Online & home tutors Registered business name: itute ABN: 96 297 924 083

2018

Mathematical Methods

Trial Examination 2
(2 hours)

SECTION A Multiple-choice questions

Instructions for Section A

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this examination are **not** drawn to scale.

Question 1 Which one of the following is the correct solution set when $x(x+1)=(x+1)$ is solved for x?

A. $\{0,-1\}$

B. $\{-1,1\}$

C. $\{0,1\}$

D. $\{1\}$

E. $\{-1\}$

Question 2 Which one of the following is **not** a correct solution when $(\sin x)(\log_e x^2)=0$ is solved for x?

A. π

B. 1

C. 0

D. -1

E. $-\pi$

Question 3 The domain of the inverse of $f:(0,4)\to R,\ f(x)=(x-1)(x-3)+1$ is

A. $[0,4)$

B. $(0,4)$

C. $[0,2]$

D. $(-4,0]$

E. $\{4\}$

Question 4 The equation of the **inverse** of $y = e^{(\log_e x - \log_e 2)}$ is

A. $y = 2x$

B. $y = \frac{x}{2}$

C. $y = \frac{2}{x}$

D. $y = x - 2$

E. $y = x + 2$

Question 5 The graphs of $y = mx + 2$ and $x^2 + y^2 = 1$ intersect at exactly one point if

A. $m = -\sqrt{3}$ only

B. $m = -\frac{1}{\sqrt{3}}$ or $\sqrt{3}$

C. $m = -\sqrt{3}$ or $\sqrt{3}$

D. $m = -\sqrt{3}$ or $\frac{1}{\sqrt{3}}$

E. $m = \sqrt{3}$ only

Question 6 If $f(x) - f(-x) = 0$ for all $x \in R$, $f(x)$ **cannot** be

A. $\sin\left(x - \frac{\pi}{2}\right)$

B. $\cos(x - \pi)$

C. $\tan\left(x + \frac{\pi}{2}\right)$

D. $\cos(x + \pi)$

E. $\sin\left(\frac{3\pi}{2} + x\right)$

Question 7 The possible number of stationary points that real function $f(x)=(x^2-a)(x^2+b)$ can have if $a \ge b > 0$ is

A. 0 or 1 only

B. 1 or 2 only

C. 2 or 3 only

D. 1 or 3 only

E. 1, 2 or 3

Question 8 The solution to the **two** simultaneous equations $y = x+1$ and $y=\begin{cases} 2 & \text{for} \quad 0 \le x < 2 \\ 4 & \text{for} \quad x \ge 2 \end{cases}$ is

A. $(1, 2)$ only

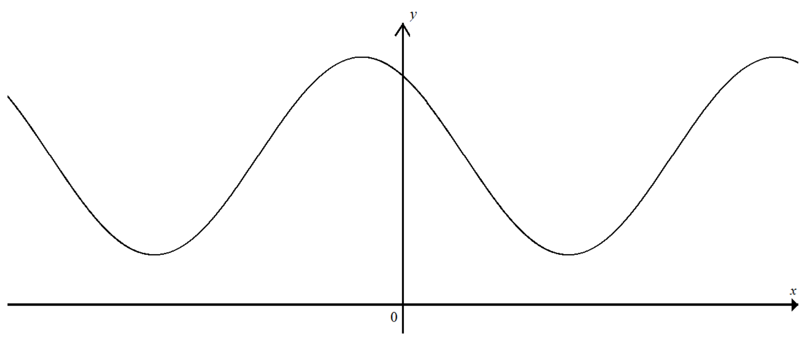
B. $(1, 3)$ only

C. $(3, 4)$ only

D. $(1, 2)$ or $(3, 4)$

E. $(1, 2)$, $(1, 3)$ or $(3, 4)$

Question 9 **A section** of the graph of $y = 3 + 2\cos\left(\frac{x}{2}+\frac{\pi}{5}\right)$, $x \in R$ is shown below.



The average value of $y = 3 + 2\cos\left(\frac{x}{2}+\frac{\pi}{5}\right)$ for $x \in R$ is closest to

A. 5.0

B. 4.2

C. 4.0

D. 3.7

E. 3.0

Question 10

The average rate of change of $x = \log_{10}\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)$ in the interval $100 \le t \le 10000$ is closest to

A. 0.00071

B. 0.00023

C. 0.00017

D. 0.00016

E. 0.00010

Question 11 The two lines $ax + by + c = 0$ and $a^2x + b^2y + c^2 = 0$ are perpendicular when

A. $a = \pm b$

B. $a^2 + b^2 = 1$

C. $a = b$

D. $a^3 - b^3 = 0$

E. $a^3 + b^3 = 0$

Question 12 If $\int_0^a f(x)dx = b$ where $a, b \in R^+$ and $f(x) > 0$ for $x \in [0, a]$, then $\int_0^{2a}\left(2f\left(\frac{x}{2}\right) + 1\right)dx =$

A. $2(a + 2b)$

B. $2(a + b)$

C. $2a + b$

D. $a + 2b$

E. $a + 4b$

Question 13 The graph of $y = f(x)$ undergoes the following sequence of transformations.
Firstly the graph is translated in the positive x-direction by b units, then the resulting graph is reflected in the y-axis, and lastly dilated from the y-axis by factor a.

The equation of the graph after the sequence of transformations is

A. $y = a\,f(b - x)$

B. $y = f(b - ax)$

C. $y = f(-a(x - b))$

D. $y = f\left(-\left(\dfrac{x}{a} + b\right)\right)$

E. $y = f\left(-\left(\dfrac{x}{a} - b\right)\right)$

Question 14 If $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} c \\ d \end{bmatrix}$ and $T^{-1}T = I$, then

A. $T^{-1}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} b & 0 \\ 0 & a \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right)$

B. $T^{-1}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} b^{-1} & 0 \\ 0 & a^{-1} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$

C. $T^{-1}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a^{-1} & 0 \\ 0 & b^{-1} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$

D. $T^{-1}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \dfrac{1}{ab}\begin{bmatrix} b & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$

E. $T^{-1}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a^{-1} & 0 \\ 0 & b^{-1} \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right)$

Question 15 A probability experiment involves the tossing of a coin (head H on one side and tail T on the other side) and rolling a small cube (a different number from 1 to 6 is marked on each of the six faces).
The result of an attempt of the experiment is recorded. A possible random variable is

A. 6 appears on the uppermost face of the cube

B. a number greater than 3 appears on the uppermost face of the cube

C. tail on the uppermost face of the coin

D. the difference between the number of tails and the number of numbers greater than 3

E. the number of tails and the number of numbers greater than 3

Question 16 The probability distribution of random variable X is given by the table below.

X	a	$a+1$	$a+2$	$a+3$
$\Pr(X=x)$	0.50	a^2	0.43	$0.6a$

The value of $\overline{X}$ is closest to

A. 0.95

B. 1.05

C. 1.15

D. 3.20

E. 3.21

Question 17 The probability density function of random variable X is given by

$$f(x)=\begin{cases} e^{x+a}, & 0 \le x \le a \\ 0, & \text{elsewhere} \end{cases}$$

The value of a is closest to

A. 5

B. 3

C. 1

D. 0.5

E. 0.2

Question 18 Given $\Pr(A)=0.6$, $\Pr(B)=0.3$ and $\Pr\left((A\cup B)'\right)=0.2$, $\Pr(A\mid B)$ is closest to

A. 0.3

B. 0.5

C. 0.59

D. 0.7

E. 0.82

Question 19 Random variable X has a binomial probability distribution.

Given $\Pr(X=5)=126\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^5 \approx 0.25$, the mean and standard deviation of X (correct to 2 decimal places) are respectively

A. 4.50 and 1.47

B. 5.40 and 1.47

C. 3.60 and 1.26

D. 6.30 and 2.16

E. 2.25 and 0.65

Question 20 52.5% of a country's voting population favors a particular political party. Random samples of size 500 voters are to be selected and asked about their voting intentions. If 10 such random samples were taken, the number of samples with sample proportion favoring the particular political party greater than 0.60 is closest to

A. 4

B. 3

C. 2

D. 1

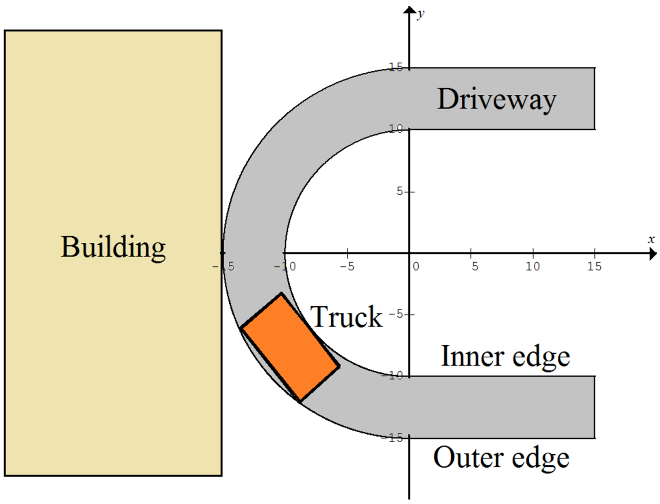
E. 0

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise stated.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this examination are **not** drawn to scale.

Question 1



The diagram above shows a U-shape driveway into a private property. The driveway consists of a semicircular section and two straight sections.

a. Write down a single relation representing the outer edge of the two straight sections. Include domain in your answer.

2 marks

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b. Write down a single relation representing the outer edge of the semicircular section. Include domain in your answer.

2 marks

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c. Write down a single transformation of part a necessary to give a relation representing the inner edge of the two straight sections. Include all required information to specify the transformation completely.

2 marks

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d. Write down the transformations of part b necessary to give a relation representing the inner edge of the semicircular section. Include all required information to specify the transformations completely.

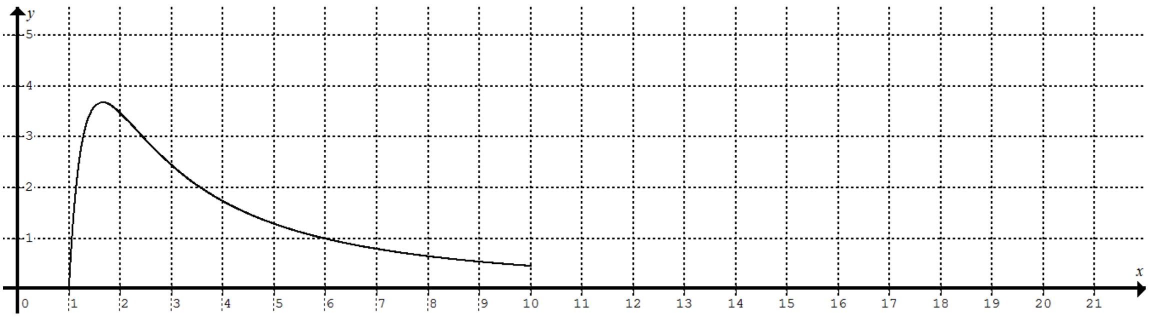
2 marks

A rectangular truck is on the semicircular section as shown on the diagram above. All parts of the truck are at or within the edges of the driveway.
Let a metres and b metres be the length and width of the truck respectively. The area of the truck is $A = ab$ square metres.

e. Express a^2 in terms of b. Hint: Draw the truck on the driveway parallel to the building. 3 marks

f. Find exact values of a and b such that area A m^2 is a maximum value. 4 marks

Question 2 A skate ramp consists of 2 curved slopes and a horizontal platform.
The following graph shows the profile of the first slope. The ground level is at $y = 0$.
Lengths (distances) are measured in metres.
The equation of the profile is $y = \dfrac{20\log_e(x)}{x^2}$ for $1 \le x \le 10$.



a. Apply the quotient rule to show $\dfrac{dy}{dx} = \dfrac{20}{x^3}(1 - 2\log_e(x))$. 2 marks

b. Hence determine the coordinates of the highest point of the profile.
Write your answer in exact form and then correct to 4 decimal places. 2 marks

c. Use CAS to find the y-coordinate of the endpoint and the gradient of the profile at the endpoint.
Correct your answer to 4 decimal places. 2 marks

The equation of the profile of the second slope is in the form $y = a(x-11)^2 + b$ for $10 \le x \le c$, where a, b and $c \in R$. The first and second profiles join smoothly at $x = 10$, i.e. they have the same gradient at $x = 10$.

d. Use the given information to set up two simultaneous equations in finding a and b. Find the values of a and b, correct to 4 decimal places. 3 marks

e. The endpoint of the second profile reaches the same height as the first profile. Determine the value of c in the interval $10 \le x \le c$, correct to 4 decimal places. 1 mark

f. The horizontal platform is joined to the endpoint in part e, and $c \le x \le 21.5$. Write down the equation of the profile of the platform. 1 marks

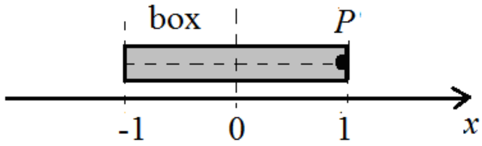
g. Sketch neatly the profiles of the second slope and the platform on the same graph shown above.

2 marks

The area above the ground and below the three profiles can be used for advertisement.

h. Find the total area (in m^2) under the skate ramp available for advertisement. 2 marks

Question 3 Particle P is made to oscillate periodically inside a box (2 metres long) from one end to the other end and back. The motion of the particle inside the box is independent of the motion of the box. The displacement of P at time t is given by $x_P = \cos(2\pi t)$.



The box is also made to oscillate horizontally and periodically with an amplitude of 1 metre. The displacement of the box at time t is given by $x_B = \sin(2\pi t)$.

The above diagram shows the box and particle P at $t = 0$ s when both oscillations begin.

The total displacement (in metres) of particle P from origin 0 at time t is given by $x = x_B + x_P$.

a. Find the total displacement of P from origin 0 at $t = 0.15$ s, correct to 2 decimal places. 1 mark

b. Find the exact maximum and minimum total displacements of P from origin 0. 2 marks

c. Find the exact value of the time interval between a maximum total displacement and the following minimum total displacements of P from origin 0.

1 mark

d. Find the exact time when particle P first moves in the negative x-direction at its maximum speed.

2 marks

e. Determine the exact maximum speed of the particle. 1 mark

f. Calculate the exact average speed of particle P when it moves in the negative x-direction.
2 marks

g. The total displacement of particle P from origin 0 can be represented by $x = \alpha \sin(2\pi t - \beta)$.
Use CAS to find the values of α and β, correct to 2 decimal places.
2 marks

h. Find the total displacement of particle P from origin 0 at time t in terms of the cosine function only.
Correct all numerical parameters to 2 decimal places. 2 marks

i. Name the two transformations together with numerical specifications, which transform $x = \sin(2\pi t)$ to the answer in part d.
2 marks

Question 4 Trees are cut down when a large parcel of land is cleared for housing development. The tree trunks and large branches are cut into logs to be sold as firewood. The length of the logs has a normal distribution with a mean of 32.0 cm and a standard deviation of 1.0 cm.

a. The logs are classified as **very short** (shorter than 28.0 cm), **short** (between 28.0 and 30.0 cm), **medium** (between 30.0 and 32.0 cm), **long** (between 32.0 and 34.0 cm) and **very long** (longer than 34.0 cm). Complete the following probability table, correct to **5** decimal places.

Length	Very short	Short	Medium	Long	Very long
Probability					

2 marks

The firewood is bundled into cubic metre lots.
The selling prices for a cubic metre of firewood are shown below:
Very short \$49; short \$50; medium \$55; long \$51; very long \$45.
The volume of firewood collected is 600 m^3 (i.e. 600 bundles).
The selling price *C* for a cubic metre of firewood is a discrete random variable.

b. Explain why *C* is a random variable and discrete. 2 marks

c. Display the probability distribution of *C* in the form of a table with the values of *C* in ascending order.

2 marks

C (\$)					
Probability					

d. Find the total amount (\$) received after all 600 m^3 of firewood are sold. 2 marks

e. How many m^3 bundles of medium length firewood are there? Correct your answer to the nearest whole number.

1 mark

f. Five m^3 bundles of firewood are selected randomly from the medium or longer length firewood. Approximate the probability that more than three of them are longer than the medium length.

2 marks

The neighboring parcel of land is much larger in area. Trees are cut down and cleared for housing development also. The tree trunks and large branches are cut into logs. The length of the logs has a normal distribution similar (but not the same) to the first parcel of land.
The firewood is bundled into m^3 lots of very short, short, medium, long and very long, just like before.
There are 2000 m^3 (2000 bundles) in total.
Many random samples of 25 m^3 bundles are selected. The mean sample proportion of firewood of medium length classification is 0.48.

g. What is the average number of m^3 bundles of medium length classification in the selected samples?

1 mark

h. Calculate the standard deviation of the sample proportion of firewood of medium length classification.

1 mark

i. Estimate the 95% confidence interval for the proportion of firewood of medium length classification in the 2000 m^3 bundles.

2 marks

End of exam 2