2018 Mathematical Methods Trial Exam 2 Solutions

SECTION A – Multiple-choice questions

1	2	3	4	5	6	7	8	9	10
B	C	A	A	C	C	D	D	E	E

11	12	13	14	15	16	17	18	19	20
E	A	D	E	D	C	D	A	B	E

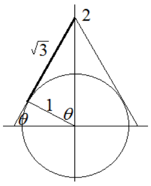
Q1 $x(x+1)-(x+1)=0$, $(x+1)(x-1)=0$, $x=\pm 1$ B

Q2 $\log_e 0$ is undefined. C

Q3 The domain of the inverse is the range of the function. A

Q4 $x=e^{\log_e\left(\frac{y}{2}\right)}$, $x=\frac{y}{2}$, $y=2x$ A

Q5 $m=\tan\theta=\sqrt{3}$ C



Q6 $f(x)-f(-x)=0$, $f(x)$ is even. $\tan\left(x+\frac{\pi}{2}\right)$ is not even. C

Q7 $f'(x)=2x\left(2x^2+b-a\right)=0$, one solution if $a=b$, three solutions if $a>b$. D

Q8 When $y=2$, $x=1$; when $y=4$, $x=3$. D

Q9 The function oscillates about 3. E

Q10 Average rate

$$=\frac{1}{10000-100}\left(\log_{10}\left(\sqrt{10000}+\frac{1}{\sqrt{10000}}\right)-\log_{10}\left(\sqrt{100}+\frac{1}{\sqrt{100}}\right)\right)$$

$$=\frac{1}{9900}\left(\log_{10}\frac{100.01}{10.1}\right)\approx 0.0001$$ E

Q11 $m_1m_2=-1$, $\frac{a}{b}\times\frac{a^2}{b^2}=-1$, $\frac{a^3}{b^3}=-1$, $a^3=-b^3$, $a^3+b^3=0$ E

Q12 $\int_0^{2a}\left(2f\left(\frac{x}{2}\right)+1\right)dx=2\int_0^{2a}f\left(\frac{x}{2}\right)dx+\int_0^{2a}1\,dx=2(2b)+2a$ A

Q13 $y=f(x)\rightarrow y=f(x-b)\rightarrow y=f(-x-b)\rightarrow y=f\left(-\frac{x}{a}-b\right)$

$y=f\left(-\left(\frac{x}{a}+b\right)\right)$ D

Q14 E

Q15 The difference between the two numbers is a possible random variable with values of 0 or 1. D

Q16 $\sum \Pr=1$, $a^2+0.6a-0.07=0$, $(a-0.1)(a+0.7)=0$, $a=0.1$

$\overline{X}=0.50a+a^2(a+1)+0.43(a+2)+0.6a(a+3)=1.15$ C

Q17 $\int_0^a e^{x+a}\,dx=1$, $\left[e^{x+a}\right]_0^a=1$, $e^{2a}-e^a-1=0$, $a\approx 0.4812$ D

Q18 $\Pr\left((A\cup B)'\right)=1-\Pr(A\cup B)$, $\Pr(A\cup B)=1-0.2=0.8$

$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$,

$\Pr(A\cap B)=0.6+0.3-0.8=0.1$, $\Pr(A\mid B)=\frac{0.1}{0.3}\approx 0.33$ A

Q19 $n=9$, $p=0.6$ and $q=0.4$

$\therefore$ $\mu=np=5.4$ and $\sigma=\sqrt{npq}\approx 1.47$ B

Q20 The mean of $\hat{P}=p=0.525$, $n=500$

The standard deviation of $\hat{P}=\sqrt{\frac{p(1-p)}{n}}\approx 0.022333$

$\Pr\left(\hat{P}>0.60\right)\approx 0.00039$ (normal approx)

Number of samples $\approx 10\times 0.00039\approx 0$ E

SECTION B

Q1a $y^2=225$, $0\le x\le 15$

Q1b $x^2+y^2=225$, $-15\le x\le 0$

Q1c Dilate parallel to the *y*-axis by a factor of $\frac{2}{3}$.

$\left(\frac{y}{\frac{2}{3}}\right)^2=225$, $y^2=100$

Q1d Dilate parallel to both axes by a factor of $\frac{2}{3}$.

$\left(\frac{x}{\frac{2}{3}}\right)^2+\left(\frac{y}{\frac{2}{3}}\right)^2=225$, $x^2+y^2=100$

Q1e $x=-(b+10)$, $y=\frac{a}{2}$, $(b+10)^2+\left(\frac{a}{2}\right)^2=225$,

$a^2=500-80b-4b^2$

Q1f $A=b\sqrt{500-80b-4b^2}$. Let $\frac{dA}{db}=0$, $2b^2+30b-125=0$

$b=\frac{-30+\sqrt{1900}}{4}=\frac{-15+5\sqrt{19}}{2}$

$a=\sqrt{500-80b-4b^2}=\sqrt{500-80b-(-60b+250)}$

$=5\sqrt{16-2\sqrt{19}}$

Q2a $\frac{dy}{dx}=\frac{(x^2)\left(\frac{20}{x}\right)-(20\log_e x)(2x)}{x^4}=\frac{20}{x^3}(1-2\log_e x)$

Q2b $1-2\log_e x=0$, $x=e^{\frac{1}{2}}=\sqrt{e}$, $y=\frac{20\log_e x}{x^2}=\frac{10}{e}$

$\left(\sqrt{e},\frac{10}{e}\right)$ or approx. $(1.6487, 3.6788)$

Q2c $x=10$, $y\approx 0.4605$, $\frac{dy}{dx}\approx -0.0721$

Q2d $y=a(x-11)^2+b$, $\frac{dy}{dx}=2a(x-11)$

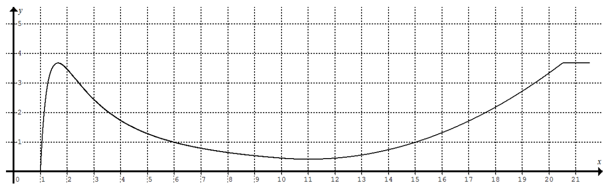
$2a(10-11)\approx -0.0721$ and $a(10-11)^2+b\approx 0.4605$

$a\approx 0.0361$ and $b\approx 0.4245$

Q2e $0.0361(c-11)^2+0.4245\approx 3.6788$, $c\approx 20.4946$

Q2f $y\approx 3.6788$, $20.4946\le x\le 21.5$

Q2g



Q2h Total area $=13.3927+14.7666+3.6987\approx 32$ m^2

Q3a $x=\sin(2\pi t)+\cos(2\pi t)=\sin(0.30\pi)+\cos(0.30\pi)\approx 1.40$

Q3b Let $\frac{dx}{dt}=0$. $2\pi\cos(2\pi t)-2\pi\sin(2\pi t)=0$, $\tan(2\pi t)=1$,

$2\pi t=\frac{\pi}{4}$, $t=\frac{1}{8}$, $\frac{5}{8}$, max $x=\sqrt{2}$, min $x=-\sqrt{2}$

Q3c $\Delta t=\frac{5}{8}-\frac{1}{8}=\frac{1}{2}$ s

Q3d $x=\sin(2\pi t)+\cos(2\pi t)$, $v=\frac{dx}{dt}=2\pi(\cos(2\pi t)-\sin(2\pi t))$

Let $\frac{dv}{dt}=0$, $-(2\pi)^2(\sin(2\pi t)+\cos(2\pi t))=0$, $\tan(2\pi t)=-1$,

$2\pi t=\frac{3\pi}{4}$, $t=\frac{3}{8}$ s

Q3e $v=2\pi\left(\cos\left(\frac{3\pi}{4}\right)-\sin\left(\frac{3\pi}{4}\right)\right)=-2\pi\sqrt{2}$, max speed $=2\sqrt{2}\pi$ m s^{-1}

Q3f Average speed $=\frac{\text{distance travelled}}{\text{time taken}}=\frac{2\sqrt{2}}{\frac{1}{2}}=4\sqrt{2}$ m s^{-1}

Q3g Use CAS to sketch $x=\sin(2\pi t)+\cos(2\pi t)$. It is a sine graph with a period of 1 and amplitude of 1.41 approximately.

$x\approx 1.41\sin(2\pi t-\beta)$, $1.41\approx 1.41\sin\left(\frac{2\pi}{8}-\beta\right)$, $\frac{2\pi}{8}-\beta=\frac{\pi}{2}$,

$\beta=-\frac{\pi}{4}\approx -0.79$ and $\alpha\approx 1.41$

Q3h $x\approx 1.41\sin(2\pi t+0.79)=1.41\cos\left(\frac{\pi}{2}-(2\pi t+0.79)\right)$

$\approx 1.41\cos(0.78-2\pi t)$ or $1.41\cos(6.28t-0.78)$

Q3i $x\approx 1.41\sin(2\pi t+0.79)=1.41\sin(2\pi(t+0.13))$

Dilate $x=\sin(2\pi t)$ parallel to the vertical axis by a factor of 1.41, and translate parallel to the horizontal axis by 0.13 (accept 0.12) to the left.

Q4a

Length	Very short	Short	Medium	Long	Very long
Probability	0.00003	0.02272	0.47725	0.47725	0.02275

Q4b Picking a bundle of firewood for sale is a random process, hence the price C is a random variable.
Note: It is incorrect to say the buyer's choice of firewood is random and hence C is a random variable.
C is discrete because the prices are single values and not an interval (or intervals) of values.

Q4c

C (\$)	45	49	50	51	55
Probability	0.02275	0.00003	0.02272	0.47725	0.47725

Q4d Total amount (\$)
$=600(45\times 0.02275+49\times 0.00003+50\times 0.02272+51\times 0.47725+55\times 0.47725)$
≈ 31650

Q4e $600\times 0.47725\approx 286$

Q4f Binomial: $n=5$, $p=\frac{0.47725+0.02275}{0.47725+0.47725+0.02275}\approx 0.51164$

$\Pr(X>3)=\Pr(X=4)+\Pr(X=5)\approx 0.16733+0.03506\approx 0.2$

Q4g $0.48\times 25=12$

Q4h The standard deviation of sample proportion

$\approx\sqrt{\frac{0.48(1-0.48)}{25}}\approx 0.10$

Q4i An approximate 95% confidence interval
$\approx(0.48-1.96\times 0.10, 0.48+1.96\times 0.10)=(0.284, 0.676)$

Please inform mathline@itute.com re conceptual and/or mathematical errors