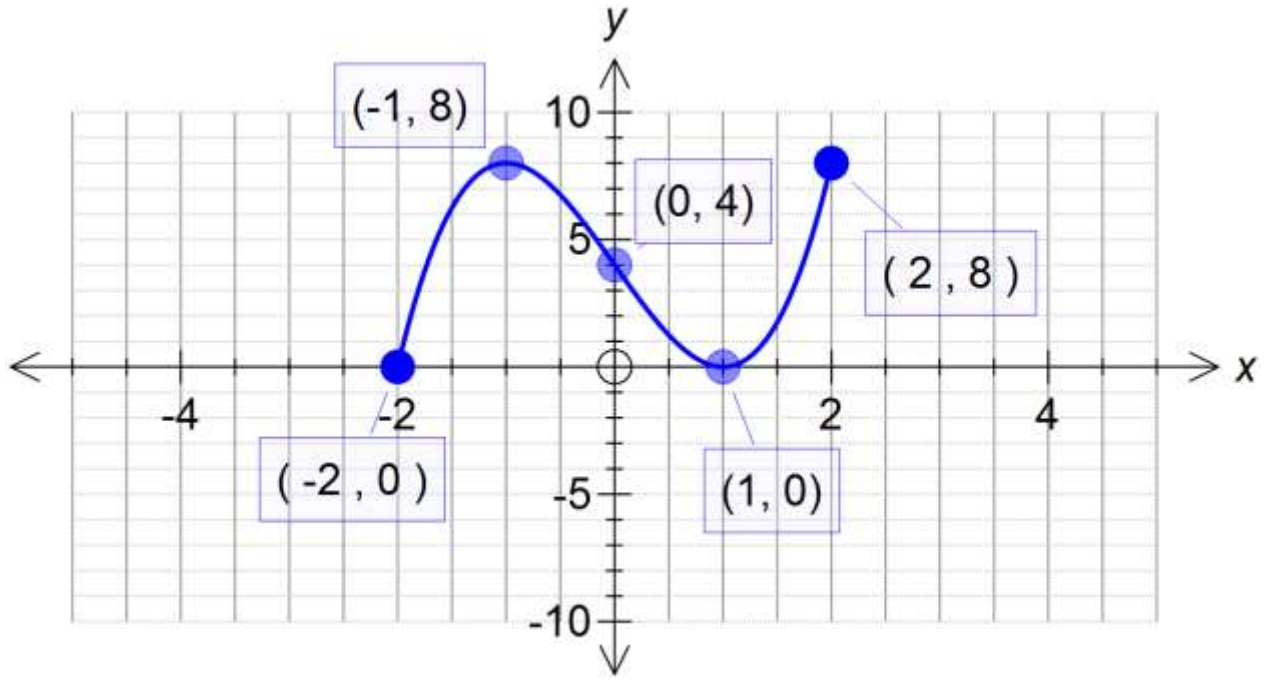
2 marks

Question 3

a. $f(x) = 2(x + 2 - 2x^2 - 4x + x^3 + 2x^2) = 2(x^3 - 3x + 2) = 2x^3 - 6x + 4$

1 mark

b.



For stationary points $f'(x) = 0 \ \ gives\ x = -1\ and\ 1$

1 mark for axis intercepts, 1 mark for stationary points and 1 mark for end points

Question 4

$\sqrt{\frac{\frac{1}{3}\times\frac{2}{3}}{n}} \leq \frac{1}{72} \rightarrow \sqrt{\frac{2}{9n}} \leq \frac{1}{72} \rightarrow \sqrt{\frac{2}{n}} \leq \frac{1}{24} \rightarrow \sqrt{\frac{n}{2}} \geq 24 \rightarrow n \geq 1152$

$n = 1152$

2 marks

Question 5

a. $\frac{1}{4}$

1 mark

b. $\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$

2 marks

c. $\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4} = \frac{1}{4}+\frac{3}{16}+\frac{9}{64} = \frac{37}{64}$ **OR** $1-\frac{27}{64} = \frac{37}{64}$

2 marks

Question 6

a. $1,\ -\frac{1}{2},\ 0$

1 mark

b. $\sin(\theta) = 1 \rightarrow \theta = \frac{\pi}{2}$
$\sin(\theta) = -\frac{1}{2} \rightarrow$ no solution in the domain
$\sin(\theta) = 0 \rightarrow \theta = 0,\ \pi$
$\theta = 0,\ \frac{\pi}{2},\ \pi$

3 marks

Question 7

a. $(-\infty, \ln(2)]$

1 mark

b.

i. Range of g must be a subset of Domain of f
Range of g: $(0, \sqrt{3-b})$, Domain of f: $(0, 1]$
$\sqrt{3-b} = 1 \rightarrow b = 2$

2 marks

ii. $f\big(g(x)\big) = f\big(\sqrt{3-x}\big) = \log_e\big(2\sqrt{3-x}\big)$

1 mark

Question 8

a. $\Pr(B|A) = \frac{\frac{1}{6}}{p} = \frac{1}{6p}$

1 mark

b. $\Pr(B'|A') = \frac{\Pr(B' \cap A')}{\Pr(A')} = \frac{1-\Pr(A \cup B)}{1-\Pr(A)} = \frac{1-\left(p+\frac{1}{3}-\frac{1}{6}\right)}{1-p} = \frac{\frac{5}{6}-p}{1-p}$

2 marks

c. $\frac{\frac{5}{6}-p}{1-p} = \frac{1}{2} \rightarrow \frac{5}{3} - 2p = 1 - p \rightarrow p = \frac{2}{3}$

1 mark

Question 9

a. Area $= \int_0^1 \left(x^{\frac{1}{2}} - x^{\frac{9}{2}}\right)\, dx = \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{11}x^{\frac{11}{2}}\right)_0^1 = \frac{2}{3} - \frac{2}{11} = \frac{16}{33}$

2 marks

b. $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{9}{2}x^{\frac{7}{2}} = \frac{1}{2\sqrt{x}} - \frac{9x^{\frac{7}{2}}}{2} = \frac{1}{2\sqrt{x}} - \frac{9x^4}{2\sqrt{x}} = \frac{1-9x^4}{2\sqrt{x}}$

1 mark

c.

i. $\frac{1-9x^4}{2\sqrt{x}} = -4 \rightarrow 1 - 9x^4 = -8\sqrt{x}$

$x = 1$ satisfies the above equation

2 marks

ii. $y = -4x + c$

$Substitute\ (1, 0)\ \ to\ get\ c = 4$

Equation of tangent is $y = -4x + 4$

2 marks

d. $y = 2(-4x + 4) + 1\ or\ \ y = -8x + 9$

2 marks