Year 2018

VCE

Mathematical Methods

Trial Examination 1

Solutions

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Question 1

a. $y = \dfrac{\cos(2x)}{2x}$ using the quotient rule

$u = \cos(2x) \qquad v = 2x$

$\dfrac{du}{dx} = -2\sin(2x) \qquad \dfrac{dv}{dx} = 2$ M1

$\dfrac{dy}{dx} = \dfrac{-4x\sin(2x) - 2\cos(2x)}{4x^2}$ A1

b. Let $y = e^{\sqrt{x}}$

$y = e^u \qquad u = \sqrt{x} = x^{\frac{1}{2}}$ chain rule

$\dfrac{dy}{du} = e^u \qquad \dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$

$\dfrac{dy}{dx} = g'(x) = \dfrac{1}{2\sqrt{x}}e^{\sqrt{x}}$

M1

$g'(4) = \dfrac{1}{2\sqrt{4}}e^{\sqrt{4}}$

$g'(4) = \dfrac{e^2}{4}$ A1

Question 2

$X \sim Bi(n = 4, p = "p")$

$\Pr(X = 1) = \binom{4}{1}p(1-p)^3 = 2\Pr(X = 2) = 2\binom{4}{2}p^2(1-p)^2$ A1

$4p(1-p)^3 = 2 \times 6p^2(1-p)^2$

$4p(1-p)^3 - 12p^2(1-p)^2 = 0$

$4p(1-p)^2(1-p-3p) = 0$

$4p = 1$ since $0 < p < 1$ M1

$p = \dfrac{1}{4}$ A1

Question 3

a. $y=\sqrt{16-x^2}=u^{\frac{1}{2}}$, $u=16-x^2$ chain rule

$\frac{dy}{du}=\frac{1}{2}u^{-\frac{1}{2}}=\frac{1}{2\sqrt{u}}$, $\frac{du}{dx}=-2x$ A1

$\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}=\frac{-2x}{2\sqrt{u}}=\frac{-x}{\sqrt{16-x^2}}$

at $x=p$ $m_T=\left.\frac{dy}{dx}\right|_{x=p}=\frac{-p}{\sqrt{16-p^2}}=\tan\left(60^0\right)=\sqrt{3}$ M1

$-p=\sqrt{3}\times\sqrt{16-p^2}$ so $p<0$

$p^2=3\left(16-p^2\right)=48-3p^2$

$4p^2=48$

$p^2=12$

$p=-2\sqrt{3}$ A1

b.i. $A=2ab$ however $b=\sqrt{16-a^2}$ A1

$A=2a\sqrt{16-a^2}$

ii. Using the product rule and **a.**

$\frac{dA}{da}=\frac{d}{da}(2a)\sqrt{16-a^2}+2a\frac{d}{da}\left(\sqrt{16-a^2}\right)$

$=2\sqrt{16-a^2}-\frac{2a^2}{\sqrt{16-a^2}}=0$ for maximum M1

$2\sqrt{16-a^2}=\frac{2a^2}{\sqrt{16-a^2}}$

$16-a^2=a^2$

$2a^2=16$

$a^2=8$ since $a>0$

$a=\sqrt{8}=2\sqrt{2}$ A1

Question 4

a. $\log_8(x+5)+\log_8(3x-1)=2$

$\log_8(x+5)(3x-1)=2$

$(x+5)(3x-1)=8^2=64$ M1

$3x^2+14x-5=64$

$3x^2+14x-69=0$

$(3x+23)(x-3)=0$

$x=-\frac{23}{3}$, 3 but $x>\frac{1}{3}$

$x=3$ as the only answer A1

b. Let $u=8^x$, $64^x=(8^2)^x=8^{2x}=(8^x)^2=u^2$

$32\times64^x-12\times8^x+1=0$

$32u^2-12u+1=0$

$(8u-1)(4u-1)=0$ M1

$u=8^x=\frac{1}{8}, \frac{1}{4}$

$x=-1, -\frac{2}{3}$ A1

Question 5

a.i number of red, 0, 1, 2, 3, in a total of 10, so $\hat{P}=\frac{x}{10}$

$\hat{P}=\{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}\}$ A1

ii. $\Pr\left(\hat{P}=\frac{1}{5}\right)=\Pr(2R)=RRO+ROR+ORR=3\times\frac{3}{10}\times\frac{2}{9}\times\frac{7}{8}$

$\Pr\left(\hat{P}=\frac{1}{5}\right)=\frac{7}{40}$ A1

b. $n=300$, $p=\frac{1}{4}$

$\text{sd}(\hat{P})=\sqrt{\frac{p(1-p)}{n}}=\sqrt{\frac{\frac{1}{4}\times\frac{3}{4}}{300}}=\frac{1}{40}$ A1

95% $\hat{p}\pm2\text{sd}(\hat{p})=\frac{1}{4}\pm2\times\frac{1}{40}=\frac{5}{20}\pm\frac{1}{20}$

$\left(\frac{1}{5}, \frac{3}{10}\right)$ A1

Question 6

a. completing the square

$g(x)=3+4x-x^2=-(x^2-4x+4)+3+4=7-(x-2)^2$

range $g=(-\infty,7]$ A1

$f(x)=\log_e(x+2)$ domain $x+2>0 \Rightarrow x>-2$, range R

	$f(x)$	$g(x)$
domain	$(-2,\infty)$	R
range	R	$(-\infty,7]$

Since range $g \not\subset$ domain f, so $f(g(x))$ does not exist. A1

b. solving $g(x)=3+4x-x^2=-2$

$\Rightarrow \quad x^2-4x-5=0$

$(x-5)(x+1)=0$ M1

$\Rightarrow \quad x=-1,5$

now $g(-1)=g(5)=-2$, $g(2)=7$, so if we now restrict the domain of g, as

domain $g(x)=D=(-1,5)=$ domain $f(g(x))$ now the range of $g=(-2,7)$

so range $g \subset$ domain f, so now $f(g(x))$ exist. A1

$f(g(x))=f(3+4x-x^2)=\log_e(3+4x-x^2+2)$

$f(g(x))=\log_e(5+4x-x^2)=\log_e((5-x)(x+1))$

$f(g(x)):(-1,5)\to R$, $=f(g(x))=\log_e(5+4x-x^2)=\log_e((5-x)(x+1))$ A1

Question 7

a. Since it is discrete probability distribution $\sum \Pr(X=x)=1$

$2\cos^2(k)+\cos(k)=1$

$2\cos^2(k)+\cos(k)-1=0$

$(2\cos(k)-1)(\cos(k)+1)=0$ M1

$\cos(k)=\dfrac{1}{2}$, $\quad \cos(k)=-1$

$k=\dfrac{\pi}{3},2\pi-\dfrac{\pi}{3} \qquad k=\pi$ since $0\le k\le 2\pi$

but $k=\pi$ is not valid as $\cos(\pi)=-1$ and each probability must be positive.

$k=\dfrac{\pi}{3}$, $\dfrac{5\pi}{3}$ are the only answers in $0\le k\le 2\pi$ A1

b. $E(X)=\sum x\,Pr(X=x)$

$E(X)=2\cos^2(k)+2\cos(k)=\left(2\cos^2(k)+\cos(k)\right)+\cos(k)=1+\frac{1}{2}=\frac{3}{2}$

$E\left(X^2\right)=\sum x^2\,Pr(X=x)$

$E\left(X^2\right)=2\cos^2(k)+4\cos(k)=\left(2\cos^2(k)+\cos(k)\right)+3\cos(k)=1+\frac{3}{2}=\frac{5}{2}$

$\text{var}(X)=E\left(X^2\right)-\left(E(X)\right)^2=\frac{5}{2}-\left(\frac{3}{2}\right)^2=\frac{5}{2}-\frac{9}{4}$

$\text{var}(X)=\frac{1}{4}$ A1

Question 8

a. The sine wave part has a length of π, and is one quarter of a cycle,

therefore one cycle $\frac{2\pi}{n}=4\pi \quad n=\frac{1}{2}$ A1

b. Since the function is continuous at $x=\pi$, $k\pi=a\sin\left(\frac{\pi}{2}\right) \Rightarrow a=k\pi$ A1

Since the total area under the curve is one.

$\int_0^{\pi} kx\,dx+\int_{\pi}^{2\pi} a\sin\left(\frac{x}{2}\right)dx=1$

$\left[\frac{1}{2}kx^2\right]_0^{\pi}+\left[-2a\cos\left(\frac{x}{2}\right)\right]_{\pi}^{2\pi}=1$ M1

$\frac{1}{2}k\pi^2-2a\cos(\pi)+2a\cos\left(\frac{\pi}{2}\right)=1$

$\frac{1}{2}k\pi^2+2a=1$ substitute $a=k\pi$, solve for k M1

$\frac{1}{2}k\pi^2+2k\pi=1$

$k\pi^2+4k\pi=2$

$k\pi(\pi+4)=2$

$k=\frac{2}{\pi(\pi+4)}$, $a=\frac{2}{\pi+4}$ A1

Question 9

a. Let $y = x^2 \log_e(2x)$ using the product rule

$$\frac{dy}{dx} = x^2 \frac{d}{dx}\left[\log_e(2x)\right] + \log_e(2x)\frac{d}{dx}\left(x^2\right)$$

$$= x^2 \times \frac{1}{x} + 2x\log_e(2x)$$

$$\frac{d}{dx}\left[x^2 \log_e(2x)\right] = x + 2x\log_e(2x)$$ A1

b. $f(x) = x\log_e(2x)$ using the product rule

$$f'(x) = x\frac{d}{dx}\left[\log_e(2x)\right] + \log_e(2x)\frac{d}{dx}(x)$$

$$= x \times \frac{1}{x} + \log_e(2x)$$

$$= 1 + \log_e(2x)$$

for turning points $f'(x) = 0 \quad \Rightarrow \quad 1 + \log_e(2x) = 0$ M1

$\log_e(2x) = -1 \quad 2x = e^{-1} \quad x = \frac{1}{2e} \quad f\left(\frac{1}{2e}\right) = \frac{1}{2e}\log_e\left(\frac{1}{e}\right) = -\frac{1}{2e}$

the minimum turning point is $\left(\frac{1}{2e}, -\frac{1}{2e}\right)$ A1

c. $A = \int_1^2 x\log_e(2x)\,dx$

from **a.** $\frac{d}{dx}\left[x^2 \log_e(2x)\right] = x + 2x\log_e(2x)$

$$\int\left(x + 2x\log_e(2x)\right)dx = \int x\,dx + 2\int x\log_e(2x)\,dx = x^2\log_e(2x)$$

$$2\int x\log_e(2x)\,dx = x^2\log_e(2x) - \int x\,dx = x^2\log_e(2x) - \frac{1}{2}x^2$$ M1

$$\int x\log_e(2x)\,dx = \frac{x^2}{4}\left(2\log_e(2x) - 1\right)$$

$$A = \left[\frac{x^2}{4}\left(2\log_e(2x) - 1\right)\right]_1^2$$ A1

$$A = \left(2\log_e(4) - 1\right) - \left(\frac{1}{4}\left(2\log_e(2) - 1\right)\right) = 4\log_e(2) - \frac{1}{2}\log_e(2) - \frac{3}{4}$$

$$A = \frac{7}{2}\log_e(2) - \frac{3}{4}$$ A1

END OF SUGGESTED SOLUTIONS