

Year 2018

VCE

Mathematical Methods

Trial Examination 1

Solutions



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9018 5376
FAX: (03) 9817 4334
kilbaha@gmail.com
<http://kilbaha.com.au>

IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Multimedia Publishing.
- The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
- For authorised copying within Australia please check that your institution has a licence from Copyright Agency Limited. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.

Reproduction and communication for educational purposes The Australian Copyright Act 1968 (the Act) allows a maximum of one chapter or 10% of the pages of this work, to be reproduced and/or communicated by any educational institution for its educational purposes provided that educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact
CAL, Level 15, 233 Castlereagh Street, Sydney, NSW, 2000

Tel: (02) 9394 7600

Fax: (02) 9394 7601

Email: info@copyright.com.au

Web: <http://www.copyright.com.au>

- While every care has been taken, no guarantee is given that these answers are free from error. Please contact us if you believe you have found an error.

Question 1

a. $y = \frac{\cos(2x)}{2x}$ using the quotient rule

$u = \cos(2x) \quad v = 2x$

$\frac{du}{dx} = -2\sin(2x) \quad \frac{dv}{dx} = 2$ M1

$\frac{dy}{dx} = \frac{-4x\sin(2x) - 2\cos(2x)}{4x^2}$ A1

b. Let $y = e^{\sqrt{x}}$

$y = e^u \quad u = \sqrt{x} = x^{\frac{1}{2}} \quad \text{chain rule}$

$\frac{dy}{du} = e^u \quad \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = g'(x) = \frac{1}{2\sqrt{x}}e^{\sqrt{x}}$

M1

$g'(4) = \frac{1}{2\sqrt{4}}e^{\sqrt{4}}$

$g'(4) = \frac{e^2}{4}$

A1

Question 2

$X \sim Bi(n=4, p="p")$

$\Pr(X=1) = \binom{4}{1}p(1-p)^3 = 2\Pr(X=2) = 2\binom{4}{2}p^2(1-p)^2$ A1

$4p(1-p)^3 = 2 \times 6p^2(1-p)^2$

$4p(1-p)^3 - 12p^2(1-p)^2 = 0$

$4p(1-p)^2(1-p-3p) = 0$

$4p = 1 \quad \text{since } 0 < p < 1$ M1

$p = \frac{1}{4}$ A1

Question 3

a. $y = \sqrt{16 - x^2} = u^{\frac{1}{2}}$, $u = 16 - x^2$ chain rule

$$\frac{dy}{du} = \frac{1}{2} u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}, \quad \frac{du}{dx} = -2x$$

A1

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{-2x}{2\sqrt{u}} = \frac{-x}{\sqrt{16 - x^2}}$$

at $x = p$ $m_T = \left. \frac{dy}{dx} \right|_{x=p} = \frac{-p}{\sqrt{16 - p^2}} = \tan(60^\circ) = \sqrt{3}$

M1

$$-p = \sqrt{3} \times \sqrt{16 - p^2} \quad \text{so } p < 0$$

$$p^2 = 3(16 - p^2) = 48 - 3p^2$$

$$4p^2 = 48$$

$$p^2 = 12$$

$$p = -2\sqrt{3}$$

A1

b.i. $A = 2ab$ however $b = \sqrt{16 - a^2}$

A1

$$A = 2a\sqrt{16 - a^2}$$

ii. Using the product rule and **a.**

$$\frac{dA}{da} = \frac{d}{da}(2a)\sqrt{16 - a^2} + 2a \frac{d}{da}(\sqrt{16 - a^2})$$

$$= 2\sqrt{16 - a^2} - \frac{2a^2}{\sqrt{16 - a^2}} = 0 \quad \text{for maximum}$$

M1

$$2\sqrt{16 - a^2} = \frac{2a^2}{\sqrt{16 - a^2}}$$

$$16 - a^2 = a^2$$

$$2a^2 = 16$$

$$a^2 = 8 \quad \text{since } a > 0$$

$$a = \sqrt{8} = 2\sqrt{2}$$

A1

Question 4

a. $\log_8(x+5) + \log_8(3x-1) = 2$

$$\log_8(x+5)(3x-1) = 2$$

$$(x+5)(3x-1) = 8^2 = 64$$

M1

$$3x^2 + 14x - 5 = 64$$

$$3x^2 + 14x - 69 = 0$$

$$(3x+23)(x-3) = 0$$

$$x = -\frac{23}{3}, 3 \text{ but } x > \frac{1}{3}$$

$$x = 3 \text{ as the only answer}$$

A1

b. Let $u = 8^x$, $64^x = (8^2)^x = 8^{2x} = (8^x)^2 = u^2$

$$32 \times 64^x - 12 \times 8^x + 1 = 0$$

$$32u^2 - 12u + 1 = 0$$

$$(8u-1)(4u-1) = 0$$

M1

$$u = 8^x = \frac{1}{8}, \frac{1}{4}$$

$$x = -1, -\frac{2}{3}$$

A1

Question 5

a.i number of red, 0, 1, 2, 3, in a total of 10, so $\hat{P} = \frac{x}{10}$

$$\hat{P} = \left\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}\right\}$$

A1

ii. $\Pr\left(\hat{P} = \frac{1}{5}\right) = \Pr(2R) = RRO + ROR + ORR = 3 \times \frac{3}{10} \times \frac{2}{9} \times \frac{7}{8}$

$$\Pr\left(\hat{P} = \frac{1}{5}\right) = \frac{7}{40}$$

A1

b. $n = 300$, $p = \frac{1}{4}$

$$\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{\frac{1}{4} \times \frac{3}{4}}{300}} = \frac{1}{40}$$

A1

$$95\% \quad \hat{p} \pm 2\text{sd}(\hat{p}) = \frac{1}{4} \pm 2 \times \frac{1}{40} = \frac{5}{20} \pm \frac{1}{20}$$

$$\left(\frac{1}{5}, \frac{3}{10}\right)$$

A1

Question 6

a. completing the square

$$g(x) = 3 + 4x - x^2 = -(x^2 - 4x + 4) + 3 + 4 = 7 - (x - 2)^2$$

range $g = (-\infty, 7]$ A1

$f(x) = \log_e(x + 2)$ domain $x + 2 > 0 \Rightarrow x > -2$, range R

	$f(x)$	$g(x)$
domain	$(-2, \infty)$	R
range	R	$(-\infty, 7]$

Since range $g \not\subset$ domain f , so $f(g(x))$ does not exist. A1

b. solving $g(x) = 3 + 4x - x^2 = -2$

$$\Rightarrow x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0$$
 M1

$$\Rightarrow x = -1, 5$$

now $g(-1) = g(5) = -2$, $g(2) = 7$, so if we now restrict the domain of g , as

domain $g(x) = D = (-1, 5) = \text{domain } f(g(x))$ now the range of $g = (-2, 7)$

so range $g \subset$ domain f , so now $f(g(x))$ exist. A1

$$f(g(x)) = f(3 + 4x - x^2) = \log_e(3 + 4x - x^2 + 2)$$

$$f(g(x)) = \log_e(5 + 4x - x^2) = \log_e((5 - x)(x + 1))$$

$$f(g(x)): (-1, 5) \rightarrow R, f(g(x)) = \log_e(5 + 4x - x^2) = \log_e((5 - x)(x + 1))$$
 A1

Question 7

a. Since it is discrete probability distribution $\sum \Pr(X = x) = 1$

$$2\cos^2(k) + \cos(k) = 1$$

$$2\cos^2(k) + \cos(k) - 1 = 0$$

$$(2\cos(k) - 1)(\cos(k) + 1) = 0$$
 M1

$$\cos(k) = \frac{1}{2}, \quad \cos(k) = -1$$

$$k = \frac{\pi}{3}, 2\pi - \frac{\pi}{3} \quad k = \pi \quad \text{since } 0 \leq k \leq 2\pi$$

but $k = \pi$ is not valid as $\cos(\pi) = -1$ and each probability must be positive.

$$k = \frac{\pi}{3}, \frac{5\pi}{3} \text{ are the only answers in } 0 \leq k \leq 2\pi$$
 A1

b. $E(X) = \sum x \Pr(X = x)$

$$E(X) = 2\cos^2(k) + 2\cos(k) = (2\cos^2(k) + \cos(k)) + \cos(k) = 1 + \frac{1}{2} = \frac{3}{2}$$

$$E(X^2) = \sum x^2 \Pr(X = x)$$

$$E(X^2) = 2\cos^2(k) + 4\cos(k) = (2\cos^2(k) + \cos(k)) + 3\cos(k) = 1 + \frac{3}{2} = \frac{5}{2}$$

$$\text{var}(X) = E(X^2) - (E(X))^2 = \frac{5}{2} - \left(\frac{3}{2}\right)^2 = \frac{5}{2} - \frac{9}{4}$$

$$\text{var}(X) = \frac{1}{4}$$

A1

Question 8

a. The sine wave part has a length of π , and is one quarter of a cycle,

$$\text{therefore one cycle } \frac{2\pi}{n} = 4\pi \quad n = \frac{1}{2}$$

A1

b. Since the function is continuous at $x = \pi$, $k\pi = a \sin\left(\frac{\pi}{2}\right) \Rightarrow a = k\pi$

A1

Since the total area under the curve is one.

$$\int_0^{\pi} kx \, dx + \int_{\pi}^{2\pi} a \sin\left(\frac{x}{2}\right) dx = 1$$

$$\left[\frac{1}{2}kx^2\right]_0^{\pi} + \left[-2a \cos\left(\frac{x}{2}\right)\right]_{\pi}^{2\pi} = 1$$

M1

$$\frac{1}{2}k\pi^2 - 2a \cos(\pi) + 2a \cos\left(\frac{\pi}{2}\right) = 1$$

$$\frac{1}{2}k\pi^2 + 2a = 1 \quad \text{substitute } a = k\pi, \text{ solve for } k$$

M1

$$\frac{1}{2}k\pi^2 + 2k\pi = 1$$

$$k\pi^2 + 4k\pi = 2$$

$$k\pi(\pi + 4) = 2$$

$$k = \frac{2}{\pi(\pi + 4)}, \quad a = \frac{2}{\pi + 4}$$

A1

Question 9

- a. Let $y = x^2 \log_e(2x)$ using the product rule

$$\begin{aligned}\frac{dy}{dx} &= x^2 \frac{d}{dx}[\log_e(2x)] + \log_e(2x) \frac{d}{dx}(x^2) \\ &= x^2 \times \frac{1}{x} + 2x \log_e(2x)\end{aligned}$$

$$\frac{d}{dx}[x^2 \log_e(2x)] = x + 2x \log_e(2x) \quad \text{A1}$$

- b. $f(x) = x \log_e(2x)$ using the product rule

$$\begin{aligned}f'(x) &= x \frac{d}{dx}[\log_e(2x)] + \log_e(2x) \frac{d}{dx}(x) \\ &= x \times \frac{1}{x} + \log_e(2x) \\ &= 1 + \log_e(2x)\end{aligned}$$

$$\text{for turning points } f'(x) = 0 \Rightarrow 1 + \log_e(2x) = 0 \quad \text{M1}$$

$$\log_e(2x) = -1 \quad 2x = e^{-1} \quad x = \frac{1}{2e} \quad f\left(\frac{1}{2e}\right) = \frac{1}{2e} \log_e\left(\frac{1}{e}\right) = -\frac{1}{2e}$$

$$\text{the minimum turning point is } \left(\frac{1}{2e}, -\frac{1}{2e}\right) \quad \text{A1}$$

- c. $A = \int_1^2 x \log_e(2x) dx$

$$\text{from a. } \frac{d}{dx}[x^2 \log_e(2x)] = x + 2x \log_e(2x)$$

$$\int (x + 2x \log_e(2x)) dx = \int x dx + 2 \int x \log_e(2x) dx = x^2 \log_e(2x)$$

$$2 \int x \log_e(2x) dx = x^2 \log_e(2x) - \int x dx = x^2 \log_e(2x) - \frac{1}{2} x^2 \quad \text{M1}$$

$$\int x \log_e(2x) dx = \frac{x^2}{4} (2 \log_e(2x) - 1)$$

$$A = \left[\frac{x^2}{4} (2 \log_e(2x) - 1) \right]_1^2 \quad \text{A1}$$

$$A = (2 \log_e(4) - 1) - \left(\frac{1}{4} (2 \log_e(2) - 1) \right) = 4 \log_e(2) - \frac{1}{2} \log_e(2) - \frac{3}{4}$$

$$A = \frac{7}{2} \log_e(2) - \frac{3}{4} \quad \text{A1}$$

END OF SUGGESTED SOLUTIONS